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DEFORMATION OF THE UNIVERSAL ENVELOPING ALGEBRA OF $\Gamma(\sigma_1, \sigma_2, \sigma_3)$

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ABSTRACT. The defining relations for the Lie superalgebra $\Gamma(\sigma_1, \sigma_2, \sigma_3)$ as a contragredient algebra are discussed and a PBW type basis theorem is proved for the corresponding q-deformation.

1. **Introduction.** In this note, we study the q-analog of the universal enveloping algebra of the Lie superalgebra $G = \Gamma(\sigma_1, \sigma_2, \sigma_3)$. This Lie superalgebra is special: as a contragredient algebra, the defining matrix of G over the complex number field $\mathbb{C}$ depends on a parameter, the algebra itself already admits a one-parameter deformation. To apply the idea of Drinfeld and Jimbo to define the q-analog of the universal enveloping algebra $U(G)$, one needs to work with a non-integer defining matrix. Hence in general, the deformation is defined over some transcendental function field extension of $\mathbb{C}$ (or just the field $\mathbb{C}$, if one takes the deformation parameter to be a suitable complex number). The deformation thus defined will actually be a two-parameter family of algebras.

We discuss the defining relations for G as a contragredient algebra in Section 2. Although these defining relations are known to the experts (*cf.* the discussion in [8]), we are unable to find a suitable reference, so we provide a complete proof for these relations.

In Section 3, we define the deformation $\mathcal{U}$ of $U(G)$ and study its structure. As in the other cases of type II classical contragredient Lie superalgebras (see [4] for the definition of type II Lie superalgebras, see [5] for a definition of the q-deformation of $U\big(\mathrm{osp}(m, 2n)\big)$, the usual Drinfeld-Jimbo deformation of $U(G)$ does not contain a copy of the standard deformation of $U(G_0)$, where G_0 is the even part of G, since there are not enough group like elements in it. However, we show that in our case, one can introduce suitable elements in $\mathcal{U}$ such that a PBW type theorem (Theorem 3.3) holds for $\mathcal{U}$.

2. **The defining relations for G.** We use the notation adopted in [9]. Recall that the algebra G is defined as a contragredient Lie superalgebra with three nonzero elements $\sigma_1, \sigma_2, \sigma_3 \in \mathbb{C}$ satisfying $\sigma_1 + \sigma_2 + \sigma_3 = 0$, with generators e_i, f_i, h_i ($i = 1, 2, 3$) and the defining matrix $(a_{ij})_{3\times 3}$ given as follows:

$$\begin{pmatrix} 0 & 2\sigma_2 & 2\sigma_3 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}.$$

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The grading on G is given by

$$\deg h_i = 0, i = 1,2,3; \quad \deg e_i = \deg f_i = 0, i = 2,3; \quad \deg e_1 = \deg f_1 = 1.$$

PROPOSITION 2.1. *The defining relations for G as a contragredient Lie superalgebra are*

(1) $[h_i, h_j] = 0, \quad i,j = 1,2,3;$
(2) $[h_i, e_j] = a_{ij}e_j, \quad [h_i, f_j] = -a_{ij}f_j, \quad i,j = 1,2,3;$
(3) $[e_i, f_j] = \delta_{ij}h_i, \quad i,j = 1,2,3;$
(4) $(\operatorname{ad} e_i)^{1-a_{ij}}(e_j) = 0, \quad (\operatorname{ad} f_i)^{1-a_{ij}}(f_j) = 0, \quad i = 2,3, \quad j = 1,2,3;$
(5) $[e_1, e_1] = 0, \quad [f_1, f_1] = 0.$

Relations (1)–(5) clearly hold in $\Gamma(\sigma_1, \sigma_2, \sigma_3)$, so we assume that G is defined as a contragredient Lie superalgebra by using the given generators and these relations and show that G is isomorphic to $\Gamma(\sigma_1, \sigma_2, \sigma_3)$. The proof will be organized in several lemmas.

