CORRELATIONS OF GLOBULAR CLUSTER PROPERTIES AS CONSTRAINTS FOR DYNAMICAL EVOLUTION MODELS

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Abstract.

Correlations between globular cluster (GC) properties are reviewed, including some new work on cluster tidal radii and densities. Most of the observed correlations can be interpreted within the framework of our current understanding of their dynamical evolution. These correlations provide empirical constraints for models of GC formation and evolution.

1. Introduction

Distributions of, and correlations between various global properties of globular clusters (GCs) can provide important observational constraints for models of their evolution, or even formation. Their studies may be the only way in which we can address the *global* picture of GC evolution.

The subject has been covered extesively in several recent papers: van den Bergh (1994ab, 1995), Djorgovski & Meylan (1994; hereafter DM), Djorgovski (1995), and Bellazzini *et al.*(1995); see also Surdin, this volume. Instead of repeating much of what was covered there, we will only give a brief summary of the results from these studies in Sect. 2, and discuss new correlations of tidal-radius related quantities in Sect. 3. The reader may wish to start with the paper by DM, and references therein, and consider this review as an update to it. A comparison of GC properties with those of early-type galaxies was presented by Djorgovski (1993a).

Most GC properties in general, and relaxation times in particular, span a large dynamical range, e.g., a factor of $\sim 10^5$ in the central relaxation time, t_{rc}. That practically guarantees that a large range of dynamical evolution states will be present in the sample, which makes it possible to use

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GC properties and their correlation as direct probes of their dynamical evolution. Likewise, GCs cover a range of a factor of $\sim 10^2$ in Galactocentric radius, R_{GC}; given the flat rotation curve for the Galaxy, that translates in a comparable range in disk or bulge crossing frequencies. That makes it possible to study the differential effects of tidal shocks from disk and bulge passages over the sample as a whole.

The observed properties of GCs today are a complicated product of the initial conditions (reflecting the formative processes of GCs), and some 15 Gyr of dynamical evolution, which can be driven both by the internal instabilities, such as the core collapse and mass segregation, and the external effects of the Galactic tidal field. Internal dynamical evolution can be modulated (typically accelerated) by the tidal shocks. One thus expects a complex picture. Physical processes which affect more than one observable property will generate correlations; for example, core collapse will simultaneously change the core radii, r_c, central densities, ρ_0, or surface brightness, μ_0, and concentrations, c. A complex interplay of physical effects can then result in multivariate correlations.

Nevertheless, it is possible to interpret and disentangle some of these effects. Core properties are presumably dominated by the effects of core collapse, since $\langle t_{rc} \rangle <<$ cluster ages. Cores thus have no memory of the initial conditions. On the other hand, theory suggests that most half-light properties change little during the core collapse. Tidal effects may be found in correlations with R_{GC}, or the distance to the Galactic plane, Z_{GP}. Cluster luminosities (absolute magnitudes, M_V), orbits (i.e., R_{GC}), and velocity dispersions, σ, may be changed very little by the dynamical evolution for most clusters, and thus may reflect the formation processes (Murray & Lin 1992). Certainly the metallicities, [Fe/H], are primordial.

2. A Brief Summary of the Previous Work

DM have analysed a subset of the data compiled by Djorgovski (1993b). Some of their results are as follows:

The dynamical range the core parameters, which are presumably more affected by dynamical evolution, is much larger than for the corresponding half-light parameters. The more rapid evolution of core regions breaks the homology of cluster structures, and introduces the shape parameter in the GC sequence, the King c. This process can be contemplated in looking at Fig. 1. Presumably GCs evolve by sliding down and to the left on these correlations. Note that for a typical GC, the time between disk (or bulge) passages is $\sim 10^8$ yr, which is roughly the value of the half-mass relaxation time, t_{rh}, where the deviation $t_{rc}/t_{rh} \rightarrow 0$ really sets in. From the distribution of GC relaxation times, one can estimate the present rates of GC core

