7
Nonperturbative gluon mass and quantum solitons

7.1 Notation

In this chapter and Chapters 8 and 9, we work in Euclidean space with metric $\delta_{\mu\nu}$ so that a (real) Euclidean four-vector has $q^2 > 0$; timelike vectors in Minkowski space have negative squared length in this metric. To avoid a profusion of is and coupling constants g, we introduce the notation

$$\mathcal{A}_\mu = \frac{g}{\mathrm{i}} A_\mu = \frac{g}{\mathrm{i}} A_\mu^a t^a, \tag{7.1}$$

where (see Chapter 1) the t^a are the elements of the Lie algebra in the fundamental representation. Other changes include the following:

$$\begin{aligned} \frac{1}{q^2 - m^2 + \mathrm{i}\epsilon} &\to \frac{1}{q^2 + m^2}, \\ -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} &\to \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \\ \mathcal{D}_\mu &\to \partial_\mu + \mathcal{A}_\mu, \end{aligned} \tag{7.2}$$

where the left-hand sides refer to Minkowski space and the right-hand sides to Euclidean space. The field strength tensor is

$$\mathcal{G}_{\mu\nu} = [\mathcal{D}_\mu, \mathcal{D}_\nu] \tag{7.3}$$

so that the action is

$$S = -\frac{1}{2g^2} \int \mathrm{d}^4 x \, \mathrm{Tr}\, \mathcal{G}_{\mu\nu}^2. \tag{7.4}$$

7.2 Introduction

We have seen in previous chapters that infrared instability of NAGTs leads to generation of a dynamical gluon mass that removes the infrared singularities of

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perturbation theory. But mere removal of these singularities, important as it is, is far from the major role of gluon mass generation. In this and the next chapter, we survey the important implications of gluon mass generation for QCD-like theories. The most direct connection is that an infrared-effective action with dynamical gluon mass predicts a number of quantum solitons not present in the massless classical gauge theory. Condensates (i.e., a finite vacuum density) of these solitons have successfully explained[1] (if only qualitatively) many nonperturbative effects, as seen on the lattice, including confinement; generation of nonintegral topological charge; and chiral symmetry breaking (although we will only briefly consider this last topic).

A quantum soliton is a localized finite-energy configuration of gauge potentials arising from an effective action that summarizes quantum effects not present in the classical action; in our case, the effect is a gauge-invariant dynamical gluon mass. Like all effective actions, the one we use is not intended to substitute for the standard NAGT action, nor can it necessarily be quantized itself; it is treated classically. An effective action is merely a summary statement of a particular set of quantum effects; the underlying action of our NAGTs is always the conventional one as used, for example, in the Schwinger–Dyson equations to find the gluon mass. Perhaps the premier example in other fields of a quantum effective action is the Ginzburg–Landau scalar field action for superconductivity – which, in fact, has gluon mass generation (the Meissner effect) and solitons (the Abrikosov vortex) closely related to some of the ones we use. But the Ginzburg–Landau effective action is far removed from the original action, involving electrons and ions, that describes a superconductor.

7.2.1 Condensates and solitons

There is only one kind of soliton in classical QCD: the instanton (and its relatives, calorons and sphalerons). It is certainly possible, in principle, that there could be a finite density of instantons in the QCD vacuum. In that case, we speak of a *condensate* of instantons. Some of the properties of the instanton condensate would be captured in VEVs such as $-\langle \mathrm{Tr}\,\mathcal{G}_{\mu\nu}^2\rangle$ or other combinations of gauge-invariant field strengths that could possess a VEV, that are also referred to as condensates.[2] At a simple and incomplete level of understanding, one could find a formula relating the instanton density to the value of this VEV. But even if this VEV were to vanish,

[1] Instantons and the like in NAGTs have not proven suitable for explaining nonperturbative phenomena because it is far from clear how to tame the infrared instabilities of these solitons, which are allowed in arbitrarily large sizes classically.

[2] The phenomenological existence of such condensates was proposed long ago; see Novikov et al. [1], for example, who make reference to earlier works.

it would not in any essential way affect the possibility of the existence of one or a few instantons in the QCD vacuum.

Things are very different with the *quantum* solitons we discuss in this and the next chapter. Unless there is a condensate, as measured by a finite VEV, the solitons themselves will not exist. And once these solitons can exist, it is natural to think that these solitons themselves supply the condensate VEV. So the solitons and the condensates must exist together in a self-consistent way: it is impossible to have, for example, just one center vortex as the only soliton in the vacuum; there must be a condensate of solitons to have a condensate VEV. Although we study quantum solitons, such as center vortices, as if they were isolated, they exist only by virtue of a finite gluon mass, which summarizes the effects of a condensate whose properties we do not investigate further here. In any case, to be an important contributor to nonperturbative effects, any soliton must have a finite density in the vacuum (i.e., be part of a condensate).

One side of the self-consistent relation between quantum solitons and condensates is already clear from Lavelle's equation for the large-momentum behavior of the running gluon mass, given in Chapter 2 for the Euclidean PT propagator:

$$\hat{d}^{-1}(q) \to q^2 + c_d \frac{\langle -\mathrm{Tr}\, \mathcal{G}_{\mu\nu}^2 \rangle}{q^2}, \tag{7.5}$$

where both the constant c_d and the VEV are positive in $d = 3, 4$. If there is no condensate, there is no quantum soliton because gluon mass is essential for the soliton, and the condensate is essential for the gluon mass. The other side of the quantum soliton-condensate relation amounts to the statement that the entropy of a soliton of given action must exceed the action so that solitons can condense. There is a good thermodynamic analog. The canonical partition function $\mathcal{Z}$ of, say, a crystalline solid is

$$\mathcal{Z} = \exp[-\beta(U - TS)], \tag{7.6}$$

where β is the inverse temperature $1/T$, U the free energy, and S the entropy (so $U - TS$ is the Helmholtz free energy, minimized in equilibrium for an isolated system at constant temperature). In the QCD analog, S is still the entropy (of gauge configurations of given action), βU is the action, and g^2 is analogous to T. A strongly coupled gauge theory is like a condensed-matter system at high temperature. Clearly, a crystalline solid will melt at high-enough temperature, essentially because the entropy becomes so great as new configurations open up for the melted material that TS exceeds U. This melted phase does support a condensate of solitons (dislocations). Similarly, in QCD, we assume that the coupling is so large that the entropy of, say, a center vortex (per unit length in $d = 3$ or area in $d = 4$) exceeds the action per unit length or area. In this case, the free energy can decrease further by formation of more center vortices, and a condensate forms (its formation arrested only by interactions among the center vortices).

