Quantum Disjunctive Facts[1]

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To the memory of John D. Trimmer

This paper assesses the impact of disjunctive facts on the quantum logic read off procedure. The purpose of the procedure is to transfer a significant quantum structure to a set of propositions; its first step is an attempt to discover that structure. Here I propose that disjunctive facts as traditionally conceived have blocked the procedure at its first step and have therefore subverted the best-known attempts to read off quantum logic. Recently however Allen Stairs has proposed a view of disjunctive facts which re-establishes the possibility of reading off quantum logic. Both the traditional conception and Stairs' revision of disjunctive facts are interesting in their own right, independent of quantum propositional logic.

1. 'Quantum Logic'

Too many things are called 'quantum logic'. The term is disentangled and notation is fixed in this preliminary step which relies on basic lattice theory.[2]

A *lattice* is a partially ordered set with a greatest lower bound and a least upper bound defined on any two of its members. A *partially ordered set* is a pair $\langle \mathbf{S}, \leq \rangle$; $\mathbf{S}$ is a set with members $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and $\leq$ is the partial order relation on $\mathbf{S}$. A *least upper bound* (lub) of $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{C} = (\mathbf{A} \vee \mathbf{B})$

(lub i) is an upper bound of $\mathbf{A}$, $\mathbf{A} \leq \mathbf{C}$,
(lub ii) is an upper bound of $\mathbf{B}$, $\mathbf{B} \leq \mathbf{C}$,
(lub iii) satisfies a leastness condition: $\mathbf{C} \leq \mathbf{D}$ for $\mathbf{D}$, any other upper bound of $\mathbf{A}$ and $\mathbf{B}$.

The dual $\wedge$ is the *greatest lower bound* (glb) and is defined accordingly. Our interest is restricted to lattices which are orthocomplemented. An *ortholattice*

$$\mathbf{L} = \langle \mathbf{S}, \leq, \wedge, \vee, ^{-} \rangle$$

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is a set with a relation and three operations. An ortholattice is *Boolean (non-Boolean)* if it satisfies (fails) the distributive law.

Lattice notation is reserved for the case where **L** is an *uninterpreted* mathematical object. Alternate notation is used for interpretations or *instantiations* of **L**. Three non-Boolean instantiations are commonly called 'quantum logic'.

Two geometric instantiations of **L** are state spaces. The set of subsets of an n-dimensional Cartesian space has a Boolean structure

$$CSS = \langle S^C, \leq, \cap, \cup, ' \rangle.$$

S^C has members (represented by) X, Y, Z,... ; the relation is set-theoretic inclusion; the operators are set-theoretic intersection and union and relative complement. The (closed linear) subspaces of a Hilbert space are a non-Boolean instantiation of **L**

$$QSS = \langle S^H, \leq, \wedge, \oplus, ^\perp \rangle.$$

S^H has members M, N, O,... ; the relation is subspace inclusion; the operators are subspace intersection, span and orthogonal complement. QSS, *quantum state space*, is sometimes referred to as 'Hilbert space logic', sometimes as 'quantum logic'.

Two more instantiations are algebraic: Sets of classical properties

$$CAL = \langle S^P, \leq, \wedge, \vee, - \rangle$$

where S^P has elements *X, Y, Z*,... ; and the interpretations of the relation and operators are the same as those of CSS. And, some claim, sets of quantum properties

$$QAL = \langle S^{\mathcal{P}}, \leq, \sqcap, \sqcup, \neg \rangle.$$

$S^{\mathcal{P}}$ has elements *M, N, O*,... . Interpretations are the subject of this paper. Accommodating the practice of referring to QAL as 'quantum logic', and preserving the distinction with quantum propositional logic, we adopt *quantum algebraic logic.*

Two final instantiations are logical:

$$CPL = \langle S^S, \leq, \cdot, \vee, \sim \rangle$$

with propositions x, y, z,... , classical semantic entailment and truth-functional operators. And possibly *quantum propositional logic*

$$QPL = \langle S^{\mathcal{S}}, \leq, \cdot^*, \vee^*, \sim^* \rangle$$

with propositions m, n, o,... , quantum semantic entailment, conjunction, disjunction and negation.

2. The Read Off Dilemma

The three quantum logics — QSS, QAL and QPL — traditionally have been related by the read off procedure.[3] Step one (RO1) is an attempt

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to discover the structure of QAL or QSS. Step two (RO2) would transfer the structure to QPL. And step three (RO3) aims to establish that QPL is full-blown propositional logic.[4] Four variants of the procedure are discussed: one from QSS, three from versions of QAL.