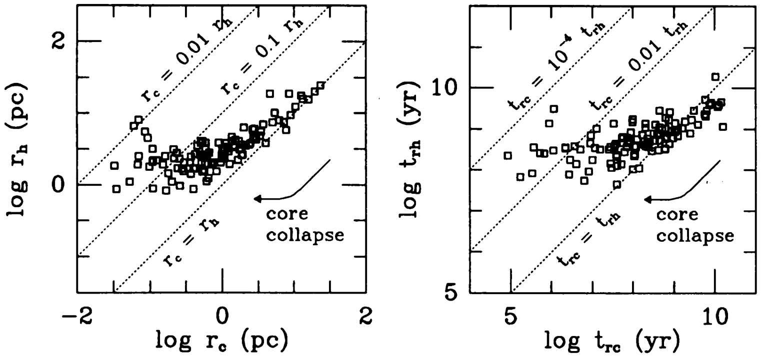

Figure 1. A comparison of core and half-light radii (left) and relaxation times (right) for Galactic GCs. At the slow evolution end (large radii and relaxation times), the core and the half-hight parameters of clusters are similar. They deviate from this ever more strongly as the relaxation times decrease, as clusters approach the core collapse.

collapses and GC evaporation in the galaxy, both being of the order of a few GCs per Gyr (Hut & Djorgovski 1992).

The best observed correlations are between the core parameters: core radius, r_c, central surface brightness, μ_0, and the concentration, c, as well as any derived quantities, such as the central relaxation time, t_{rc}. Two of them are shown in Fig. 2. The correlations are exactly as what may be expected from a population of GCs approaching the core collapse. The observed scaling relations are:

$$I_0 \sim r_c^{-1.8\pm0.2} \qquad \rho_0 \sim r_c^{-2.6\pm0.15}$$

where I_0 is the central surface brightness in linear units, rather than magnitudes, and ρ_0 is the central luminosity density, both in the V band. Note that for a single-mass-species cluster, the exponent in the second relation should be -2.23 from the standard core collapse theory (Lynden-Bell & Eggleton 1980, Cohn 1980), marginally different from what is observed; perhaps this reflects the real mass spectrum of the stars. These scaling relations imply $L_{core} \sim r_c^{0.3\pm0.2}$, possibly reflecting the accelerated evaporation or ejection of stars from a collapsing core.

The core parameters and concentrations also correlate with the position in the Galaxy, with clusters closer to the Galactic center or plane being more concentrated and having smaller and denser cores (cf. also Chernoff & Djorgovski 1989, and Djorgovski 1993a). These trends are *much* more pronounced for the fainter (less massive) clusters. This is in an agreement with a picture where tidal shocks form disk or bulge passages accelerate dynamical evolution of clusters (cf. Chernoff & Weinberg 1990, Aguilar

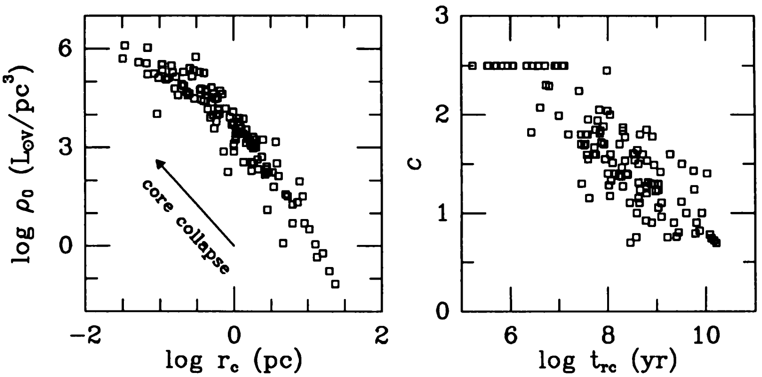

Figure 2. *Left:* The correlation between core radii (r_c) and central luminosity densities (ρ_0). *Right:* Dependence of cluster concentrations (King c) on the central relaxation time (t_{rc}). PCC clusters have been set to $c = 2.5$, and their t_{rc}'s are greatly uncertain. Both trends are expected in a simple picture of globular clusters as a collapsing sequence of King models.

1993, and references therein). The mean trends for the binned data are:

$$r_c \sim R_{GC}{}^{1.1\pm0.5} \qquad \rho_0 \sim R_{GC}{}^{-2.8\pm1.6}$$

Since for a flat Galaxy rotation curve the frequency of disk passages scales as $R_{GC}{}^{-1}$, these scaling relations imply shrinking of the GC cores roughly in a direct proportion to the frequency of tidal shocks.

It is also worth noting that for almost all clusters, half-mass relaxation times, t_{rh} are comparable to, or greater than the time intervals between the disk passages, t_{DC}, as estimated from R_{GC} by assuming a flat Galaxy rotation curve.

