THE DETERMINATION OF ANGULAR DIAMETERS OF STARS

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1. Introduction

Ideally the determination of the angular diameter of a star would include the measurement of the distribution of intensity across the stellar disc. However, direct methods of measuring angular diameters have so far lacked adequate 'signal to noise' ratio to measure the intensity distribution and it has been the custom, in the first instance, to express the measured angular diameter in terms of the angular diameter of the equivalent uniform disc (θ_{UD}). Subsequent use of the angular diameter involves the assumption of a limb-darkening law and the application of an appropriate correction to θ_{UD} to find the 'true' angular diameter (θ_{LD}) of the star (e.g. Hanbury Brown *et al.*, 1967). In this article we will discuss the determination of θ_{UD} for single stars and we will not refer further to the more difficult problems of determining intensity distributions involving limb-darkening and rotational effects and of measuring the angular parameters of binary systems.

By itself the angular diameter of a star has no intrinsic value but when it is combined with other observational data it enables basic physical properties of the star to be determined. It is then possible to make a direct comparison of the observed properties of the star with the predictions of theoretical models of stellar atmospheres and interiors. For example, the combination of an angular diameter with the absolute monochromatic flux received from the star (f_ν), corrected for interstellar extinction, yields the absolute emergent flux at the stellar surface ($\mathscr{F}_\nu$). If the spectral energy distribution for the star is known it can be calibrated absolutely by $\mathscr{F}_\nu$ and hence the effective temperature (T_e) of the star can be found (this is equivalent to knowing the bolometric correction for the star and using it with the angular diameter to find T_e). In addition to leading to the determination of T_e, the absolute surface flux distribution may be compared directly with the predicted flux distributions for theoretical model stellar atmospheres (e.g. Davis and Webb, 1970). For O and early B type stars a large fraction of the emergent flux is in the far ultra-violet and the effective temperatures cannot be determined from the, at present, incomplete empirical flux curves. In these cases it is possible to obtain an estimate of the effective temperatures by using the values of $\mathscr{F}_\nu$ to calibrate a grid of model atmospheres which have T_e as a parameter. In this way, by measuring the angular diameters of stars of different spectral types, it is possible to establish an effective temperature scale.

Another example of the importance of angular diameter determinations is afforded by stars of known parallax. An angular diameter combined with a parallax gives the linear diameter of the star thus allowing the absolute luminosity of the star to be

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calculated from its radius and effective temperature. This enables the star to be plotted as an empirical point in the theoretical *H–R* diagram. If the mass of the star is also known a completely empirical test of stellar evolutionary models is possible.

2. Methods of Measuring Angular Diameters of Stars

In principle angular diameters can be obtained from eclipsing spectroscopic binaries of known parallax. Apart from YY Gem (M1V) for which the parallax is reasonably well known [±6% p.e. (Jenkins, 1963)] the remaining cases are now principally of historical interest. Although μ^1Sco(B1.5V) and β Aur(A2V) provided anchor points for the effective temperature scale for early type stars (Kuiper, 1938; Harris, 1963) they have now been superseded by the more accurate Narrabri intensity interferometer results (Hanbury Brown *et al.*, 1967). In view of this the present discussion will be confined to the measurement of angular diameters of single stars with an interferometer or by means of lunar occultations. As this subject has been reviewed recently by Hanbury Brown (1968) only a brief resume of the principal points will be given here.

A. THE MICHELSON INTERFEROMETER

The first determination of the angular diameter of a star was made by Michelson and Pease (1921) when they measured α Ori using a 20 ft Michelson interferometer mounted on the 100 in. telescope. Following this achievement the angular diameters of several stars were obtained with the 20 ft instrument and the results for 7 stars were published by Pease (1931). The result for α Her was preliminary and was omitted from a later list (Kuiper, 1938). The angular diameters for the remaining 6 stars are listed in Table I. These are the basic data, together with that for YY Gem, by means of which the temperature scale for late type stars has been placed on an observational foundation (Kuiper, 1938; Popper, 1959; Harris, 1963).

The original measurement of α Ori was estimated to have an uncertainty of ±10% by Michelson and Pease (1921) but an assessment of the accuracy of the remaining results was not published. From an examination of the various values for the 7 stars measured by Pease, which are distributed throughout the literature over a period of many years, Hanbury Brown (1968) has concluded that the standard error increases with the baseline required to resolve a star and probably lies in the range 10 to 20%.