Various estimates, not given here, suggest that this dominance of entropy over action does take place for QCD in $d = 4$, and we can actually prove the existence of a condensate in $d = 3$, as shown in the next chapter. So $d = 3$ QCD-like theories confine just as do $d = 4$ theories.

We now take up the connection between a condensate of center vortices and confinement. Then, in the next chapter, we discuss the role of nexuses, monopole-like objects whose world lines lie on center vortex surfaces (and disorient them), in generation of nonintegral topological charge as well as the QCD sphaleron, a saddlepoint soliton associated with tunneling just as the usual sphaleron describes a tunneling barrier in electroweak theory.

7.2.2 What does confinement really mean?

Any proposed mechanism of confinement must be tested in detail; it is not enough to say vaguely that the mechanism keeps quarks and gluons from actually materializing. We are fortunate that computers allow us to look (at least in principle) at detailed tests not readily available in the real world, where Wilson loops are not much larger than the QCD scale length, quark loops can be broken by pair production, and the area law forces (linearly rising potential) may in certain cases be far from the most important part of the QCD binding forces.

To conduct these tests, we imagine an idealized world of NAGT gauge potentials but no matter fields at all. Confinement will be probed with large Wilson loops whose relevant length scales are large compared to the NAGT length scale in various group representations. Following is a list of some of the important tests of confinement. To keep this book to a manageable length, we will discuss just the first four criteria in this chapter (and finite-temperature issues in Chapters 9 and 10). For the other four, it is not always the case that both a detailed center-vortex picture and lattice simulations exist; for example, although there are lattice simulations of k-string tensions, there is yet no good theoretical understanding of the details of such string tensions in any confinement picture, and there are no lattice simulations on multiple Wilson loops, as called for in item 7. Where a center-vortex picture and lattice simulations both exist for a given criterion, there is good agreement. It would take us far too much space to detail this agreement. For a comprehensive review of this and other results up to 2003, see Greensite[2].

1. The leading term in the VEV of a fundamental-representation Wilson loop, all of whose scale lengths are large compared to the QCD length scale, is of the form $\exp[-K_F A]$, where A is the area of a minimal surface spanning the loop and K_F is the usual string tension.
2. For $SU(3)$ baryons made of static quarks, the baryonic area law is a Y-law, with the three-quark Wilson loop of the form $\exp\{-K_F[\sum A_i]\}$, where K_F is the fundamental mesonic string tension and the A_i, $i = 1, 2, 3$, are minimal areas of three Wilson loops having a common line. This is a nontrivial statement because the string tensions could have been a sum of pairwise forces. (The analogs for $SU(N)$, $N > 3$, depend on certain as yet uncalculated properties of k-string tensions; see criterion 5.)
3. The Wilson loop for the adjoint representation (or other representations with N-ality[3] zero) shows no area law extending to arbitrarily large distance; instead it shows *screening*. This means that the leading term is a perimeter term of the form $\exp[-ML]$ for some mass $M \sim m$ and loop perimeter L. However, there is an area law and a so-called breakable string for intermediate distances. At a distance roughly corresponding to storing enough energy in the string to allow formation of a gluon pair, the string breaks as this pair is formed by tunneling. Because it takes an energy of about $2m$ for gluon-pair formation, this breaking occurs at some finite distance, and the breakable string plays a role up to this distance.
4. It should explain the general characteristics of hybrids, which differ from qqq or $\bar{q}q$ states by having one or more extra valence gluons. The center-vortex picture is in good agreement with lattice simulations.
5. There should be plausible explanations for finite-temperature phenomena, including deconfinement, the mechanism for the thermal phase transition, chiral symmetry restoration, and generation of a magnetic mass in $d = 3$.
6. For $SU(N)$ with $N > 3$, there are k-string tensions K_k, subject to $K_k = K_{N-k}$, depending only on the N-ality k. The real test is to calculate the K_k, which has not yet been done accurately for any picture of confinement. (For center vortices, there are different areal densities $\rho_k = \rho_{N-k}$ that have yet to be calculated; their knowledge is tantamount to knowing the K_k.)
7. The exceptional Lie groups G_2, F_4, and E_8, whose centers are trivial, have no confined representations, only screened ones. In the center vortex picture,

[3] The N-ality is an integer equal to the number $n - \bar{n}$ mod N, where n is the number of fundamental representations and $\bar{n}$ is the number of antifundamental representations of $SU(N)$ whose product forms the given representation.

these groups each have a vortex solution that corresponds to the trivial element (the identity) of the gauge group; these behave just as explained in item 3 for N-ality = 0 representations in the gauge group $SU(N)$.[4]

8. A confinement mechanism should explain certain special cases such as for two disjoint, parallel, fundamental-representation Wilson loops as a function of separation between them: does the compound Wilson loop behave as if it were rings with soap films stretched on them?

There is not space here to detail all these criteria, and we concentrate on the first four. As for the rest; some but not all finite-temperature effects will be discussed in Chapters 9 and 10. It is worth pointing out here the basic reason for a finite-temperature transition to a deconfined phase in the center vortex picture [3]. As the length β of the Euclidean time direction shrinks with increasing temperature, the vortices of size m^{-1} are squeezed in this direction. When (up to a factor we will not estimate) $m \sim T$, vortices tend to point in the time direction because the action of spacelike vortices increases, and their entropy decreases because of the squeezing. Vortices pointing in the time direction cannot link to a Polyakov loop, so there is a transition to deconfinement. These vortices are not entropically constrained in the three spacelike directions and can condense, providing for mass generation in this $d = 3$ space.[5] Essentially, the same explanation was given later and confirmed by lattice simulations: confining vortices fail to condense (percolate) above a critical temperature [4, 5]. For interesting but not definitive work on k-string tensions, see Greensite and Olejnik [6] and [7], who show that center vortices are consistent but not yet predictive with what is known (from the lattice) about such tensions. The exceptional gauge group G_2 is discussed by Holland et al. [8, 9] and Pepe [10]. The last criterion – essentially about the tension in surfaces spanning Wilson loops – is particularly challenging for center vortices because the basic topological confinement mechanism of linkages of loops and center vortices is independent of any surface spanning a Wilson loop. Some progress has been made, using center vortices, on this problem (see also [11] and [12]), but there are no lattice results for comparison.

[4] Aside from the triviality of the center, there is a physical explanation why there is screening but not confinement in such theories. Consider G_2; all of its representations can be decomposed into sums of $SU(3)$ representations. The gluons lie in $8 \oplus 3 \oplus \bar{3}$, and so a string in any G_2 representation can break by production of $3, \bar{3}$ gluons. It takes three gluons to shield a G_2 "quark."