Note that by the Jacobi identity, we have

$$\big[e_1, [e_1, e_i]\big] = 0, \quad \big[f_1, [f_1, f_i]\big] = 0, \quad i = 2,3.$$

Let

$$\begin{aligned} e_0 &= (2\sigma_1)^{-1}\Big[e_1, \big[e_3, [e_2, e_1]\big]\Big] \\ &= (2\sigma_1)^{-1}\big[[e_1, e_3], [e_2, e_1]\big], \\ f_0 &= (2\sigma_1)^{-1}\Big[f_1, \big[f_3, [f_2, f_1]\big]\Big] \\ &= (2\sigma_1)^{-1}\big[[f_1, f_3], [f_2, f_1]\big], \\ h_0 &= [e_0, f_0]. \end{aligned}$$

LEMMA 2.2. *The subalgebra $\langle e_0, f_0, h_0\rangle$ of G generated by e_0, f_0, h_0 is isomorphic to* sl(2)*, and $\langle e_i, f_i, h_i; i = 0,2,3\rangle \cong$* sl(2) $\oplus$ sl(2) $\oplus$ sl(2).

PROOF. A straightforward computation shows that

$$h_0 = (2\sigma_1)^{-1}(2\sigma_2 h_2 + 2\sigma_3 h_3 - 2h_1).$$

Hence $[h_0, e_0] = 2e_0$, $[h_0, f_0] = -2f_0$, and $\langle e_0, f_0, h_0\rangle \cong$ sl(2). For the second statement, first we note that by the definitions of e_0 and f_0, we have

$$[e_0, f_i] = 0, \quad [f_0, e_i] = 0, \quad i = 2,3.$$

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Then we note that

$$\begin{aligned}[e_2,e_0] &= (2\sigma_1)^{-1}\Big[e_2,\big[[e_1,e_3],[e_2,e_1]\big]\Big]\\ &= (2\sigma_1)^{-1}\Big[\big[e_2,[e_1,e_3]\big],[e_2,e_1]\Big]\\ &= -(2\sigma_1)^{-1}\Big[\big[e_3,[e_2,e_1]\big],[e_2,e_1]\Big]\\ &= -(4\sigma_1)^{-1}\Big[e_3,\big[[e_2,e_1],[e_2,e_1]\big]\Big]\\ &= -(4\sigma_1)^{-1}\Big[e_3,\big[e_2,[e_1,[e_2,e_1]]\big]\Big]\\ &= 0,\end{aligned}$$

and similarly

$$[e_3,e_0]=0,\quad [f_2,f_0]=0,\quad [f_3,f_0]=0.$$

Now the lemma follows from these identities.

Let $G_0 = \langle e_i,f_i,h_i;\ i=0,2,3\rangle$ (Lemma 2.4 below will show that G_0 is indeed the even part of G and thus justify our notation).

LEMMA 2.3. *Let $e_{111}=\big[e_3,[e_2,e_1]\big]$, then as a G_0-module via the adjoint representation, the submodule (e_{111}) generated by e_{111} is isomorphic to $\mathbb{C}^2\otimes\mathbb{C}^2\otimes\mathbb{C}^2$, where $\mathbb{C}^2$ is the two-dimensional natural representation of* sl(2).

PROOF. By the definition of e_{111}, $e_{111}\neq 0$. Note that we have $[e_i,e_{111}]=0$, $i=2,3$. Note also that $[e_0,e_1]=0$, so since $[e_i,e_j]=0$ for $i,j\neq 1$, we see that

$$\begin{aligned}[e_0,e_{111}] &= \Big[e_0,\big[e_3,[e_2,e_1]\big]\Big]\\ &= \Big[e_3,\big[e_2,[e_0,e_1]\big]\Big]\\ &= 0.\end{aligned}$$

Thus e_{111} is a highest weight vector. Now since $[h_i,e_{111}]=e_{111}$ and $(\mathrm{ad} f_i)^2(e_{111})=0$ $(i=0,2,3)$, by the representation theory of the semisimple Lie algebras, the lemma follows as desired.

Define the following elements of G:

$$e_{112}=[f_3,e_{111}],\quad e_{121}=[f_2,e_{111}],$$

$$f_{212}=[f_3,f_1],\quad f_{221}=[f_2,f_1],\quad f_{222}=\big[f_3,[f_2,f_1]\big].$$

Then e_1,f_1, together with the e_{ijk} and the f_{rst} form a basis of the G_0-module (e_{111}).