The QSS read off procedure is uncontroversial at RO1 which establishes the structure of QSS; the result was stated when QSS was defined. But at RO2 a dilemma arises. One option transfers the QSS structure to a set of statements where one-dimensional members of H (e.g., ψ) are said to be members of subspaces:

'm' is true iff ψ lies in M.
'm v* n' is true iff ψ lies in M $\oplus$ N.

This option is precise but sterile.[5] The second option is empirical, but contentious : the QSS structure is transferred to "basic physical propositions" (e.g., Putnam 1969).

A related dilemma[6] asks if QSS is a Hilbert space *or* a quantum state space. On the first option, disjunctive facts gain ontological status only by mathematical realism; M $\oplus$ N represents a fact, a real independently existing mathematical object. The second option allows a more robust realism and a more prestigious ontological status for facts.

The second horn of the two dilemmas is the basis for the objections of the remainder of the paper. If the read off procedure is not to rest on a sterile mathematical realism, it must have an empirical basis. Attempts to provide that basis have run afowl of the following Theorem.

The *Mapping Theorem* is a lattice reformulation of Theorem 1 of Kochen and Specker (1967).[7] If QSS is the lattice of subspaces of a Hilbert space of dimension ≥ 3 and L(2) is the two-element Boolean lattice defined on the set S(2),

Mapping Theorem: There is no lattice homomorphism from QSS onto L(2).

In general, a homomorphism is a structure preserving transformation; a lattice homomorphism preserves $\leq$, $\wedge$, and $\vee$. Here only $\leq$ is preserved; we will focus on the Theorem's denial of a *V-homomorphism* or a transformation from S^H of QSS to S(2) of L(2) such that for every M, N in S^H

$$h(M \oplus N) = h(M) \vee h(N).$$

By the Theorem these are inconsistent: (i) a set has a QSS structure and (ii) each member is assigned one of two values and the structure is preserved. A *violation* attempts to either: i) assign one of two values to each member of a QSS set while preserving its structure or ii) impose a QSS structure on a set each member of which has two values. The most frequent use of the Theorem has been to restrict admissable valuations for QPL. Here I am concerned with its impact on QAL. Specifically, the QAL read off procedure in each of its best-known versions has committed if not one then the other of these two violations. Or so I am next going to propose.

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3. The QAL Read Off Procedure

The QAL read off procedure originated with von Neumann (1935, sec. III.5): a system has a set of properties, it is a *property* of a system "that a certain quantity takes value λ." (p. 249). And the set of properties has a "sort of logical [QAL] calculus." (p. 253). The property that magnitude A takes value a(i) admits three significant interpretations: that A now measured has a(i) (measured-valued property), that upon measurement A will be found to have a(i) (measurable-value property), and that in the absence of measurement A has a(i) (possessed-value property). The three interpretations generate three corresponding versions of the QAL read off procedure to which we now turn.

4. The QAL of Tests

Macroscopic disjunctive facts, necessary for R01 of the QAL of tests (or of measured-value properties), present an insurmountable obstacle to reading off quantum logic from the QAL of tests.

A measured-value property can be associated with the experiment which reveals it. And in principle, any experiment can be reduced to tests or yes/no experiments (Jauch 1968). So the measured-value read off procedure can be regarded as a test procedure.

From the beginning it was realized that there was a problem with *conjunctive* facts; there is, in general, no test for property O:T(O) = T(M) $\sqcap$ T(N) (the glb of S^T the set of tests) which gives yes when T(M) and T(N) each do. The lattice-tail wagged the empirical-dog: the missing test was either postulated or invented as an idealization.

For disjunctive facts no such *ad hoc* rescue is possible. The CAL of tests includes a (set of) test(s) T(Z) which gives yes if and only if either T(X) does or T(Y) does. The *if* clause gives (lub i) and (lub ii); the *only if* clause assures leastness (lub iii).[8] For QAL an important (class of) test(s) T(O) gives yes if T(M) does, if T(N) does and in other cases also (cf. Finkelstein (1972) or Bub (1979)). To assure (lub iii) for these tests it must be established *empirically* that *no* T' (O) exists which fails to give yes when T(O) does. Things get worse.