[5] For bosons, dynamical mass generation as discussed here is not damped by finite-temperature effects, whereas for fermions mass generation is damped, so the superconducting mass gap for superconductors vanishes above a certain temperature.

7.3 The quantum solitons

Three fundamental types of solitons are extrema of the NAGT effective action. The first is the center vortex of the present chapter.[6] In the next chapter, we present its close relative, the nexus [15, 16, 17, 18, 19, 20, 21], and modified versions of the instanton [14] and sphaleron [22]. Of these, the most important are the center vortex and the nexus, a monopole-like object whose world line is in fact embedded in a center vortex and therefore does not exist independently of center vortices.

All the solitons have idealized three-dimensional (static) realizations displayed by dropping the time integration in the $d = 4$ effective action S_{eff} and extremalizing the resulting $d = 3$ massive effective Hamiltonian[7]:

$$H_{\mathrm{eff}} = -\frac{1}{2g^2}\int \mathrm{d}^3x\, \mathrm{Tr}\,\mathcal{G}_{ij}^2 - \frac{m^2}{g^2}\int \mathrm{d}^3x\, \mathrm{Tr}\left[U^{-1}\mathcal{D}_i U\right]^2. \tag{7.7}$$

The coupling g^2 is evaluated at some low mass scale such as m^2 itself. Rescale the potential as done in Derrick's theorem: $\mathcal{A}_i(\vec{x}) \to (1/\rho)\mathcal{A}_i(\rho\vec{x})$. Then the first term of H_{eff} scales like $1/\rho$ and the mass term scales like $m^2\rho$. Using ρ as a variational parameter shows that S_{eff} must have solitons of size $\rho \sim m^{-1}$ and that H_{eff} itself scales like m/g^2. Note that the value of H_{eff} scales with the length of the z-axis, or more generally with the length of the closed string defining the vortex. Similarly, in $d = 4$, the vortex effective action scales with the area of the closed vortex surface (because of the reinstated integral over the time direction). All the solitons have $d = 4$ counterparts, whose form we will discuss as needed. However, it is in their $d = 3$ realization that we see most clearly their topological structure.

The gauge potential and the U-matrix are coupled at large distances. So that the energy of a soliton can be infrared finite, the integrand of both the $\mathcal{G}^2$ term and the mass term in S_{eff} must vanish at infinity. These requirements imply that $\mathcal{A}_i$ approaches a pure gauge at infinite distance, and this gauge is precisely U itself:

$$x_i \to \infty: \qquad \mathcal{A}_i \to U\partial_i U^{-1}, \tag{7.8}$$

as one sees from Eq. (7.11). Generally, the subleading terms vanish exponentially rapidly near infinity. The topological properties of a soliton are characterized by the behavior of U at large distance. These topological properties are most succinctly

[6] The center vortex was introduced as a mathematical construct by 't Hooft [13]. The first dynamical model based on gluon mass is given in [14]. For a review of the lattice properties of center vortices, see Greensite [2].

[7] This has the same form as the effective action $S_{\mathrm{eff}(3)}$ of $d = 3$ gauge theory, except that the coupling g_3^2 has dimensions of mass; thus, $S_{\mathrm{eff}(3)}$ is dimensionless.

expressed as certain homotopies, whose mathematical truth we take for granted in this book.[8] For us, the interesting question is the physical realization of these homotopies. So for each soliton, we begin by postulating a form for U that not only carries the homotopy but also provides instructions for the kinematic structure of the soliton in spatial and group coordinates.

All we need to know about homotopies at this point is that the homotopy group Π_1, which applies to the center vortex, describes the ways in which a closed one-dimensional string can be mapped on to a topological space. In physical terms, this is the homotopy that tells us the consequences of the topological linkage of the closed string of a center vortex (at a given time) with a Wilson loop. We find these consequences by studying the usual Wilson loop for a closed curve Γ but without taking its trace:

$$\mathcal{W}_\Gamma = P \exp\left[\oint_\Gamma \mathrm{d}x_\mu \, \mathcal{A}^\mu(x)\right]. \tag{7.9}$$

To preserve the single-valuedness of the gauge potential $\mathcal{A}_\mu(x)$ in the presence of vortices, we must – as shown later in this chapter – quantize the magnetic flux carried by center vortices, which causes the group matrix $\mathcal{W}_\Gamma$ to lie in the center of the group (the necessarily Abelian subgroup that commutes with all group elements). If $\mathcal{W}_\Gamma$ is in the fundamental representation, its elements are of the form of powers of $\exp[2\pi \mathrm{i}/N]$ times the $N \times N$ identity matrix for $SU(N)$. But in representations with N-ality zero, such as the adjoint representation, the elements of $\mathcal{W}_\Gamma$ are only the identity matrix itself.

Because the center group is Abelian, center vortices themselves appear at first glance to be simply Abelian objects and so are described by group matrices in the Cartan subalgebra of $SU(N)$. To the extent that center vortices are Abelian, they very much resemble earlier vortex constructions in physics, notably the Abrikosov vortex of type II superconductors and its relativistic generalization, the Nielsen–Olesen vortex of the Abelian Higgs model. In fact, as one might expect for a non-Abelian group, center vortices are much more complicated than these earlier vortices and actually must be thought of as participating fully in non-Abelian processes. But we need not study this now; it is the subject of the next chapter.

[8] Roughly speaking, homotopy groups $\Pi_n(X)$ classify the different ways an n-sphere can be mapped to a topological space X. The group Π_1 – the fundamental group – tells about the ways of mapping closed $d = 1$ loops into the space. For us, such a closed loop is a Wilson loop. As we will see in Chapter 8, Π_2 is relevant for nexus-vortex intersections (a source of topological charge) and Π_3 for various carriers of topological charge. For a review of homotopies and other topological subjects aimed at physicists, see Mermin [23].

7.4 The center vortex soliton

The field equations for S_{eff} are

$$[\mathcal{D}_j, \mathcal{G}_{ji}] - m^2\widetilde{\mathcal{A}}_i = 0 \tag{7.10}$$
$$[\mathcal{D}_i, \widetilde{\mathcal{A}}_i] = 0,$$

where the first equation comes from varying $\mathcal{A}_i$, the second equation from varying U, and $\widetilde{\mathcal{A}}_i$ is defined by

$$\widetilde{\mathcal{A}}_i = \mathcal{A}_i + (\partial_i U)U^{-1}. \tag{7.11}$$

It is important to note that the U equation of motion is not really independent; it is required by the identity

$$[\mathcal{D}_i, [\mathcal{D}_j, \mathcal{G}_{ij}] \equiv 0. \tag{7.12}$$

The equation for U has an interesting interpretation. After some algebra, one finds that it can be written

$$\partial_i(U\mathcal{A}_i U^{-1} + U\partial_i U^{-1}) = 0. \tag{7.13}$$

(The analogous equation also holds in $d = 4$.) This says that if the original gauge potential is in the Landau gauge (as, e.g., happens for center vortices), there is a gauge transformation U that preserves this gauge – in other words, a Gribov ambiguity. So if there is a condensate of center vortices, there is automatically a condensate of Gribov ambiguities.

