

not just to convenient regions of them. Some knowledge of topology is needed for an understanding of this chapter, and Dr Willmore makes it clear that he has not attempted to give a complete account. Nevertheless this chapter is both interesting and important. The material in it should, if possible, be included in any course on differential geometry for Honours undergraduates. Indeed, Part I of the book forms an excellent basis for a course which should be undertaken by all candidates for an Honours Mathematics degree.

The second part of the book deals with differential geometry in  $n$  dimensions. A chapter on tensor algebra precedes the introduction of tensor calculus as applied to differential manifolds. Some may find the definition of tensor a little remote for practical purposes. But at least there *is* a definition of tensor, which is more than can be said for those text-books which purport to define tensors but only define tensor components. The invariance of a tensor is the most important thing about it, and Dr Willmore is quite right to stress this from the beginning. A chapter on Riemannian geometry follows, and this includes mention of some fairly recent results on topics such as harmonic Riemannian spaces. There is also a readable account of Élie Cartan's methods of investigating problems in Riemannian geometry, and this should prove most valuable to post-graduate students. Finally, there is an account of tensor methods applied to surface theory, and the greater power of these methods as compared with the vector methods of the earlier part of the book is illustrated.

This is an important, interesting and well-written book. The style is clear and unpretentious and the printing conforms to the high standard set by the Oxford University Press.

E. M. PATTERSON

BIRKHOFF, G. D., AND BEATLEY, R., *Basic Geometry* (Chelsea Publishing Co., New York, 1958), 294 pp., \$3.95.

This textbook has been published, following classroom experience with an experimental edition, by two professors from Harvard University. It is designed to cover a year's work in an American high school, and seems to be aimed at an age group of about 16. The emphasis is on logical deduction rather than on a list of geometrical theorems, it being assumed that this facility will be carried over to arguments dealing with "real life".

The geometry is developed from five assumptions as follows: (1) points on any straight line can be numbered so that number differences measure distances, (2) there is one and only one straight line through two points, (3) all half-lines having the same end-point can be numbered to measure angles, (4) all straight angles have the same measure, (5) the (S.A.S.) case of similar triangles.

The number system is defined to contain all real numbers, rational or irrational, and from this basis seven theorems are derived. The geometry is then developed rapidly in the following order: (a) parallel lines and networks, (b) the circle and regular polygons, (c) ruler-compass constructions, (d) area and length, (e) continuous variation, (f) loci; the whole being rounded off by brief references to power, coaxial circles, inversion and projection.

There are numerous exercises, many being standard theorems in British textbooks. All are expected to be done, though the aid of the teacher would obviously be required in many of them. Although this book is not suitable as a textbook in this country, it has great interest in showing a development quite foreign to ideas current in our schools, and could prove very useful in the training colleges.

G. ALLMAN

KEMENY, J. G., AND SNELL, J. L., *Finite Markov Chains* (D. van Nostrand Co. Ltd., London, 1960), 210 pp., 37s. 6d.

If this book is regarded from the point of view of the undergraduate at whom

primarily it seems to be aimed, then the opening chapter, on mathematical prerequisites, is rather frightening. This is probably due partly to its condensed nature and partly to an unfamiliar philosophical ring in the language and notation used—thus the authors speak of “the truth set of a statement” rather than “the set representing an event”. It would be unfortunate if this acted as a deterrent to the reader because the book continues with a very readable account of the theory of finite Markov chains.

The treatment is very systematic and problems are well isolated so that they can be discussed without extraneous difficulties. In this respect the device of making ergodic states absorbing is used with great effect. Moreover the mathematics introduced is kept at as elementary a level as possible. One would expect that use of a fairly wide range of matrix theory would be required in this subject, because a finite Markov chain is completely defined by an initial probability vector and a transition matrix. But the authors succeed in carrying the theory through on elementary matrix operations, without using any canonical reduction theory. And fairly simple matrix expressions are arrived at for the quantities of practical interest such as means and variances of passage times.

Early in the text a long list of examples of simple Markov chains is given and these are used throughout for illustrative purposes. There are exercises for the reader at the end of each chapter, and in the final chapter applications in a wide variety of subjects are considered.

This would be a very useful text-book for an undergraduate course on finite Markov chains. It is self-contained and as elementary as is consistent with mathematical precision. As a reference book it suffers from the disadvantage of having no index, and a non-mathematical reader would probably have considerable difficulty in following the book in its entirety.

S. D. SILVEY

KAC, MARK, *Statistical Independence in Probability, Analysis and Number Theory* (*Carus Mathematical Monographs No. 12*) (Math. Association of America, and John Wiley & Sons, 1959), xiv+93 pp., \$3.

It used to be fashionable to say that “probability theory is just measure theory plus the concept of statistical independence”. In a sense this is true; we can equate probabilities with measures, random variables with measurable functions, and expectations with integrals, and after defining the new concept of statistical independence we are then in a position to state and prove the strong law of large numbers, which relates probabilities to frequency ratios and so puts measure theory at the service of the statistician. One of the aims of this book is to show that “statistical” independence is not after all a new notion, but one already active below the surface in many branches of pure mathematics; thus the book opens with the remark that Vieta’s formula

$$\frac{\sin x}{x} = \prod_{k=1}^{\infty} \cos \frac{x}{2^k}$$

can lead one straight to the statistical independence of the Rademacher functions and thence to the now standard mathematical model for coin tossing. The author then discusses “normal” numbers (e.g. Champernowne’s number 0.1234567891011...), the convergence of  $\sum \pm c_k$ , Weyl’s equidistribution theorem, and results in number theory due to Davenport, Erdős, Halberstam, Kac, Renyi, Schoenberg and Turan. A final section links ergodic theory with continued fractions (Khinchine’s theorem is proved by Ryll-Nardzewski’s method, using the individual ergodic theorem of G. D. Birkhoff). This final section helps to dissipate any feeling the reader might otherwise have acquired, that probabilists are *only* interested in independence situations. In