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Probing Reynolds stress models of convection with numerical simulations: II. Non-locality and third order moments.

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Abstract. We provide results from an extended numerical simulation study of Reynolds stress models of stellar convection and probe the modelling of third order moments and non-locality.

Keywords. Convection, turbulence, stars: interiors

1. Introduction and motivation

In Kupka & Muthsam (2007a) we introduce our study of Reynolds stress models for turbulent convection. Here, we investigate the modelling of non-locality in these models, which requires the computation of third order moments (TOMs) of vertical velocity and temperature fields. We compare independent solutions of Reynolds stress models with averages obtained from 3D numerical simulations. We also perform consistency tests in which second order moments and mean quantities are taken from the simulations and put into the approximations used in the Reynolds stress models to evaluate the TOMs. The simulations have been performed with the ASCIC code (Muthsam *et al.* 1995, 1999) as in the case of Kupka & Muthsam (2007a), where we provide some details on the different configurations we have studied, the assumed microphysics, numerical resolution, and boundary conditions. Physical quantities such as the flux of kinetic energy in units of the total (bottom input) flux or the TOMs $\langle w^3 \rangle$ and $\langle w\theta^2 \rangle$ are key ingredients for a successful non-local Reynolds stress model of convection (w and θ denote the fluctuations of vertical velocity and temperature relative to their horizontal, ensemble averaged mean).

2. Third order moment models

To compute independent solutions of Reynolds stress models we use a modified version of the code of Kupka (1999). We have also used this code to evaluate the equations for the TOMs on their own, if second order moments and mean structure quantities (temperature, pressure, etc.) are taken from the ensemble averaged simulation runs. Since the simulations are relaxed before computing ensemble averages, such a procedure provides a test whether an approximation is consistent with the numerical simulation. No time integration for relaxation to a stationary state of the complete Reynolds stress model is performed in this case.

The TOM model equations considered are first (a variant of) the down-gradient approximation (DGA), also described in Canuto & Dubovikov (1998) (CD98) and Xiong (1978, 1986). Contrary to CD98 slightly simplified coefficients are used for the turbulent conductivity and turbulent visosity (different variants of this model were found to provide little overall improvement). The second model is from Canuto *et al.* (2001) (C2001). It differs from the original proposal of Canuto (1993) by a damping factor $1/(1+0.04N^2\tau^2)$ applied in all occurrences of the time scale τ in the TOM equations in case of unstable stratification, as a better model of eddy damping (N denotes the Brunt-Vaisälä frequency). The third model ('full TOM') is just a variant of this approach with a damping factor $1/(1+0.0008N^4\tau^4)$ as used in Kupka & Montgomery (2002), because of its more smooth behaviour near the convective boundary (its coefficient was determined using the simulation '3J' shown here). Finally, we show the result of using the Gryanik & Hartmann (2002) approximation (GH2002; see Kupka & Robinson 2007) for $\langle w\theta^2\rangle$ in consistency tests (see middle row of Fig. 1). In a few cases, we changed the models for the pressure flux $\langle p'w\rangle$ (p' denotes pressure fluctuations, see also Kupka & Muthsam 2007b).

3. Discussion of results and conclusions

Already from the few tests shown in Fig. 1 it is evident that pure down-gradient type models for TOMs fail in the interior of convection zones, even though they may work in the transition regions between stable and unstable stratification (upper part of '3J'). This is in contrast with the simplicity and numerical robustness that diffusion type models would offer in stellar structure calculations. For shallow convection zones, the full dynamical equations for TOMs closed by the eddy-damped quasi-normal approximation (Canuto 1992, 1993; Canuto & Dubovikov 1998) and supplemented by more refined prescriptions for damping (Canuto *et al.* 2001) appear more satisfactory. In the modified version by Kupka & Montgomery (2002) and Montgomery & Kupka (2004), who computed realistic models of envelopes of A-stars and hot DA/DB white dwarfs (and also neglected the pressure flux $\langle p'w\rangle$), they provide a good estimate for $\langle w^3\rangle$ for the shallow zone ('3J'), apart from small 'pathological' features (kinks). However, the models tested here become inappropriate for the deep, efficient convection zone ('155X'). Tuning of parameters, if attempted, would be useless, as there are evidently problems in the overall functional form of the TOMs. For $\langle w\theta^2\rangle$ neither the down-gradient, nor the eddy-damped models are satisfactory in either case. For A-stars and hot DA/DB white dwarfs this deficiency was acceptable, because temperature fluctuations were less important (see also the discussion in Kupka & Muthsam 2007a). Note that the GH2002 model expression, as in Kupka & Robinson 2007, yields remarkable results in consistency tests for both configurations (for applications in Reynolds stress models, see, however, Kupka 2007). We would also like to point out here that the prediction of local convection models for any of the quantities considered here is simply '0', something which is evidently incorrect. Whether even a very coarse approximation is more desirable than complete neglection is a separate question which we expect to have case dependent answers. For A-stars the answer has clearly been positive (KM2002). But we consider our results a strong indication that more advanced models would be needed for the Sun.