There are solutions to the U equation of motion in perturbation theory, but they are of no interest for the solitons. They do, however, exhibit the long-range longitudinally coupled scalar excitations that are essential if a gauge-invariant gluon mass is to be generated. Because the gauge potential incorporates a power of the coupling g, this is an expansion in powers of $\mathcal{A}_i$ itself. Write $U = \exp\omega$, where ω is an anti-Hermitean function on the Lie algebra, and find [24]:

$$\omega = -\frac{1}{\nabla^2}\partial\cdot\mathcal{A} + \frac{1}{\nabla^2}\left\{\left[\mathcal{A}_i, \partial_i\frac{1}{\nabla^2}\partial\cdot\mathcal{A}\right] + \frac{1}{2}\left[\partial\cdot\mathcal{A}, \frac{1}{\nabla^2}\partial\cdot\mathcal{A}\right] + \cdots\right\}. \tag{7.14}$$

Clearly, this is a very complicated object because of the non-Abelian nature of the group. But for the standard Abelian center vortex that we now take up, U is a diagonal matrix that commutes with all its derivatives.

7.4.1 The standard Abelian center vortex

The simplest case [14] for the center vortex is an Abelian thick string extending along the entire z-axis. (We consider that the z-axis is a closed string, identifying the points at $\pm\infty$.) In this case, the center vortex is (with some important limitations) the Nielsen–Olesen vortex of the Abelian Higgs model in the limit of infinite Higgs-field mass. Only the massless Goldstone fields survive, to be eaten by the gauge field. The appropriate ansatz for U (equivalent to the angular, or Goldstone, part of the Higgs–Goldstone field) is

$$U = \exp[\mathrm{i}Q\phi] \qquad U\partial_i U^{-1} = \frac{Q}{\mathrm{i}\rho}\widehat{\phi_i}, \tag{7.15}$$

where ϕ, ρ are cylindrical coordinates and $\widehat{\phi_i}$ is a unit vector in the ϕ direction; Q is diagonal and one of a set of matrices in the Cartan subalgebra whose detailed properties we will soon give. Because Q is the only group matrix appearing in the standard center vortex, this vortex has no obvious non-Abelian properties; Q itself is equivalent to an Abelian charge and U to a simple phase factor.

To make the connection to the Abelian Higgs model, note that in that model, the Higgs–Goldstone field ψ is of the form $\psi = \chi\exp[\mathrm{i}\phi]$. The real (Hermitean) field $\chi(\rho)$ is the Higgs field; it has a vacuum expectation value v. The Goldstone field is ϕ and is massless. The coupling to the canonical Abelian gauge potential A_j is

$$\frac{1}{2}|\partial_j\psi - \mathrm{i}GA_j\psi|^2, \tag{7.16}$$

with G standing for the Abelian charge of the Higgs field; G is replaced by the diagonal matrix Q in the NAGT version. Suppose we freeze the field χ by taking the limit of infinite Higgs self-coupling λ at fixed v (or equivalently, at infinite Higgs mass). In the scalar coupling term $\lambda(|\psi|^2 - v^2)^2$, the limit tells us that $\chi = v$ everywhere. This is a singular limit because for finite λ, the equations of motion say that the field χ vanishes at the center of the vortex (along the z-axis, in our case). But in this limit, the Higgs-gauge potential coupling simply reduces to the Abelian version of the mass term in Eq. (7.7), with gauge particle mass[9] Gv. In the case of NAGTs with no Higgs fields, the effective vanishing of the Higgs field along the vortex axis is replaced by the requirement, coming from solving the Schwinger–Dyson equations, that the dynamical gluon mass vanish at large momentum or short distance. For the sake of brevity, we will not actually impose this condition on the solitons of this book; failure to do so leaves some logarithmic divergences in the gauged nonlinear sigma model part of the action that will be resolved when the short-distance vanishing of the mass is accounted for.

As is the case with all gauge solitons, the form of the gauge potential $\mathcal{A}_i$ follows from the form of the pure-gauge part. Equation (7.15) suggests the following simple

[9] In the non-Abelian case, it is also possible to interpret the gauged nonlinear sigma model mass term as coming from N frozen Higgs fields in the fundamental representation of $SU(N)$ [14].

kinematics for $\mathcal{A}_i$:

$$\mathcal{A}_i = \frac{Q}{\mathrm{i}}\widehat{\phi}_i F(\rho). \tag{7.17}$$

With this ansatz, $\partial_i \mathcal{A}_i = 0$, so the potential is in the Landau gauge; of course, any other gauge can be reached by a regular gauge transformation.

The corresponding pure-gauge potential $U\partial_i U^{-1}$ has no gauge fields almost everywhere, but there is a Dirac string (in this case, the z-axis) where there is a singular field strength.[10] We will see that the short-range parts of $\mathcal{A}_i$, the only parts depending on the mass, precisely cancel this string so that there is no such string in the full gauge potential $\mathcal{A}_i$. But the would-be Dirac string is essential for describing the topology and homotopy.

The U equation is

$$\nabla^2\phi = 0 \tag{7.18}$$

and is satisfied by the usual polar angle, except for the z-axis.[11] However, because ϕ is discontinuous along the z-axis, there is a singular magnetic field along this axis, or in other words, a Dirac string:

$$B_i = \epsilon_{ijk}\left[\nabla_j \times \frac{Q}{\mathrm{i}\rho}\widehat{\phi}_k\right] = \left(\frac{Q}{\mathrm{i}}\right) 2\pi\widehat{z}_i\delta(x)\delta(y). \tag{7.19}$$

There must be such a singularity to this solution of Laplace's equation somewhere in three-space. We now show that the equations of motion automatically cancel this singularity with an equal and opposite contribution from the short-range parts of $\mathcal{A}_i$. These equations are as follows:

$$\nabla^2\mathcal{A}_i = m^2(\mathcal{A}_i - \partial_i\phi), \tag{7.20}$$

and they have the solution

$$\mathcal{A}_i = \frac{Q}{\mathrm{i}}\widehat{\phi}_i\left[-mK_1(m\rho) + \frac{1}{\rho}\right], \tag{7.21}$$

where K_1 is the Hankel function of the first kind with an imaginary argument. As advertised, near the Dirac string ($\rho \simeq 0$), the singularity of the Hankel function exactly cancels the $1/\rho$ term; moreover, at large ρ, the K_1 term vanishes exponentially, and $\mathcal{A}_i$ approaches the required pure gauge.