There are no good correlations of other parameters with luminosity, although more luminous clusters tend to be more concentrated (cf. also Djorgovski 1991, and van den Bergh 1994c). When data are binned in luminosity, several trends emerge: more luminous clusters tend to have smaller and denser cores. The corresponding scaling relations are:

$$r_c \sim L^{-0.5\pm0.25} \qquad \rho_0 \sim L^{2\pm1}$$

where L is the cluster total luminosity in the V band. Possibly this may reflect the initial conditions (Bellazzini *et al.*1995).

Cluster metallicities do not correlate with any other parameter, including luminosity and velocity dispersion (a behavior strikingly different from that of elliptical and dwarf galaxies; see Djorgovski 1993a). The only detectable trend is with the position in the Galaxy, probably reflecting Zinn's

(1980) disk-halo dichotomy. Along with their great chemical homogeneity and the narrowness of the main sequences, this suggests that GCs were not self-enriched systems, but that they formed from a pre-enriched material within larger structures (former dwarf galaxies or protogalactic fragments).

Central velocity dispersions, σ, show excellent correlations with luminosity and surface brightness. Their origin is not well understood, but they may well reflect initial conditions of cluster formation. The corresponding scaling relations are:

$$\sigma \sim L^{\,0.60\pm0.15} \qquad \sigma \sim I_0^{\,0.50\pm0.10} \qquad \sigma \sim I_h^{\,0.45\pm0.05}$$

where I_h is the mean central surface brightness within the r_h, in linear units. Core radii and concentrations play a role of a "second parameter" in these correlations. Djorgovski (1995) has extended this analysis to obtain bivariate correlations analogous to the "Fundamental Plane" (FP) of elliptical galaxies. For the core parameters, the bivariate scaling relation is:

$$r_c \sim \sigma^{\,2.0\pm0.1}\, I_0^{\,-1.1\pm0.1}$$

which is exactly what may be expected from the virial theorem, if all GC cores have a similar structure (which they do), and uniform (M/L) ratios. For the half-light parameters, the bivariate scaling relation is:

$$r_h \sim \sigma^{\,1.45\pm0.2}\, I_h^{\,-0.85\pm0.1}$$

which is very close to the corresponding FP relation for elliptical galaxies. Since the (M/L) ratios are unlikely to vary a lot among GCs, the culprit is clearly their variety of density profiles, with the concentration correlated with other parameters. This is an important lesson for the physical interpretation of the FP of ellipticals itself.

A multivariate statistical analysis of the entire data set shows that the global manifold of cluster properties has a high statistical dimensionality, $D > 4$ (cf. also Djorgovski 1981). However, a subset of structural, photometric, and dynamical parameters forms a statistically 3-dimensional family, as expected from objects following King (1966) models, which DM propose to call the King Manifold.

Clusters with post-core-collapse (PCC) morphology (Djorgovski & King 1986) participate in the same correlations as the clusters with King model (KM) morphology. Operationally, their core radii were set to the observed HWHM in the surface brightness profiles, which is just an upper limit. It may be fortuitous that seeing effects move the data points roughly along the observed correlations.

This analysis did not reveal presence of any distinct subgroups within the Galactic GC system, beyond the disk-halo dichotomy, and the possible

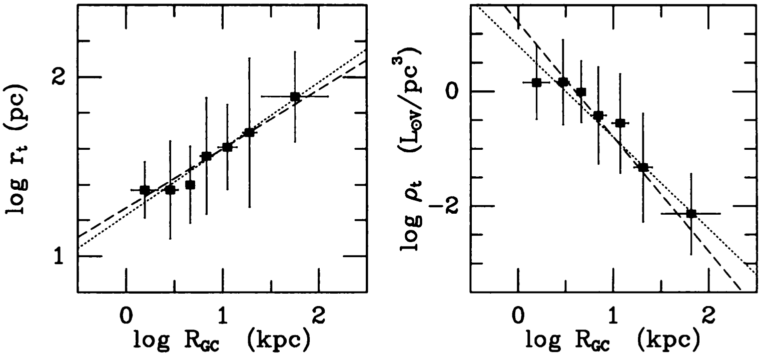

Figure 3. Median estimated tidal radii, r_t, and mean luminosity densities within the r_t, ρ_t, in bins of 21 clusters each (fewer in the last bin). The "error" bars indicate quartile-estimated sigma for each bin, in both coordinates. The best fit scaling laws, $r_t \sim R_{GC}{}^{0.37}$ and $\rho_t \sim R_{GC}{}^{-1.6}$ are indicated with dotted lines. Alternative, nearly as good fits, $r_t \sim R_{GC}{}^{1/3}$ and $\rho_t \sim R_{GC}{}^{-2}$ are indicated with dashed lines.

groups identified by van den Bergh (1993). This is curious, given that many GCs may have been acquired or produced in accretion events in Galaxy's history. Apparently, dynamical evolution processes, which should be universal, drive most of the observed correlations.