LEMMA 2.4. *Table I in [9] holds for the elements we defined above, where (ijk) corresponds to the e_{ijk} or the f_{ijk} with $(122)\leftrightarrow e_1$ and $(211)\leftrightarrow f_1$.*

PROOF. We only verify that $e_{111}^2=0$, the other relations can be verified similarly. Since $[e_1,e_1]=0$, we have

$$\big[e_2,[e_1,e_1]\big]=2\big[[e_2,e_1],e_1\big]=0,$$

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and hence by applying ad e_2 and using (4) in Proposition 2.1, we have

$$\big[[e_2,e_1],[e_2,e_1]\big]=0.$$

Therefore

$$x=:\Big[\big[e_3,[e_2,e_1]\big],[e_2,e_1]\Big]=1/2\Big[e_3,\big[[e_2,e_1],[e_2,e_1]\big]\Big]=0,$$

and thus

$$[e_{111},e_{111}]=[e_3,x]=0.$$

PROOF OF PROPOSITION 2.1. The proposition follows from Lemma 2.2-Lemma 2.4 with G_0 being the even part of $\Gamma(\sigma_1,\sigma_2,\sigma_3)$, (e_{111}) being the odd part of $\Gamma(\sigma_1,\sigma_2,\sigma_3)$ (for the structure of $\Gamma(\sigma_1,\sigma_2,\sigma_3)$, see [9, Section 2]).

3. **Deformation of $U(G)$ and a PBW type theorem.** Let q be a variable over $\mathbb{C}$, and let $q_1=q^{-1}$, $q_i=q^{2\sigma_i}$ $(i=2,3)$ (the q_i are well defined complex value functions as long as $q\neq 0$). Let $\mathcal{A}=\mathbb{C}[q^{\pm1};q_i^{\pm1},i=2,3]$, and let $\mathcal{F}$ be the quotient field of $\mathcal{A}$.

We define the algebra $\mathcal{U}$ to be the $\mathbb{Z}_2$-graded associative algebra with 1 over $\mathcal{F}$ generated by the elements E_i, F_i, $K_i^{\pm1}$ $(i=1,2,3)$, with grading given by

$$\deg E_i=\deg F_i=0,\quad i=2,3;\quad \deg K_i^{\pm1}=0,\quad i=1,2,3;$$
$$\deg E_1=\deg F_1=1,$$

and with the following generating relations:

$$K_iK_j=K_jK_i,\quad K_iK_i^{-1}=K_i^{-1}K_i=1,\quad 1\le i,j\le 3, \tag{3.1}$$

$$K_iE_jK_i^{-1}=q_i^{a_{ij}}E_j,\quad K_iF_jK_i^{-1}=q_i^{-a_{ij}}F_j,\quad 1\le i,j\le 3, \tag{3.2}$$

$$\begin{gathered}E_iF_j-(-1)^{ab}F_jE_i=\delta_{ij}\frac{K_i-K_i^{-1}}{q_i-q_i^{-1}},\\ a=\deg E_i,\quad b=\deg F_j,\quad 1\le i,j\le 3.\end{gathered} \tag{3.3}$$

$$E_2E_3=E_3E_2,\quad F_2F_3=F_3F_2, \tag{3.4}$$

$$\begin{gathered}E_i^2E_1-(q_i+q_i^{-1})E_iE_1E_i+E_1E_i^2=0,\quad i=2,3,\\ F_i^2F_1-(q_i+q_i^{-1})F_iF_1F_i+F_1F_i^2=0,\quad i=2,3,\end{gathered} \tag{3.5}$$

$$E_1^2=F_1^2=0. \tag{3.6}$$

The algebra $\mathcal{U}$ is a $\mathbb{Z}_2$-graded Hopf algebra with comultiplication Δ, antipode S and counit ε defined by

$$\Delta E_i = E_i \otimes 1 + K_i \otimes E_i, \quad \Delta F_i = F_i \otimes K_i^{-1} + 1 \otimes F_i, \quad \Delta K_i = K_i \otimes K_i; \tag{3.7}$$

$$SE_i = -K_i^{-1} E_i, \quad SF_i = -F_i K_i, \quad SK_i = K_i^{-1}; \tag{3.8}$$

$$\varepsilon E_i = 0, \quad \varepsilon F_i = 0, \quad \varepsilon K_i = 1. \tag{3.9}$$