[10] In other dimensions d, there is a Dirac hypersurface of dimension $d-2$, that is, of codimension 2.
[11] There are no perturbative contributions from the series in Eq. (7.14).

7.4.2 The general center vortex

To describe a center vortex generally, specify a closed line (surface) in $d = 3(4)$[12] and a matrix in the Cartan subalgebra that gives rise to a center element on parallel transport of the vortex's gauge function around a closed loop. We will only describe $k = 1$ vortices explicitly here because higher-k vortices are in some sense composed of $k = 1$ vortices.

In $d = 3$, the generalized center vortex is

$$\mathcal{A}_i(x; j) = \frac{2\pi Q_j}{\mathrm{i}} \epsilon_{iab} \oint_\Gamma \mathrm{d}z_a\, \partial_b \left[\Delta_m(x - z) - \Delta_0(x - z)\right], \tag{7.22}$$

where $\Delta_{m(0)}$ is the free $d = 3$ propagator for mass $m(0)$ and Γ is a closed curve; the coordinate z traces out the curve. It may not be obvious, but the massless propagator term is the generalization of the pure gauge U of Eq. (7.18) but with a Dirac string along the closed curve Γ.[13] With the identification of the Δ_0 term with the U term, it is easy to check that the field equations are satisfied by the vortex of Eq. (7.22). A simple calculation gives the field strength:

$$\mathcal{G}_{ab} = \frac{2\pi Q_j}{\mathrm{i}} \epsilon_{abc} \oint_\Gamma \mathrm{d}z_c\, m^2 \Delta_m(x - z). \tag{7.23}$$

This has an integrable logarithmic singularity on the Dirac string, which would be removed if we were to account for the short-distance vanishing of the mass. There is no singularity in the classical $\mathcal{G}^2$ action term, but the mass term has a logarithmic short-distance divergence that would be cured by the short-distance vanishing of the mass.

The generalization to $d = 4$ is equally simple:

$$\mathcal{A}_\mu(x; j) = \frac{2\pi Q_j}{\mathrm{i}} \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \partial_\nu \oint_\Gamma \mathrm{d}\sigma_{\alpha\beta}(z) \left[\Delta_m(x - z) - \Delta_0(x - z)\right], \tag{7.24}$$

where now the integral is over a closed two-surface Γ of surface element $d\sigma_{\alpha\beta}(z)$ and the propagators are four-dimensional. The field strengths are

$$\mathcal{G}_{\mu\nu}(x; j) = \frac{2\pi Q_j}{\mathrm{i}} \oint_\Gamma \mathrm{d}\,\widetilde{\sigma}_{\mu\nu}(z) m^2 \Delta_m(x - z), \tag{7.25}$$

where $\mathrm{d}\widetilde{\sigma}_{\mu\nu}$ is the dual surface element.

The massive propagators, in either dimension, approach the massless ones in the limit $m|x - z| \ll 1$, so the propagator singularities cancel on the would-be Dirac string or surface. (This shows, by the way, that for finiteness, one must choose

[12] For general dimensions, the Dirac singularity of the pure-gauge part of a center vortex has codimension 2 or ordinary dimension $d - 2$. So in $d = 2$, vortices are point particles.
[13] A function F_i can be both a gradient and a curl, provided it satisfies Laplace's equation.

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the same string or surface for the massive and massless terms in the vortex.) This cancellation gets rid of the Dirac string-surface in the full soliton. However, it will prove very useful for us later to have expressions for these Dirac string-surface field strengths because they give the cleanest view of the topology of these vortices and their interactions. From the field strengths in Eqs. (7.23) and (7.25) and from the propagators expressed as Fourier integrals, the Dirac would-be singularities emerge in the limit $m \to \infty$ – or otherwise said, the limit of zero vortex thickness – and give

$$d = 3: \qquad \mathcal{G}_{ab} = \frac{2\pi Q_j}{\mathrm{i}} \epsilon_{abc} \oint_\Gamma \mathrm{d}z_c\, \delta_3(x - z) \tag{7.26}$$

$$d = 4: \qquad \mathcal{G}_{\mu\nu}(x; j) = \frac{2\pi Q_j}{\mathrm{i}} \oint_\Gamma \mathrm{d}\widetilde{\sigma}_{\mu\nu}(z)\, \delta_4(x - z),$$

so they are just integrals of delta functions over the string-surface.

So far we have not explained why the string-surface must be closed. Suppose otherwise and consider the integral in $d = 3$:

$$\mathcal{A}_i(x) = \epsilon_{ij3}\, \partial_j \int_{-\infty}^{0} \mathrm{d}z\, \Delta_0(x - z), \tag{7.27}$$

which represents part of the center vortex but with an open string. Do the integral to find precisely the gauge potential of a Dirac monopole with a magnetic field $\sim 1/r^2$ and a Dirac string along the negative z-axis. We know there are no such long-range monopoles in QCD, so to exclude them, we use a closed string. (The same argument works for $d = 4$, using a surface with boundary rather than a closed surface. The boundary is a closed loop, the world line of the monopole.) This argument is certainly correct for Abelian vortices but not for non-Abelian vortices, as we describe in the next chapter for vortex junctions.

7.4.3 The Q-matrices and the center-vortex homotopy

Relation of confinement to homotopy Now we can see how to associate a homotopy with the group carried by Wilson loops themselves. A Wilson loop depends on a closed curve and a map of this curve to group elements:

$$\mathcal{W}_\Gamma^R = P \exp\left[\oint_\Gamma \mathrm{d}x_i\, \mathcal{A}_i\right], \tag{7.28}$$

where Γ is a closed curve and R labels the group representation for the loop. The argument in the exponent, $\mathcal{A}_i[x(t)]$, is a map of the closed curve described by $x_j(t)$, as t runs from 0 to 1, say, to an element of the Lie algebra of the gauge group. Exponentiation turns this into a map of the loop to the gauge group, and so

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the homotopy is the group of (Wilson) loops to the gauge group G, or $\Pi_1(G)$. Of course, Π_1 tells us to look for a wrapping of closed strings around this gauge group. One must be careful to appreciate that the gauge group G for gluons is not $SU(N)$ but $SU(N)/Z_N$, where Z_N, the group of integers, is the center of the gauge group. As far as gluons go, any group element of $SU(N)$ transforming it has exactly the same effect as if any element of the center group in this element were discarded. Equivalently, for the adjoint representation, every element of the center group is represented by the identity. So the homotopy associated with the center vortex is

$$\Pi_1(SU(N)/Z_N) \simeq Z_N, \tag{7.29}$$

which is not at all the same as $\Pi_1(SU(N)) \simeq \mathbb{I}$. These equations are not entirely trivial, but we will not pause to prove them here by the usual mathematical techniques. Instead, we give a physical demonstration.