Distinguish two tests. One would give yes for property M, for N and for other properties as well; this test T($M \sqcup N$) would measure the property related to M and N as M $\oplus$ N is related to M and N. Another, T(O) = T(M) $\sqcup$ T(N), would give yes when T(M) does, when T(N) does and when tests which are in its 'span' (tests related to T(M) and T(N) as M $\oplus$ N is related to M and N) do. The *property* $M \sqcup N$ (necessary for the claim that the set of properties has a QSS structure) is, in the best case, an intuitive challenge. The *test* T(M) $\sqcup$ T(N) (required if S^T has a QSS structure) is an absurdity.

The absurdity rests on a Mapping Theorem violation. Both Jauch (1968) and Putnam (1969) following Finkelstein attempted to impose a QSS structure on the set of tests. And both made a technical mistake at the same point although from opposite directions. Jauch defined the lub nonclassically but imposed it on tests while Putnam mistakenly claimed the lub could be classical and still have a QSS structure. In either case there would be a **V**-homomorphism violation. If $\sqcup$ defined on S^T has the properties of $\oplus$ and also is classical, there would be a transformation from S^T to (yes, no)

$$h(T(M) \sqcup T(N)) = h(T(M)) \vee h(T(N)).$$

The search for macroscopic disjunctive facts, RO1 of the test-QAL read off procedure, and QPL read off the QAL of tests all receive their coup de grace with this result.[9]

5. The QAL of Measurable Properties

The measurable-value read off program (at least at RO1) can be traced to von Neumann (1935)[10] and Mackey (1963). They assigned 1/0 or yes/no at measurement for measurable-value properties and proposed a QSS structure for the set — in apparent violation of the Mapping Theorem. A defense[11] might allow that *prior to* measurement quantum theory only assigns probable yes/no values (*then* properties have a QSS structure), while *at* measurement properties have actual values (*then* properties have a L(2) structure). The Mapping Theorem can be avoided in this way, but only by paying the price of generating an inconsistent triad: (i) the set of measurable-value properties has a QSS structure, (ii) test results have a L(2) structure and, (iii) tests do not disturb or change the value of the measurable-value property. A similar triad looms large for the QAL possessed-value (hereafter QAL-*P*) program. The following analysis of that program may also be regarded as an assessment of the prospect of a QAL measurable-value read off program.

The minimal claim of the QAL-*P* program is that there exists sets of measurement-independent, possessed-value properties with a QSS structure.[12] Within Putnam's "Is Logic Empirical?" is a triad composed of the minimal claim, *No Disturbance* (ND) - a perfect measurement neither disturbs nor creates but merely reveals the possessed value i.e., the measured value is the same as the possessed value at the time of measurement, and (tacitly) *Macroscopic Precise Value* (MPV) - every macroscopic observable has a precise or definite value.

Putnam's triad was soon challenged as an inconsistent violation of the Mapping Theorem. Although Friedman and Glymour (1972) focused on QPL semantics, they posed the "tentative" *algebraic* challenge that the triad could not be sustained. Gardner (1972) stressed that, given MPV, minimal QAL-*P* and ND, but not minimal QAL-*P* alone, violate the Mapping Theorem. These challenges can be sharpened as the denial of a QAL-*P* onto CAL-P **V**-homomorphism: both

$$h(M \sqcup N) = h(M) \vee h(N)$$

and

$$h(M \sqcup N) = X \vee Y$$

fail (M and N are the possessed-value properties which a perfect measurement reveals as X and Y, the measured values). So if there is some disjunctive fact, a possessed-value property, represented by $M \sqcup N$, it is empirically inaccessible, at least in the straightforward sense that it could be revealed at measurement.[13] (Schrödinger's cat would

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have celebrated its fifth birthday last year.[14])

6. The QAL of Precise Possessed-Value Properties

The triad of the previous section appeared more plausible (but was no less inconsistent) because an additional claim was included: *Microscopic Precise Value* (PV) - every observable has a precise, well-defined or definite value in every state. If the measurement claims ND and MPV are abandoned and PV is added to minimal QAL-*P* we get the concept of precise-valued quantum disjunctive facts, even if never measured. These too give no hope to those who would read off quantum logic.

Ten years after "Is Logic Empirical?" Friedman and Putnam (1978), then Bub (1982) defended the *pair*: (i) minimal QAL-*P* and (ii) (a version of) PV while taking cognizance of the Mapping Theorem. Here I recast[15] their defense and show the pair is inconsistent.