It is now becoming possible to conduct similar global statistical studies for the GC systems in the Magellanic Clouds (cf. Meylan & Djorgovski 1987), in M31, where HST resolution is necessary (cf. Fusi Pecci *et al.*1994), or even beyond. Comparative studies of GC systems in other, nearby galaxies can add considerably to our understanding gained so far from the Galactic GC system alone.

3. Correlations Involving Tidal Radii and Densities

There are very few measured tidal radii for globular clusters: this is an extremely difficult task. A recent attempt to measure some was made by by Grillmair *et al.*(1995). Lacking real measurements, very rough estimates may be obtained as $\log r_t = \log r_c + c$, using the data from Djorgovski (1993b).

Mean cluster densities ρ_t within r_t can then be obtained from their total luminosities. These must be higher than the local dark halo density at the perigalacticon:

$$\rho_t^{\mathrm{cluster}} \approx \alpha\, \rho^{\mathrm{halo}}(R_{peri})$$

where $\alpha \approx 5.5$ (see below). Innanen *et al.*(1983) find that:

$$\frac{r_t}{R_{peri}} = \frac{2}{3}\left[\frac{m_{cluster}}{(3+\epsilon)M(R_{peri})}\right]^{1/3}$$

where ϵ is the orbital eccentricity, and $M(R_{peri})$ is the enclosed Galactic mass within R_{peri}. For a Galaxy with a flat rotation curve $V = const.$, $M(R)/R = V^2/G$. Assuming $\epsilon \approx 0$, it is easy to derive:

$$R_{peri} = \alpha\, V\, (4\pi G)^{-1/2}\, \rho_t^{-1/2}$$

where $\alpha = (243/8)^{1/2}$. For simple estimates,

$$R_{peri} \approx 0.95\, \alpha\, \rho_t^{-1/2} \approx 5.2\, \rho_t^{-1/2}$$

where R_{peri} is in kpc, and ρ_t is in $M_\odot/pc^3$.

Trends of r_t and ρ_t with the Galactocentric radius are shown in Fig. 3 for binned data. Tidal radii and the mean densities within r_t decline at smaller Galactocentric radii, as expected. The mean trends are:

$$r_t \sim R_{GC}{}^{0.37\pm0.05} \quad (\approx R_{GC}{}^{1/3})$$

$$\rho_t \sim R_{GC}{}^{-1.6\pm0.2} \quad (\approx R_{GC}{}^{-2}?)$$

The second relation nearly mimics the average density law of the galaxy, implied by the flat rotation curve, $\rho \sim R_{GC}{}^{-2}$, which is gratifying to see. Also, at a given R_{GC}, more luminous clusters tend to have larger r_t's.

The ratios of the present Galactocentric distances R_{GC} and the derived perigalactic radii R_{peri} are a statistical measure of the shapes of globular cluster orbits: for nearly circular orbits, $R_{peri} \approx R_{GC}$; for eccentric/plunging orbits, $R_{peri} < R_{GC}$. Such an estimate may be very uncertain, especially for the PCC clusters, whose derived tidal radii, and thus the perigalactic radii are more uncertain, or the clusters near the Galactic center, say at $R_{GC} < 2$ kpc, where the errors in distances and the R_{GC}/R_{peri} ratios become too high. A "clean" sample of clusters with $c < 2.1$ and $R_{GC} < 2$ kpc may be safer for this analysis.

¿From the distribution shown in Fig. 4, it appears that most clusters are on nearly circular orbits (or at least not very eccentric ones).