Let the curve Γ be any closed path in the presence of a center vortex with Dirac string along the z-axis, as in the standard form of Eq. (7.21), and let all parts of the loop be far from the z-axis. At the Wilson loop, the only surviving part of the gauge potential is its pure-gauge part, as in Eq. (7.8), in which case, $\mathcal{W}_\Gamma^R$ is the product $U(i)U^{-1}(f)$, where i, f are the (identical) initial and final points of the contour Γ, respectively. These are the same points, but different curves between them gives different results. If the curve does not enclose the z-axis, then $\mathcal{W}_\Gamma^R = \mathbb{I}$, where $\mathbb{I}$ is the identity matrix. But if the curve does enclose it (is linked to it), the result is

$$\mathcal{W}_\Gamma^R = \exp[2\pi \mathrm{i} Q], \tag{7.30}$$

where Q is the representative of the group generator in representation R.

Here is the physics of the homotopy group. One of the most basic requirements we can place on the gauge potentials of an NAGT is that they be single valued. Imagine transporting a localized gluon wave function once around a curve Γ linked to the z-axis (and thus to the center vortex of Eq. (7.21)) and back to the starting position. This has to yield the same wave function. When Γ is far from the z-axis, the only contribution comes from the pure-gauge function U. As the gluon wave function is transported around the closed loop, it suffers a gauge transformation:

$$\mathcal{A}_i \to V\mathcal{A}_i V^{-1} + V\partial_i V^{-1}, \tag{7.31}$$

where V is the group element

$$V = P\exp\left[\oint \mathrm{d}x_i U\partial_i U^{-1}\right]. \tag{7.32}$$

With $U = \exp[\mathrm{i}Q\phi]$, this phase factor is just $V = U(\phi = 2\pi)U^{-1}(0) = \exp[2\pi\mathrm{i}Q]$. The only elements of $SU(N)$ that commute with all other elements

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are elements of the center, and we conclude that $\exp(2\pi \mathrm{i} Q)$ is in the center of the group.

The Q-matrices What matrices Q realize the center of the group in the fundamental representation? For $SU(N)$, the center is generated by the matrix[14] $Z = \exp(2\pi \mathrm{i}/N)$ and contains the elements $Z(k) \equiv Z^k$, $k = 1, 2, \dots N$. There are center vortices for every value of k, which we call k-vortices. The magnetic flux carried by a k-vortex is just k (in units of $2\pi/N$); because multiples of 2π are irrelevant, we need only define flux *mod* N. There are many traceless diagonal matrices Q for any value of k, and we need indices to distinguish them. For $k = 1$, the generator Z is the exponential of any of the matrices Q_j, $j = 1 \dots N$:

$$Q_j = \mathrm{diag}\left(\frac{1}{N},\ \frac{1}{N}, \dots \frac{1}{N},\ -1 + \frac{1}{N},\ \frac{1}{N}, \dots\right)\ j = 1, 2, \dots N, \tag{7.33}$$

with the –1 in the jth position.[15] Each of these matrices obeys

$$\exp[2\pi \mathrm{i} Q_j] = \exp[2\pi \mathrm{i}/N] \tag{7.34}$$

and so has unit flux. Because each Q_j can be transformed into any other by an element of the permutation group, one may think of the index j as a label for group collective coordinates of the vortex. These matrices are linearly dependent; the sum of all N of them vanishes. They obey the trace formula

$$\mathrm{Tr}\,(Q_i Q_j) = \delta_{ij} - \frac{1}{N}. \tag{7.35}$$

It might be thought that for a k-vortex, one replaces Q_j by kQ_j; however, the corresponding vortex has higher energy than one with the same space-time configuration but using the matrix $Q_j(k)$ defined as the sum of any k distinct Q_j. The energy of a $k = 1$ vortex (or action, in $d = 4$) scales with the factor

$$\frac{1}{g^2}\mathrm{Tr}\,Q_j^2 = \frac{N-1}{Ng^2}. \tag{7.36}$$

A k-vortex described by $Q_j(k)$ has flux k and energy proportional to

$$\frac{1}{g^2}\mathrm{Tr}\,Q_j(k)^2 = \frac{k(N-k)}{Ng^2}. \tag{7.37}$$

Note that this is symmetric under the exchange $k \leftrightarrow N - k$, which is equivalent to replacing a k-vortex by its antivortex, or conjugate vortex with flux $-k$ mod N.

[14] The identity matrix $\mathbb{I}$ is understood.
[15] Various useful properties of these matrices are given in [7].

This antivortex is just the $N - k$-vortex. Note also that this action factor is less than the action factor of k widely separated unit vortices, so in some sense, k-vortices are bound composites of unit vortices.

In $SU(N)$ for odd N, there are $(N-1)/2$ vortices plus their antivortices or conjugate vortices identical in all dynamical properties (except for their fluxes, which are opposite in sign) to those of their vortex partners. For even N, there is in addition a self-conjugate vortex. This property has special implications for $SU(2)$ and $SU(3)$, which are the only unitary groups with exactly one dynamically distinct vortex. For $N > 3$, there is always more than one type with generically different space-time density and other dynamical properties. Little is known about center vortices for $N > 3$.

One might think that center vortices are irrelevant at large N because their action S_C increases with N in the 't Hooft limit of Ng^2 fixed, and so $\exp(-S_C)$ vanishes strongly. However, for the leading N behavior, the collective-coordinate integral over the gauge group grows with N at exactly the rate needed to compensate for the action [25] (this also happens for other solitons with action growing like a power of N such as instantons [26]). What happens at nonleading orders remains to be settled, but there can well be the necessary cancellations that cause center vortices to persist at large N. We will, for other reasons, only discuss $N = 2, 3$ explicitly, and so the large-N behavior is a secondary issue.

7.4.4 Confinement

Confinement is a *topological* property of center vortices, with the homotopy of Eq. (7.29) directly realized as a linking of closed center-vortex Dirac strings with a Wilson loop.