During the late 1920's a 50 ft Michelson interferometer was constructed at Mt. Wilson but although some observational data appear in the Carnegie Yearbooks for 1933, 1935 and 1937, the final results were not published. The 50 ft instrument was apparently a very difficult instrument to operate and when Pease died his valuable experience was lost. The main difficulties were in meeting the severe requirements in the mechanical rigidity and guidance of the instrument and in making accurate measures of fringe visibility in the presence of atmospheric scintillation.

A proposal for building a large Michelson interferometer using modern servo control techniques and photoelectric detectors has been put forward (Miller, 1966) and reports

TABLE I

List of published angular diameters

BS	Star	Sp.	Lum.	Angular diameter of uniform disc $\theta_{UD} \pm \sigma$ (10^{-3} sec of arc)		Ref.	Technique
3207	γ^2Vel	WC8[a]		0.44	0.05	[1]	Intensity Interferometer
3165	ζ Pup	05f		0.42	0.03	[2]	
1903	ε Ori	B0	Ia	0.70	0.05	[3]	
4853	β Cru	B0.5	IV	0.705	0.025	[3]	
5056	α Vir	B1.5[a]	IV–V	0.87	0.04	[4]	
2618	ε CMa	B2	II	0.78	0.05	[3]	
1790	γ Ori	B2	III	0.74	0.05	[3]	
7790	α Pav	B3	IV	0.77	0.06	[3]	
472	α Eri	B5	IV	1.86	0.07	[3]	
8425	α Gru	B5	V	0.98	0.07	[3]	
3982	α Leo	B7	V	1.33	0.07	[3]	
1713	β Ori	B8	Ia	2.57	0.14	[3]	
7001	α Lyr	A0	V	3.31	0.15	[3]	
2491	α CMa	A1	V	5.85	0.10	[3]	
8728	α PsA	A3	V	1.98	0.13	[3]	
7557	α Aql	A7	IV, V	2.79	0.14	[3]	
2326	α Car	F0	Ib–II	6.48	0.39	[3]	
2943	α CMi	F5	IV–V	5.31	0.36	[3]	
5340	α Boo	K2	IIIp	20	± 10–20 %	[5]	Michelson Interferometer
1457	α Tau	K5	III	20		[5]	
2061	α Ori	M1–M2	Iab	47 34		[5] [5]	
6134	α Sco	M1–M2	Iab	40		[5]	
8775	β Peg	M2	II–III	21			
681	o Cet	M6e	III	47		[5]	
6134	α Sco	M1–M2	Iab	41 38	1 6	[6] [7]	Lunar occultation
8698	λ Aqr	M2	III	7.4	0.4	[8]	
2286	μ Gem	M3	III	23	?	[9]	

[a] Primary

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of proposed experimental work have been published (Beavers, 1963; Twiss, 1965). So far no new angular diameter determinations have been reported.

B. THE INTENSITY INTERFEROMETER

Since the first angular diameter determination with an intensity interferometer by Hanbury Brown and Twiss (1956) a large instrument has been constructed and operated for seven years at Narrabri in Australia. This instrument has been used to establish an effective temperature scale for early type stars from the angular diameters of 15 stars (Hanbury Brown *et al.*, 1967). Additional angular diameters for the WC8 component of γ^2Vel (Hanbury Brown *et al.*, 1970), for ζ Pup (Davis *et al.*, 1970) and the primary of α Vir (Herbison-Evans *et al.*, 1970) have been published. Table I contains a list of the published angular diameters. The observational programme planned for the Narrabri instrument includes the completion of measurements of an additional 14 angular diameters by the end of 1971.

An intensity interferometer has two major advantages compared with a Michelson interferometer. Firstly, very long baselines can be used without the need for extreme mechanical precision and guidance. This is because the tolerance in the relative delay in the two arms of the intensity interferometer can be of the order of one million times greater (Hanbury Brown, 1968) being set by the electrical bandwidth for the intensity interferometer and by the optical bandwidth for the Michelson interferometer. The second advantage of an intensity interferometer is that it is essentially unaffected by atmospheric scintillation (Hanbury Brown and Twiss, 1958; Hanbury Brown *et al.*, 1967). However, the intensity interferometer suffers from the disadvantages that it is relatively insensitive and does not work satisfactorily for cool stars.