The QAL-*P* claim gains credence by a QSS—QAL-*P* association: possessed-value properties are represented by subspaces. Bub claims that a particular physical example[16] is associated with the subspace (projection operator)

(1) M(1) vel M(2) vel M(3)

which is the QSS unit element. It would represent the possessed-value property

(2) $M(1)$ vel $M(2)$ vel $M(3)$

(the physical example and (1) provide that i=1, 2, 3 are all one quantity's possible values).

PV is expressed as "physical quantities have well-defined values, but [only] in a somewhat Pickwickian sense" (Bub, p. 404). The value of (2) is well-defined *if* 'vel' "denotes exclusive disjunction" (p. 404). Then for some i, M(i) is *the* property of the system. That's how exclusive disjunction works. So PV is defended. However (1), and therefore (2), have no **V**-homomorphism onto L(2). And no assignment of 1 or 0 to each M(i) (or M(i)) gives exactly one 1. That's not how exclusive disjunction works. So PV is Pickwickian.

Both (1) and (2) with 'vel' as exclusive disjunction are doubly mischievous category mistakes. The M(i) represent *subspaces*, so (1) misrepresents the QSS

(1') M(1) ⊕ M(2) ⊕ M(3),

and still assuming the QSS—QAL-*P* association (2) misrepresents the QAL-*P*

(2') $M(1) \sqcup M(2) \sqcup M(3)$.

And if m(i) is the QPL proposition read off M(i), both (1') and (2') must be distinguished from the QPL expression

(3') m(1) v* m(2) v* m(3).

The mischief of exclusive disjunction both imported a *logical* connective into a geometric and then algebraic structure and used a *classical* connection in a nonclassical context.

If so, there is a V-homomorphism from neither (1) nor (2) to L(2), and while QAL-*P* alone is not challenged, QAL-*P* and PV are inconsistent. The lesson is not (as before) that ND is indefensible, that the measurement problem reappears, and that there is no immediate empirical access to quantum disjunctive facts (all of which is true), but that (2') represents what Stairs aptly flaunts as an irreducible disjunctive fact, the sort of *thing* which was once called "an entirely new idea ... to which one must get accustomed ... without having any detailed classical picture". (Dirac 1947, p. 12).

With this result the traditional read off procedure which sets out to *discover* a significant quantum structure stands before a door over which is inscribed *lasciate ogni speranza, voi ch'entrate.*

7. The QAL of Irreducible Disjunctive Facts

Stairs has recently mounted a sustained defense of irreducible disjunctive facts (cf. especially Stairs (1982, 1983, 1985)).[17] After comparing what I have written with his position on two points, I will relate his position to the read off procedure.

I have been far less congenial than Stairs concerning the compatibility of QAL-*P* (or "realist quantum logic") and PV. He acknowledges "a particular tension" and defends an interpretation which denies PV.[18] Yet, by accepting an exclusive or interpretation, he allows that (2) "seems to" support PV. Consequently, he regards his position as the better of two options, while I see it as the only tenable position.

Throughout I have strived steadfastly to demarcate disjunctive *facts* and *propositions* or QAL (and QSS) and QPL. And I have maintained that QSS or QAL are necessary for QPL, if read off. But QSS and QAL are prior to and conceptually independent of QPL as Bub and Demopoulos (1974, p. 98) long ago stressed: "we maintain a sharp distinction between [QSS] and [QPL] ... . The choice of [QSS] is directly related to quantum theory. [QPL] raises a completely different set of problems." Two corollaries follow. Both concern the impact of disjunctive facts on the read off program; one concerns RO1, the other RO3.

Stairs is quite up-front that his QAL-*P* proposal is *now* merely a very robust ontological posit. He stakes his claim on a research program which may *eventually* provide empirical warrant. This RO1 program is conceptually prior to QPL: QAL problems cannot be addressed by "tinkering with" QPL. Stairs' QAL program is not concerned with bivalence failures, metalanguage adjustments or theories of truth.[19]

If the QAL RO1 program succeeds, a second distinct program must then address "a completely different set of problems" at RO3 if quantum propositional logic is to be read off quantum theory.

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We can share Stairs' enthusiasm for the R01 program and acknowledge the necessity of embarking on an R03 program and still realize that whatever will eventually come to be there is now no reason to think quantum logic can be read off quantum theory.

The read off program is dead. Long live the read off program. Maybe.

Notes

[1]It is a pleasure to acknowledge commentary on a previous draft by Paul Beem, Arthur Fine, Bas van Fraassen and Linda Wessels. This paper is a narrowly-focused continuation of an earlier dialogue: McGrath (1978) and Bugajski (1980).