If center vortices are to be responsible for confinement, there must be a condensate of them – that is, a finite density of vortices.[16] Such a condensate forms when the configurational (and other) entropy of vortices per unit length (in $d = 3$)[17] exceeds their action per unit length, in which case, a finite fraction of the vortices will have infinite length. A little thought shows that the vortex density is an *areal* density with the same codimension as the vortices. So in $d = 2$, k-vortices, which are point particles there, have a density ρ_k per unit area. By imagining that $d = 2$ is projected from higher dimensions, one sees that, in all dimensions, the vortex

[16] Recall that as long as the entropy exceeds the action, the vortices grow bigger and bigger; at some point, they fill enough of the space so that the entropy of an additional vortex segment diminishes because of the unavailability of unoccupied space. If this new segment were to try to occupy space that already had a vortex, the action would increase. The result is that growth terminates at a finite vortex density where the increase in action just balances the decrease in entropy.

[17] In other dimensions, substitute a closed hypersurface of codimension 2.

density is an area density. Although it is true that the density ρ_k of vortices and the density $\rho_{-k} \equiv \rho_{N-k}$ of conjugate vortices are always equal, there is no reason for the density to be independent of k; however, there are in fact good reasons for ρ_k to depend nontrivially on k. In $SU(2)$ and $SU(3)$, there is only one density, so these two groups give the easiest description of confinement (and fortunately, both are relevant to the real world).

The simplest semirealistic picture of center-vortex confinement begins with a flat Wilson loop, whose boundary is the flat curve Γ, *in the fundamental representation* of $SU(N)$; we drop the superscript R indicating this representation. Take this loop to be a sum of touching squares, each of side λ, and Γ is similarly approximated by a polygon of sides λ. The length λ is the correlation length of vortices such that there is only one vortex per λ-square. Any such square is pierced by a single vortex with probability p, and clearly

$$p = \rho\lambda^2. \tag{7.38}$$

Now assume that piercings in different λ-squares are statistically independent, which is not really true. However, correcting for this effect gives a quantitative but not a qualitative change in the picture of confinement. We wish to calculate the VEV $\langle W_\Gamma \rangle$ of this flat Wilson loop, bounded by the (also flat) curve Γ. Our interest is only in the area law part of this VEV, which comes solely from the long-range pure-gauge parts of the vortex (the massless propagator terms in Eqs. (7.22) and (7.24)). The massive propagator terms at best contribute perimeter terms to the VEV.

Begin by calculating the fundamental-representation Wilson loop (not yet its VEV) in the presence of a single k-vortex, described by the matrix $Q_j(k)$ and closed contour C. We need the trace of the path-ordered exponential:

$$W_\Gamma = \frac{1}{N}\mathrm{Tr}_F\, \mathcal{W}_\Gamma = \frac{1}{N}\mathrm{Tr}_F\, P \exp\left[\oint_\Gamma \mathrm{d}x_i\, \mathcal{A}_i\right], \tag{7.39}$$

and with the help of Eq. (7.22), we find

$$W_\Gamma = \mathrm{e}^{2\pi \mathrm{i} Q_j(k) Lk}, \tag{7.40}$$

where

$$Lk = \oint_\Gamma \mathrm{d}x_i\, \epsilon_{ijk} \oint_C \mathrm{d}y_j\, \partial_k \Delta_0(x-y). \tag{7.41}$$

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The expression for Lk happens to be the canonical expression for the Gauss linking number of two closed curves in $d = 3$ (which is why we call it Lk).[18] It is an integer counting the signed number of times that the curve C is linked to the curve Γ (see Kaufmann [27]).

The linking number has another interesting interpretation as an Abelian Chern–Simons term (see Chapter 9). The result, then, for the Wilson loop is

$$W_\Gamma = Z(k)^{Lk}. \tag{7.42}$$

Only the link number mod N has any significance for the Wilson loop, which, for a specific vortex, is a certain element of the center group. This is the dynamical realization of the homotopy in Eq. (7.29) that we sought.

For an ensemble of vortices, under the assumption of vortex independence, the Wilson loop VEV is an average of center-group elements, each of which is a product over the vortices in a particular member of the ensemble of the form $\prod_i Z_i^{Lk_i}$, where Z_i is an element of the center of the gauge group, as specified by the properties of the ith vortex (and the group representation of the loop itself).

The rest of this section, deals explicitly only with $SU(2, 3)$ for reasons given earlier. For the fundamental Wilson loop in $SU(2)$, the only nontrivial element of the center has $Z_i = -1$. In the preceding product, Lk_i is the Gauss linking number of this vortex with the Wilson loop. The Gauss linking number, a topological invariant, can be written through an integration by parts as an intersection number of the vortex and any surface spanning the Wilson loop; its (integral) value is independent of the choice of surface.[19] In the $SU(2)$ case, the necessary average is

$$\left\langle \exp\left[i\pi \sum_i Lk_i \right] \right\rangle. \tag{7.43}$$

We now make explicit the assumption that p is the probability that a vortex is actually linked once to a flat Wilson loop. When an odd number of vortices is linked once, the Wilson loop has value –1, and when an even number is linked, the value is +1. If the assumption is true, the area law follows from multiplying the probabilities $\bar{p} - p$ for all the λ-squares of the spanning surface so that

$$\langle W_\Gamma \rangle = (\bar{p} - p)^{A_\Gamma/\lambda^2} = \exp\left[-|\ln(1-2p)| \frac{A_\Gamma}{\lambda^2} \right], \tag{7.44}$$

where A_Γ/λ^2 is the number N_Γ of λ-squares in the Wilson loop. Another useful way of expressing this area law is to write out the combinatorics for vortex occupancy

[18] In other dimensions, the Wilson loop for a center vortex yields the Gauss linking number for linking of a closed 1-curve with a $d - 2$-dimensional closed hypersurface.

[19] Which raises the interesting question of why a *specific* surface occurs in the VEV; for a flat Wilson loop, the obvious and correct choice is the flat surface spanning the loop.

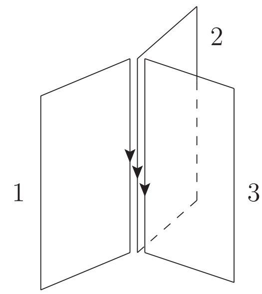

Figure 7.1. A baryonic Wilson loop in $SU(3)$ is composed of three simple Wilson loops sharing a common central line (expanded in the figure). The central line is invisible to $SU(3)$ center vortices.

of N_Γ sites of a surface spanning a Wilson loop Γ,

$$\langle W_\Gamma \rangle = \bar{p}^{N_\Gamma} - N_\Gamma \bar{p}^{N_\Gamma - 1} p + \frac{N_\Gamma(N_\Gamma - 1)}{2} \bar{p}^{N_\Gamma - 2} p^2 + \cdots$$
$$= (\bar{p} - p)^{N_\Gamma} = (1 - 2p)^{A_\Gamma/\lambda^2}, \tag{7.45}$$

as before. Here each term represents the number of ways of arranging empty and once-filled λ-squares.