There exists a $\mathbb{C}$-algebra anti-automorphism θ of $\mathcal{U}$ given by

$$\theta E_i = F_i, \quad \theta F_i = E_i, \quad \theta K_i = K_i^{-1}, \quad \theta q = q^{-1} \tag{3.10}$$

and $\theta(uv) = \theta(v)\theta(u)$, for all $u, v \in \mathcal{U}$.

The adjoint action of $\mathcal{U}$ on itself is given by

$$\mathrm{ad}_q x(y) = \sum (-1)^{\deg(b_i)\deg(y)} a_i y S(b_i), \tag{3.11}$$

where $\Delta x = \sum a_i \otimes b_i$. Note that by using the adjoint action, relations (3.4) and (3.5) can be replaced by

$$(\mathrm{ad}_q E_i)^{1-a_{ij}} E_j = 0, \quad i = 2, 3, \quad 1 \leq j \leq 3. \tag{3.12}$$

Introduce the following elements of $\mathcal{U}$:

$$\begin{aligned} E_{121} &= \mathrm{ad}_q E_3(E_1) = E_3 E_1 - q_3^{-1} E_1 E_3, \\ E_{112} &= \mathrm{ad}_q E_2(E_1) = E_2 E_1 - q_2^{-1} E_1 E_2, \\ E_{111} &= \mathrm{ad}_q E_3 \mathrm{ad}_q E_2(E_1) = \mathrm{ad}_q(E_3 E_2)(E_1), \\ E_0 &= (q_2 + q_2^{-1}) E_1 E_{111} + (q_3 + q_3^{-1}) E_{111} E_1 + (q_3 q_2^{-1} - q_3^{-1} q_2) E_{121} E_{112}, \end{aligned} \tag{3.13}$$

and let

$$F_{212} = \theta E_{121}, \quad F_{221} = \theta E_{112}, \quad F_{222} = \theta E_{111}, \quad F_0 = \theta E_0. \tag{3.14}$$

LEMMA 3.1. *The following formulas hold in* $\mathcal{U}$:

(1) $E_{ijk}^2 = 0$, $F_{ijk}^2 = 0$,

(2) $E_1 E_{121} + q_3 E_{121} E_1 = 0$, $F_1 F_{212} + q_3 F_{212} F_1 = 0$,

(3) $E_1 E_{112} + q_2 E_{112} E_1 = 0$, $F_1 F_{221} + q_2 F_{221} F_1 = 0$,

(4) $E_{112} E_{111} + q_3 E_{111} E_{112} = 0$, $F_{221} F_{222} + q_3 F_{222} F_{221} = 0$,

(5) $E_{121} E_{111} + q_2 E_{111} E_{121} = 0$, $F_{212} F_{222} + q_2 F_{222} F_{212} = 0$,

(6) $E_1 E_{111} + q^{-2\sigma_1} E_{111} E_1 + q_2 E_{112} E_{121} + q_3 E_{121} E_{112} = 0$.

PROOF. We only need to prove those formulas involving E, those involving F can then be obtained by applying θ. Formulas (2) and (3) are clear, (4) and (5) can be verified by using $E_{111} = \mathrm{ad}_q E_2(E_{121})$ or $E_{111} = \mathrm{ad}_q E_3(E_{112})$, (6) can be verified by using (2) and (3). To verify (1), note that formulas (3.5) and (3.6) imply that

$$E_1 E_i^2 E_1 = (q_i + q_i^{-1}) E_1 E_i E_1 E_i = (q_i + q_i^{-1}) E_i E_1 E_i E_1, \quad i = 2, 3.$$

Thus $E_{121}^2 = 0$, $E_{112}^2 = 0$. Similarly, using $E_{112}^2 = 0$ and $(\mathrm{ad}_q E_3)^2 E_{112} = 0$ instead of (3.5) and (3.6), we get

$$E_{111}^2 = \left(\mathrm{ad}_q E_3(E_{112})\right)^2 = 0.$$

The proof of the lemma is now complete.