We can now explore correlations for clusters with statistically different orbits. Using the "clean" sample of clusters with $c < 2.1$ and $R_{GC} < 2$ kpc, and dividing the sample as: (1) $R_{GC}/R_{peri} \leq 1.5$, clusters with nearly circular orbits, and (2) $R_{GC}/R_{peri} > 1.5$, clusters with plunging/eccentric orbits, we find the following trends:

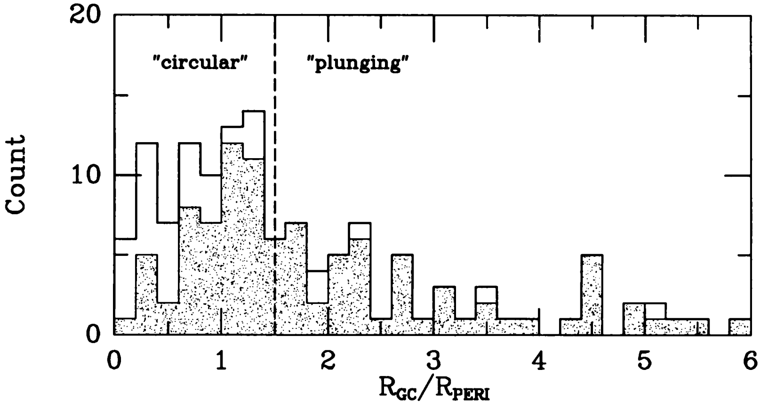

Figure 4. Distribution of the estimated R_{GC}/R_{peri} ratios for Galactic GCs. Clusters on more elongated orbits should have on the average larger R_{GC}/R_{peri}. The open histogram is for all GCs in the samlpe, and the shaded histogram for the "clean" sample with $c < 2.1$ and $R_{GC} < 2$ kpc.

At a given R_{GC}, "plunging" clusters tend to have smaller radii and higher densities, both core and half-light, and tend to be slightly more concentrated. Also, most correlations with R_{GC} are better for the "circular" sample; i.e., the "plunging" sample shows more scatter. A possible interpretatin of these trends is that the effects of tidal shocks appear to be slightly stronger for the "plunging" clusters, both in terms of their tidal truncation, and the acceleration of their dynamical evolution towards the core collapse.

Correlations with velocity dispersion tend to be slightly better for the "circular" sample (there is even a correlation with the cluster concentration). There are no obvious differences in the (non)correlations involving cluster metallicities.

A similar analysis was presented by van den Bergh (1995), who claims that there are correlations between the cluster perigalactic radii and metallicities or ages. Van den Bergh (1994a) also suggested that GCs on retrograde orbits tend to have half-light radii lower than average at the same R_{GC}, whereas GCs with circular orbits tend to have larger half-light radii.

In this analysis we made gross approximations, such as the circular orbits. The reality may be much more complicated, as suggested, e.g., by the recent study by Dauphole *et al.*(1995). More realistic estimates of both GC tidal radii and perigalactica may reveal more interesting correlations.

4. Concluding Remarks

The situation seems encouraging: the observed correlations largely bear out the expectations from our theories, at least qualitatively, and in many cases quantitatively as well. What remains to be done is to compare the predictions with the observations in some detail. Modeling the effects of tidal shocks and GC evolution in a realistic tidal field of our Galaxy may be quite interesting. There are also correlations, most notably those involving velocity dispersions, which were not clearly expected from theory, and are not yet really understood.

Observationally, the greatest need remains for better distances to most clusters. Other parameters can also be substantially improved in many cases. Actual measurements of GC tidal radii and envelopes (as opposed, e.g., to crude estimates made in this work) are also a high priority; the methodology used by Grillmair *et al.*(1995) is very promising for this task. Direct observational constraints on GC orbits, e.g., from proper motions, will also be very useful for modeling of tidal effects.

The structure of collapsed cores remains murky, and core radii for highly evolved (concentrated) clusters are not well defined, even with the HST data. Cores for PCC clusters may never be well defined. It may be better then to institute a new radial scale for the "core" regions of GCs, which would be operationally well defined. One possibility is to use fixed fraction of the total light radii, e.g., radii enclosing 10% of the total light (in projection). Similarly, tidal radii, which may be equally difficult to define and measure, could be replaced with the radii enclosing 90% of the total light. Their ratios could define new concentration index, $c_* = \log(r_{90}/r_{10})$. This can be done, e.g., with the profiles compiled by Trager *et al.*(1995).

This review did not address the correlations between stellar populations and dynamical structure of GCs (Djorgovski *et al.*1991, Fusi Pecci *et al.*1993; see Djorgovski & Piotto 1993 for a review and references). This also includes the possible dependence of the formation rates of millisecond pulsars and LMXBs on the cluster parameters, the origins of blue stragglers, etc. This is a fascinating area of GC research, in which much progress yet remains to be made, both observationally (especially with the HST; see several excellent papers in this volume) and theoretically, through detailed simulations of stellar collisions and tidal interactions in dense cluster cores.

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