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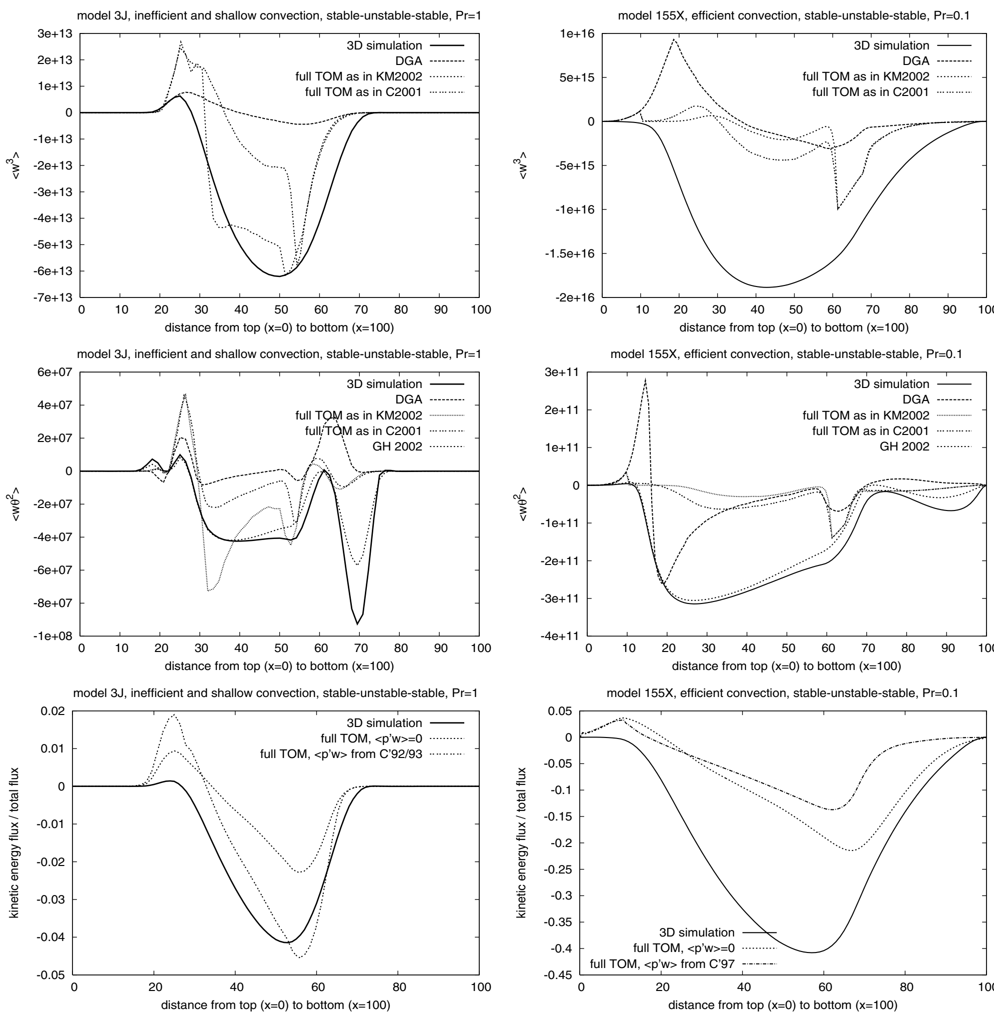

Figure 1. For cases '3J' (left column panels) and '155X' (right column panels) consistency tests are shown for the TOMs $\langle w^3 \rangle$ (top row) and $\langle w\theta^2 \rangle$ (middle row). Input data for TOM models are taken from simulations and compared against direct computation. The bottom row compares the relative kinetic energy flux from simulations to results from closed Reynolds stress models (with input data for TOM models obtained as part of the complete solution).

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