The result for $SU(3)$ is also an area law but with a different string tension because of different center-group phase factors. It is easy to see that $\bar{p} - p$ should be replaced by $\bar{p} + \cos(2\pi/3)p = 1 - (3/2)p$, where $\cos(2\pi/3)$ is the average of the vortex and conjugate vortex center elements.

We have assumed no correlations other than mutual repulsion between vortices, but in fact, other correlations do exist. In particular, a vortex has a finite chance of reentering a Wilson-loop spanning surface a few steps after piercing it the first time; this dilutes the effective density of actually linked vortices below the density ρ of vortices piercing the flat spanning surface, as shown in [11]. This dilution has been observed in lattice simulations with center vortices [28].

Another issue for $SU(3)$ is the form of the area law for baryons formed from infinitely heavy static quarks; the corresponding Wilson loop is shown in Figure 7.1.

There are a priori two interesting possibilities: a sum of areas between each of the three quark pairs (e.g., surface 4, spanning quark lines 1 and 3, with folds at the two dotted lines) or a three-bladed minimal area joining each quark line to a central Steiner line, often termed the *Y-area law*. In Figure 7.1, the Steiner line is shown as three coincident lines from each of three elementary Wilson loops. A straightforward extension of the linking arguments already given for the elementary Wilson loop shows that the second possibility is correct [29]. Note that in $SU(3)$,

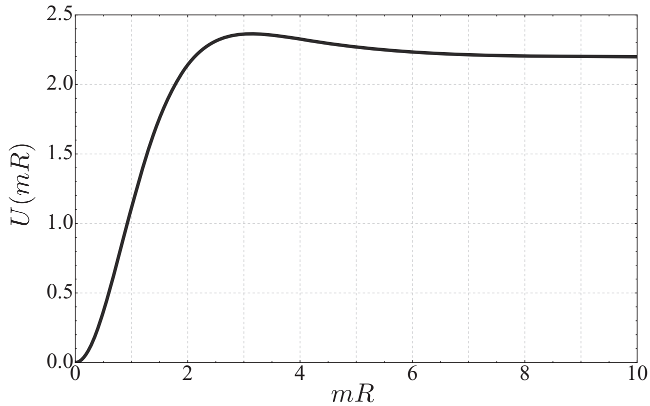

Figure 7.2. Adjoint potential $U(mR)$ in a $d = 2$ center-vortex model.

these three coincident lines always give the identity element of the center group. The Y-area law has been confirmed on the lattice [30, 31, 32, 33].

7.4.5 Screening

Physically, screening means that gluons or other N-ality zero fields have a string between them, as do quarks, but the string breaks when enough energy has been stored in it to materialize another gluon pair. Of course, this is the same thing that happens for quarks when they are included as dynamical fields. A model of this adjoint string breaking has been worked out in [25], with results for the potential $V(R) = (K_F/m)U(mR)$ shown in Figure 7.2. The model is in $d = 2$, but the NAGT action used is not the conventional $d = 2$ action; instead, it has a mass term added (and so is the same as Eq. (7.7), except for the integration, which is over two-space, and the coupling, which has dimensions of mass). The center vortex is a point particle described by the wave function

$$\mathcal{A}_i(x) = \left(\frac{2\pi Q}{\mathrm{i}g}\right)\epsilon_{ik}\partial_k[\Delta_M(x - x_0) - \Delta_0(x - x_0)], \tag{7.46}$$

where x_0 is the center of the vortex. A condensate of these is simply a condensate of point particles in the plane. A simple closed Wilson loop is linked to a vortex if the vortex is inside the loop and is unlinked otherwise. However, if the Wilson loop is in the adjoint representation, linkages contribute trivially to the loop VEV, and we can drop the massless pure-gauge part of Eq. (7.46). The short-range massive parts contribute only if they are within a distance $\sim 1/m$ of the loop, whether inside it or outside. Consider now the contribution of these vortices to an adjoint Wilson rectangle with spatial width R. After integrating over all vortex positions x_0 and

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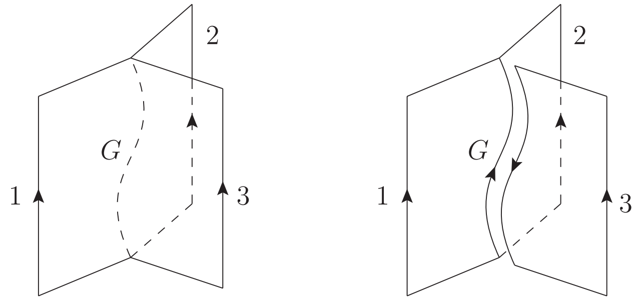

Figure 7.3. (*Left*) Wilson loop for a $qqqG$ configuration (quarks labeled 1, 2, and 3; gluon labeled G). (*Right*) The same loop with the gluon line decomposed into another quark and antiquark line, with quark line 3 singled out, as discussed in the text.

other collective coordinates [34, 25], the adjoint potential of Figure 7.2 emerges. It is the same for $SU(2)$ and $SU(3)$. Note that the center-vortex potential is always positive, is roughly linear in R up to distances $\sim 1/m$, and then breaks (becomes constant) at a distance of about 1 fm. The breaking height of about $2.4K_F/m$ should be of order $2m$ to materialize a gluon pair at breaking, suggesting that $m \simeq K_F^{1/2}$ – a value that is hardly quantitative but in the right ballpark.

Studying the adjoint potential on the lattice has been difficult, apparently because of a poor overlap between the physical gluonic state and the corresponding adjoint Wilson loop and because of the large distance to string breaking. But it is claimed that breaking has been observed, at least in $d = 3$ $SU(2)$ gauge theory (see, e.g., Kratochvila and de Forcrand [35]).

7.4.6 Hybrids

Finally, the role of a dynamical gluon mass (along with center vortices) is apparent in lattice simulations [36] of a heavy-quark baryonic hybrid, a bound state of three infinitely heavy quarks and a single valence gluon. The corresponding Wilson loop is shown in Figure 7.3.

In Figure 7.3 (*left*), the gluon occupies what would be the Steiner line for an ordinary baryon, but of course, the gluon world line, not being infinitely massive, fluctuates. This gluon line can be decomposed into a $q\bar{q}$ pair, as shown on the right, indicating mixing between the hybrid and a baryon plus meson. In the lattice simulations [36], the authors claim a clear signal of a mass of some 600 MeV for the valence gluon; estimates [37] based on a modified form of the Y-law for normal baryons, plus shorter-distance corrections, are in good agreement with the lattice data over a range of separations of the three quarks.

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