The following lemma provides some formulas involving the element E_0.

LEMMA 3.2. *The following formulas hold in* $\mathcal{U}$:

(1) $E_0 E_2 = E_2 E_0$, $E_0 E_3 = E_3 E_0$, $E_0 F_2 = F_2 E_0$, $E_0 F_3 = F_3 E_0$,

(2) $E_0 E_1 - q^{2\sigma_1} E_1 E_0 = q_2(1 - q^{4\sigma_1}) E_1 E_{111} E_1$,

(3) $E_0 E_{121} = E_{121} E_0$, $E_{112} E_0 = E_0 E_{112}$, $E_0 E_{111} = E_{111} E_0$.

PROOF. The proofs for those formulas involving only the E's are just direct applications of Lemma 3.1. To verify the last two formulas in (1), we use the following formulas

$$\begin{aligned} F_2 E_{112} - E_{112} F_2 &= E_1 K_2^{-1}, \\ F_2 E_{121} &= E_{121} F_2, \\ F_2 E_{111} - E_{111} F_2 &= E_{121} K_2^{-1}, \end{aligned} \tag{3.15}$$

$$\begin{aligned} F_3 E_{112} &= E_{112} F_3, \\ F_3 E_{121} - E_{121} F_3 &= E_1 K_3^{-1}, \\ F_3 E_{111} - E_{111} F_3 &= E_{112} K_3^{-1}. \end{aligned} \tag{3.16}$$

REMARK. Compare with the corresponding formulas in $U(G)$, one would like to have a vector E_0 which satisfies (1) in Lemma 3.2 and has a better commutation relation with E_1, but this does not seem to be possible, since a search along this line will lead to the left hand side of (6) in Lemma 3.1, which is 0.

Let $\mathcal{U}_{\mathcal{A}}$ be the $\mathcal{A}$-subalgebra of $\mathcal{U}$ generated by E_i, F_i, $K_i^{\pm 1}$ and

$$[K_i; 0] = \frac{K_i - K_i^{-1}}{q_i - q_i^{-1}}, \quad i = 1, 2, 3.$$

For $\epsilon \in \mathbb{C}^\times$, let $\mathcal{U}_\epsilon = \mathcal{U}_{\mathcal{A}}/(q-\epsilon)\mathcal{U}_{\mathcal{A}}$. Then the algebra $\mathcal{U}_1$ is an associative algebra over $\mathbb{C}$ with generators E_i, F_i, K_i, $H_i = [K_i; 0]$ $(i = 1, 2, 3)$ and the defining relations (which can be verified easily):

$$K_i \text{ are central elements with } K_i^2 = 1, \tag{3.17}$$

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(3.18) $$[E_i, F_j] = \delta_{ij} H_i, \quad [H_i, E_j] = a_{ij} K_i E_j, \quad [H_i, F_j] = -a_{ij} K_i F_j,$$

(3.19) $$(\operatorname{ad} E_i)^{1-a_{ij}}(E_j) = 0, \quad (\operatorname{ad} F_i)^{1-a_{ij}}(F_j) = 0, \quad i = 2,3, \quad j = 1,2,3,$$

(3.20) $$E_1^2 = 0, \quad F_1^2 = 0.$$

Therefore, $\mathcal{U}_1/(K_i - 1; i = 1,2,3) \cong U(G)$, the universal enveloping algebra of G. Note that the image of E_0 in $U(G)$ is $2e_0$, where e_0 is defined in Section 2.

Let $\mathcal{U}^+$, $\mathcal{U}^-$, $\mathcal{U}^0$ be the subalgebras of $\mathcal{U}$ generated by the E_i, the F_i, and the $K_i^{\pm 1}$ $(i = 1,2,3)$ respectively. Then just as in the Lie algebra case (see [7]), one can show that $\mathcal{U} = \mathcal{U}^- \mathcal{U}^0 \mathcal{U}^+$ and (use the comultiplication) that $\mathcal{U} \cong \mathcal{U}^- \otimes \mathcal{U}^0 \otimes \mathcal{U}^+$ as $\mathcal{F}$-vector spaces.