[2]I regard the lattice formulation analysis of this paper to be applicable to partial Boolean algebra representations (cf note 7). Views of others are recast in lattice terminology, if necessary, to provide a uniform exposition.

[3]The read off procedure used by followers of van Fraassen's semantic analysis moves in the opposite direction. It starts with elementary sentences of a language, then seeks out a satisfaction set (which may be *structured like* QSS or QAL) which captures important features of the language (thereby revealing the language to be QPL). So there are no *facts* involved and the analysis of this paper does not apply to such a procedure.

[4]The three steps are elaborated in McGrath (1978).

[5]It was one purpose of 'acrobat logic' (McGrath 1978) to show that the read off procedure can be 'abused' in this trivial way and consequently does not, by R01 and R02 alone, generate a *propositional* logic.

[6]Bell and Hallett (1982, p. 369) pose a similar dilemma.

[7]The Theorem reformulates the algebraic result that $B(H) = \langle S^H, \circ, \curlywedge, \oplus, \perp, 1\rangle$ the partial Boolean algebra of closed linear subspaces of a Hilbert space of dimension > 3 has no algebraic homomorphism onto Z(2) the two-element Boolean algebra. Since Z(2) = L(2) and an algebraic and lattice homomorphism preserve the same operators ($\curlywedge$ and $\oplus$), we need only show that $B(H)$ = QSS, i.e. that $B(H)$ has $<$ and that QSS has $\circ$. S^H is partially ordered by subspace inclusion so $B(H)$ is a transitive partial Boolean algebra in the sense of Kochen and Specker. QSS trivally has $\circ$: when $\circ \subseteq S^H \times S^H$: $(M,N) \in \circ$, there are elements O, P, Q, of S^H which are mutually orthogonal with $M = O \oplus P$ and $N = P \oplus Q$ (Kochen and Specker 1967, p. 65).

[8]The partial order relation on the set of classical tests is defined so that $T'(X) < T(X)$ if whenever $T'(X)$ gives yes so does $T(X)$.

[9]Gardner (1972) issued Putnam a QAL Mapping Theorem citation. Dummett (1976) and Hellman (1980) later pinpointed the technical mistake.

[10]For a recent elaboration of the von Neumann program see Demopoulos (1976, sec. 5).

[11]My interpretation of Mackey relies on his Axiom VII and commentary on pp. 72-73. A commentator on an earlier draft suggested that a more sympathetic reading would allow this defense.

[12]Stairs (1983) recently isolated this ontological claim as the core of the realist quantum logic program. It is fair to read 'QAL-*P*' as 'realist quantum logic'.

[13]Nor will it help to propose an *ensemble* of measurable-value properties (which I understand to be the strategy of Bugajski (1980)) for reasons elaborated by Schrödinger (1935, sec. 4) and reiterated in McGrath (1980, sec. 3 and sec. 6).

[14]Schrödinger argued (1935, sec. 5) that if one accepted the triad : (i) a "blurred model" of atomic reality which (ii) "becomes transformed into" (iii) "macroscopically tangible and visible things, for which the term 'blurring' seems simply wrong", then "One can even set up quite ridiculous cases ... ." In McGrath (1980) the triad was isolated as Rule VAL, Rule Ø INT and Rule MR; the contradiction of sec. 5 represents "ridiculous".

[15]The argument is recast in lattice form and simplified from "*all* physical quantities" to one (i.e. I disregard the intersection over the indices). I follow Bub's presentation; Friedman and Putnam give a technically equivalent argument. Bub's comments should be compared to Bub (1979).

[16]The physical example is detailed by Friedman and Putnam (1978, p. 312) and Bub (1982, p. 404) and is explicated by Stairs (1983, p. 584).

[17]It is instructive to compare Stairs' conception to Fine's position (cf., e.g., (1976)) that quantum theory underdetermines the values of or simply has nothing to say about the values of [irreducible disjunctive facts].

[18]One conjunct of the conjunction P (Stairs 1983, p. 585) corresponds to (2).

[19]This is not a trivial point. For example, Friedman and Putnam (1978, p. 312) pass too easily from the *algebraic* claim of PV to the observation that (2) "is true when interpreted quantum logically, but inconsistent when interpreted classically." Stairs (1983), from my point of view, becomes needlessly embroiled in QPL "responses" to his QAL thesis. And, elaborating note 13, there is no possible rescue from a problematic QAL ensemble of measurable-value properties by introducing a mapping from the ensemble to "'probabilistic' truth values" of a multi-valued QPL.

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