Second, I argue that there is a close connection between dynamic epistemic logic and logical geometry. The latter is the systematic investigation of extensions and variants of the well-known Aristotelian square of opposition. I show that dynamic epistemic logics give rise to some very interesting Aristotelian diagrams (squares of opposition, but also many other, more complex diagrams). As a further illustration of the philosophical significance of logical geometry, I also develop a theoretical account of the information levels of the Aristotelian relations and diagrams. This account can then be applied to the Aristotelian diagrams for dynamic epistemic logic that were mentioned above.

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Rafael Zamora, ***Separation Problems of Analytic Relations*** (***Problèmes de séparation des relations analytiques***), Université Pierre et Marie Curie, France, 2015. Supervised by Dominique Lecomte. MSC: Primary 03E15, Secondary 26A21, 54H05. Keywords: Borel class, Wadge class, separation, product space.

Abstract

This thesis is about descriptive set theory. One of the main problems in this area is related to the complexity of subsets of a Polish space, with respect to several hierarchies. Two of the main hierarchies studied are the Borel and the Wadge hierarchies.

A more general way to see the problem of the complexity of a set is to ask the question: "Given two analytic subsets A, B of a Polish space X, when can you find a third subset C in a class Γ, such that $A \subseteq C$ and $C \cap B = \emptyset$?". This was first answered by Lusin, taking Γ as the class of Borel sets.

For the Borel classes Louveau and Saint-Raymond solved it, expanding on work by Hurewicz. They found a minimal example, under a certain quasi-order, in the class of pairs of analytics subsets of a Polish set that cannot be separated by a set in $\mathbf{\Sigma}^0_\xi$. Finding small basis, i.e., antichain basis, is a powerful characterization which has been looked for in several contexts.

If we consider analytics subsets A, B of $X \times Y$ for X, Y Polish spaces, we can consider a lot more classes. There are also several notions of comparison for which knowing an antichain basis is interesting.

In the first part of this thesis, we consider the question for the class $\Gamma \times \Gamma'$ of subsets of the form $C \times D$ for $C \in \Gamma, D \in \Gamma'$. Again, for a certain quasi-order, we find small antichain basis in the class of pairs of sets that are not separable by a set in $\Gamma \times \Gamma'$ for Γ, Γ' of small Borel complexity.

In the second part, we consider the classes of subsets of the form $\text{Pot}(\Gamma)$. This class was defined by Louveau and consists of the subsets A of a product space that can be made in Γ by refining the topology, allowing only products of Polish topologies (so, for example, the diagonal of an uncountable Polish space is not potentially open).

A minimum example in the class of pairs of sets that are not separable by a $\text{Pot}(\Gamma)$ set was previously found by Lecomte for all Wadge classes of Borel sets. In this thesis, we focused on classes of small Wadge rank. For several of those, we find conditions under which there are small antichain basis, for a stronger quasi-order involving injectivity.

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Athar Abdul-Quader, ***Interstructure Lattices and Types of Peano Arithmetic***, The Graduate Center, City University of New York, USA, 2017. Supervised by Roman Kossak. MSC: 03C62, 03H15. Keywords: Enayat models, interstructure lattices, coded sets.

Abstract

The collection of elementary substructures of a model of PA forms a lattice and is referred to as the substructure lattice of the model. In this thesis, we study substructure and interstructure lattices of models of PA. We apply techniques used in studying these lattices to other problems in the model theory of PA.

In Chapter 2, we study a problem that has its origin in Stephen G. Simpson's, "Forcing and models of arithmetic." *Proceedings of the American Mathematical Society* 43.1 (1974): p. 193–214. Simpson used arithmetic forcing to show that every countable model of PA has an expansion to a model of PA^* that is pointwise definable. Ali Enayat "Undefinable classes and definable elements in models of set theory and arithmetic." *Proceedings of the American Mathematical Society* 103.4 (1988): p. 1216–1220 later showed that there are models $\mathcal{M}$ with the property that for each undefinable class $X \subseteq M$, and in particular those for which $(\mathcal{M}, X) \models \mathsf{PA}^*$, $(\mathcal{M}, X)$ is pointwise definable. We use techniques involved in representations of lattices to show that there is a model of PA with this property which contains an infinite descending chain of elementary cuts.

In Chapter 3, we study the question of when sets can be coded in elementary end extensions with prescribed interstructure lattices. This problem originated in Haim Gaifman's, "Models and types of Peano's arithmetic." *Annals of Mathematical Logic* 9.3 (1976): p. 223–306, who showed that every model of PA has a conservative, minimal elementary end extension. That is, every model of PA has a minimal elementary end extension which codes only definable sets. Roman Kossak and Jeffrey Paris "Subsets of models of arithmetic." *Archive for Mathematical Logic* 32.1 (1992): p. 65–73 showed that if a model is countable and a subset X can be coded in any elementary end extension, then it can be coded in a minimal extension. James Schmerl, in "Subsets coded in elementary end extensions." *Archive for Mathematical Logic* 53.5–6 (2014): p. 571–581 and "Minimal elementary end extensions." *Archive for Mathematical Logic* 56.5–6 (2017): p. 541–553, extended this work by considering which collections of sets can be the sets coded in a minimal elementary end extension. We extend this work to other lattices and in particular study two questions. Given a countable model $\mathcal{M}$, which sets can be coded in an elementary end extension such that the interstructure lattice is some prescribed finite distributive lattice; and, given an arbitrary model $\mathcal{M}$, which sets can be coded in an elementary end extension whose interstructure lattice is a finite Boolean algebra?

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