For $\delta = (\delta_1, \delta_2, \delta_3, \delta_4)$, $\delta_i = 0$ or 1; $m = (m_1, m_2, m_3), m_i \in \mathbb{Z}_+$, let

(3.21) $$E^{(\delta,m)} = E_{111}^{\delta_1} E_{121}^{\delta_2} E_{112}^{\delta_3} E_1^{\delta_4} E_0^{m_1} E_2^{m_2} E_3^{m_3},$$
$$F^{(\delta,m)} = F_{222}^{\delta_1} F_{212}^{\delta_2} F_{221}^{\delta_3} F_1^{\delta_4} F_0^{m_1} F_2^{m_2} F_3^{m_3}.$$

For $t = (t_1, t_2, t_3)$, $t_i \in \mathbb{Z}$, let

(3.22) $$K^t = K_1^{t_1} K_2^{t_2} K_3^{t_3}.$$

Then the K^t form a basis of $\mathcal{U}^0$, and we have the following theorem:

THEOREM 3.3. *The elements of the form $E^{(\delta,m)}$ (resp. $F^{(\delta,m)}$) form a basis of $\mathcal{U}^+$ (resp. $\mathcal{U}^-$), and the elements of the form*

$$F^{(\delta,m)} K^t E^{(\delta',m')}$$

form a basis of $\mathcal{U}$.

PROOF. We only need to prove that the elements of the form $E^{(\delta,m)}$ form a basis of $\mathcal{U}^+$, since the statement about $\mathcal{U}^-$ will follow from symmetry and the statement about $\mathcal{U}$ will follow from the fact that $\mathcal{U} \cong \mathcal{U}^- \otimes \mathcal{U}^0 \otimes \mathcal{U}^+$. We first show that these elements span $\mathcal{U}^+$, that is, by using the commutation relations in $\mathcal{U}^+$ we can express any monomial of $\mathcal{U}^+$ as a linear combination of these elements. In fact, Lemma 3.1 and Lemma 3.2 along with the defining relations of $\mathcal{U}$ provide the commutation relations we need. In particular, to bring the terms $E_{112}E_{121}$ and $E_1 E_{111}$ to the right order, we use formula (6) in Lemma 3.1 together with the definition of E_0. Then, we show that these elements are linearly independent over $\mathcal{F}$. Note that by (3.13), these elements are in fact in $\mathcal{U}_{\mathcal{A}}$. So if we have a linear relation

(3.23) $$\sum_{i=1}^{r} c_i E^{(\delta_i, m_i)} = 0,$$

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with $0 \neq c_i \in \mathcal{F}$ $(1 \leq i \leq r)$, then by multiplying a suitable element from $\mathcal{A}$, we may assume that $c_i \in \mathcal{A}$. Now if there exists a c_i such that $c_i(1) \neq 0$, then the image of the right hand side of (3.20) gives a nontrivial linear relation in $U(G)$. But by the PBW theorem of $U(G)$, the images of the $E^{(\delta,m)}$ in $U(G)$ form a basis of $U(G)$, and we have a contradiction. If $c_i(1) = 0$ for all $1 \leq i \leq r$, then by the results in [1, Ch. 3, Section 3], we may assume that the order of 1 for c_i is n_i, and set $n = \min\{n_i : 1 \leq i \leq r\}$. Then $\lim_{q\to 1} c_i/(q-1)^n \neq 0$ for some i, hence by (3.20) we have

$$\lim_{q\to 1}\Big(\frac{1}{(q-1)^n}\sum_{i=1}^{r} c_i E^{(\delta_i,m_i)}\Big) = \sum_{i=1}^{r}\Big(\lim_{q\to 1}\frac{c_i}{(q-1)^n}\Big)E^{(\delta_i,m_i)} = 0,$$

which provides a nontrivial linear relation in $U(G)$ contradicting the PBW theorem for $U(G)$. Hence the elements of the form $E^{(\delta,m)}$ are linearly independent, and the proof of the theorem is now complete.

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