C. LUNAR OCCULTATIONS

The angular size of a star occulted by the Moon can be found in principle from the degree of smoothing it produces on the fringe pattern which results from diffraction at the lunar limb. The resolution is limited by the calculable effects of the finite sizes of the telescope aperture, the optical bandwidth and the time constant of the recording system.

The effects of irregularities at the limb of the Moon and of atmospheric scintillation will also set limits to the resolving power. Although efforts have been made to estimate the effects of the limb irregularities (Diercks and Hunger, 1952; Evans, 1955a, 1957 and 1970), it remains to establish the magnitude of the effects by observation. This may be done by simultaneous observations of the same occultation through different apertures either at the same or at different sites, by observations of different occultations of the same star, by observations of occultations of unresolved stars or by means of two colour observations as suggested by Nather (Evans, 1970).

In spite of pioneering work by Whitford (1939, 1946) the first significant set of angular diameter measurements were those carried out by Cousins and Guelke (1953), Evans *et al.* (1953) and Evans (1955b) of the bright M1 supergiant Antares (α Sco). The results have been summarised by Evans (1957) and re-analysed by Taylor (1966).

In addition, Evans (1959) has published an angular diameter for μ Gem (M3III) and recently an accurate measurement of λ Aqr (M2III) has been made by Nather *et al.* (1970). The published angular diameters obtained from lunar occultations are listed in Table I.

3. The Angular Diameters of Stars

The published angular diameters of a total of 27 stars determined by interferometric and occultation techniques are listed in Table I. The available data illustrate the fact that the intensity interferometer is the instrument to use for early type stars while the Michelson interferometer and lunar occultations are more suitable methods for measuring late type stars. This reflects the higher resolving power of the intensity interferometer on the one hand, and the higher sensitivity of the other two techniques for cooler sources, for which the higher resolving power is not required, on the other.

It is noted that the accuracy of the angular diameters for the early type stars is generally of the order ± 5 to 7% whereas for the later type stars the accuracy is poorer and generally only ± 10 to 20%. An exception is provided by λ Aqr whose angular diameter was measured with the occultation technique.

Figure 1 shows the distribution of stars whose angular diameters have been published as a function of spectral type and luminosity class. Also included are an

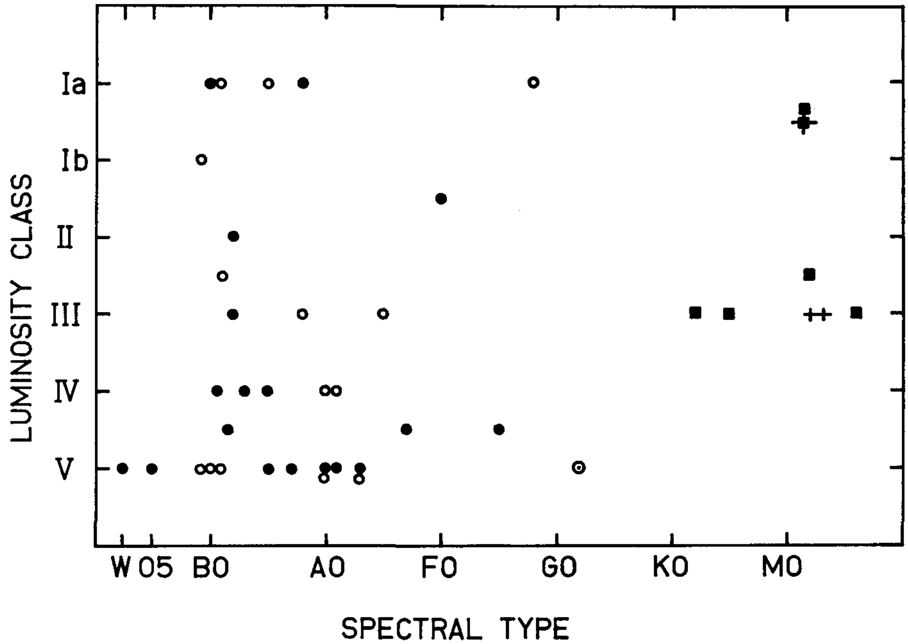

Fig. 1. The distribution of stellar angular diameter determinations with spectral type and luminosity class. Dots – intensity interferometer (from Table I); circles – intensity interferometer (measured or to be measured in 1970–1971); squares – Michelson interferometer (from Table I); crosses – lunar occultations (from Table I).

additional 14 stars which have already been measured or which it is planned to measure with the Narrabri intensity interferometer during 1970–1971.

It can be seen from the Figure that when the proposed observational programme at Narrabri is completed the intensity interferometer will have provided a reasonable coverage of the luminosity class – spectral type array down to late-A spectral type. For later spectral types the coverage is generally poor. Apart from α CMi (F5IV-V), the Sun (G2V) and YY Gem (M1V) there are no main-sequence stars of spectral types F, G, K and M with measured angular diameters. The M and K giants and M supergiants are represented in Figure 1 although generally with much lower accuracy than for the early type stars. The remainder of the diagram is essentially empty and this illustrates the need for angular diameters of representative types to fill in the gaps.

4. The Future of Angular Diameter Determinations

In terms of proven ability to obtain angular diameters of single stars the intensity interferometer is the most successful technique and it is appropriate to consider its future first. The Narrabri instrument was designed to measure stars down to $B = +2.5$ and it is now nearing the completion of this task. Undoubtedly a far more sensitive intensity interferometer could be built and this has been discussed by Hanbury Brown (1968) and Twiss (1969). Such an instrument would be large, complex and costly but it is not unreasonable to think in terms of an improvement up to a factor of the order of one hundred times over the Narrabri interferometer which would allow stars brighter than $B = +7.5$ to be measured. An improvement in sensitivity of this magnitude would open up a number of possibilities of great astrophysical interest including the measurement of the limb darkening law for various types of stars, the intensity distributions of rotating stars, the variations of apparent angular size of Cepheid variables, angular parameters of double stars, and so on. In terms of the determination of angular diameters of single stars there would be of the order of 10^4 stars within the sensitivity reach of the instrument. This would allow stars to be selected for a far more detailed study of intrinsic differences due to luminosity class, spectral type and other parameters such as metal content, rotation, etc. than is possible with the present instrument. The advent of phototubes with quantum efficiencies of the order of 10% at λ 6500 and suitable design of the instrument would make it possible to extend the measurement of angular diameters to stars of spectral type K with an intensity interferometer. As detectors improve in the infra-red it should be possible to extend this limit to M type stars.

The future possibilities of a Michelson interferometer are not so easy to predict. While it would appear that the formidable difficulties encountered with the Mt. Wilson instruments should not be insuperable when modern techniques and detectors are applied to the problem, it is difficult to envisage the maximum baselines which might be feasible. Miller (1966) has speculated on a 1 km Michelson interferometer but it would seem more plausible at present to consider an instrument with a maximum baseline in the range 50 to 100 m. Such an instrument would be capable of making a

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significant contribution to our knowledge of the cooler stars. It would be complementary to the intensity interferometer and would provide a valuable overlapping of results. It is clearly desirable that some stars should be measured independently by the two techniques.

The lunar occultation technique is a relatively simple and inexpensive method when compared to the complexities and cost of interferometers. It does, of course, suffer from the obvious disadvantage of being tied to objects which lie in the path of the moon with the possibility of an observation, which cannot be repeated at will, being lost due to weather or other unavoidable circumstances. Nevertheless, it appears that lunar occultations will provide angular diameters for a number of cool stars and in the absence of a modern version of the Michelson interferometer or a more sensitive intensity interferometer it may prove to be the only practicable method of getting these data. But even if these instruments are built, lunar occultations may still provide a valuable independent check on interferometric data. An example already exists in the case of α Sco which has been measured with the original Michelson interferometer and by the occultation method. The agreement between the results is excellent, as can be seen from Table I, but in view of the relatively large uncertainties it must be regarded as fortuitous. Observations of occultations of α Vir and α Leo would provide valuable cross-checks between the present intensity interferometric results and the occultation method. It does remain, however, to establish the full potential and limitations of the lunar occultation technique.

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