Chapter 1

INTRODUCTION

This book is a contribution to *descriptive inner model theory*, which is the area of set theory that lies between descriptive set theory as developed in [24] and inner model theory. The main goal of this book is to advance the descriptive inner model theoretic methods to the level of the Largest Suslin Axiom (LSA), which is a strong determinacy axiom asserting that there is a largest Suslin cardinal and that the largest Suslin cardinal is a member of the Solovay sequence. In more concrete terms, our goal is twofold: Firstly develop methods for analyzing the minimal model of LSA, and secondly, develop methods for building the minimal model of LSA under various hypotheses such as the Proper Forcing Axiom or Large Cardinals. Since the introduction of Steel's recent book [63], the expository paper [28] and the introduction of [36] contain all the introductory information we need, here we will not introduce the subject matter of this book and instead will hope that the reader has consulted these sources.

The first problem is an instance of the problem Steel mentions on page xii of [63] where he writes: "The most important of the remaining open problems is whether, assuming determinacy, there actually are mouse pairs at every appropriate level of logical complexity". Theorem 10.1.2 shows that the aformentioned problem has a positive solution in the minimal model of LSA. As explained in any of the sources cited above, the goal for doing this is to show that letting Θ be the least ordinal that is not a surjective image of the reals, V_Θ^{HOD} as computed inside a determinacy model is a hod premouse. The above sources explain the importance of having a hod premouse representation of V_Θ^{HOD}.

The second problem amounts to advancing the Core Model Induction to the level of LSA. Corollary 12.0.22 constructs the minimal model of LSA assuming PFA. More dramatically, the paper [36], which extends the methods of this book, demonstrates that the Core Model Induction, in its current form, cannot be used to go much further than LSA.

Corollary 12.0.22 also builds the minimal model of LSA directly from large cardinals, namely strongly compact cardinals. However, Theorem 10.3.1 shows that LSA is weaker than a Woodin cardinal that is a limit of Woodin cardinals, and so strong compactness seems to be much more than needed. Nevertheless, while it is widely believed that strongly compact cardinals are consistency wise stronger

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than a Woodin cardinal that is a limit of Woodin cardinals, this is not yet known. Still we strongly believe that the methods developed in this book, the methods of [1] and the main theorem of [26] can be used to show that assuming the existence of a Woodin cardinal that is a limit of Woodin cardinals, the minimal model of LSA exists (cf. Definition 1.1.4).

Historically, LSA was introduced by Woodin in [68, Remark 9.28], and it features prominently in Woodin's Ultimate L framework (see [69, Definition 7.14] and Axiom I and Axiom II on page 97 of [69][1]). Theorem 10.3.1 is historically the first proof of the consistency of LSA relative to large cardinals. Cramer and Woodin established the consistency of LSA from large cardinals in the region of I_0 (see [5, Theorem 65]).

1.1. The technical content of the book

1.1.1. The Largest Suslin Axiom. LSA is a determinacy theory whose underlying theory is Woodin's AD^+. Chapter 9.1 of [68] provides a quick overview of AD^+, and Larson's recent manuscript [20] provides more details. Perhaps the most important consequence of AD^+ is the fact that assuming $V = L(\wp(\mathbb{R}))$, the fragment of V coded by the Suslin, co-Suslin sets of reals is Σ_1 elementary in V (see Theorem 9.7 of [68]).

We will need the following concepts to introduce LSA. A cardinal κ is OD-inaccessible if for every $\alpha < \kappa$ there is no surjection $f : \wp(\alpha) \to \kappa$ that is definable from ordinal parameters. A set of reals $A \subseteq \mathbb{R}$ is κ-*Suslin* if for some tree T on κ, $A = p[T]$[2]. A set A is *Suslin* if it is κ-Suslin for some κ; A is *co-Suslin* if its complement $\mathbb{R}\backslash A$ is Suslin. A set A is *Suslin, co-Suslin* if both A and its complement are Suslin. A cardinal κ is a *Suslin cardinal* if there is a set of reals A such that A is κ-Suslin but A is not λ-Suslin for any $\lambda < \kappa$. Suslin cardinals play an important role in the study of models of determinacy as can be seen by just flipping through the Cabal Seminar Volumes ([14, 15, 16, 17]). LSA is then the following theory.

DEFINITION 1.1.1. The Largest Suslin Axiom, abbreviated as LSA, is the conjunction of the following statements:

1. $\mathsf{ZF} + \mathsf{AD}^+$.
2. There is a largest Suslin cardinal.
3. The largest Suslin cardinal is OD-inaccessible.

LSA can also be defined in terms of the *Solovay sequence*.

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[1] The requirement in these axioms that there is a strong cardinal which is a limit of Woodin cardinals is only possible if $L(A,\mathbb{R}) \models \mathsf{LSA}$.

[2] Given a cardinal κ, we say $T \subseteq \bigcup_{n<\omega} \omega^n \times \kappa^n$ is a *tree* on κ if T is closed under initial segments. Given a tree T on κ, we let $[T]$ be the set of its branches, i.e., $b \in [T]$ if $b \in \omega^\omega \times \kappa^\omega$ and letting $b = (b_0, b_1)$, for each $n \in \omega$, $(b_0 \restriction n, b_1 \restriction n) \in T$. We then let $p[T] = \{x \in \mathbb{R} : \exists f((x,f) \in [T])\}$.

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DEFINITION 1.1.2. The Solovay sequence is a sequence $(\vartheta_\alpha : \alpha \leq \Omega)$ such that

1. $\vartheta_0 = \sup\{\beta : \exists f : \wp(\omega) \to \beta (f \text{ is an } OD \text{ surjection})\}$,
2. if $\vartheta_\alpha < \Theta$ then $\vartheta_{\alpha+1} = \sup\{\beta : \exists f : \wp(\vartheta_\alpha) \to \beta (f \text{ is an } OD \text{ surjection})\}$,
3. for limit $\lambda \leq \Omega$, $\vartheta_\lambda = \sup_{\alpha<\lambda} \vartheta_\alpha$.
4. $\vartheta_\Omega = \Theta$.

REMARK 1.1.3. LSA is then equivalent to the conjunction of the following axioms:

1. $\mathsf{ZF} + \mathsf{AD}^+$.
2. For some ordinal α, $\Theta = \vartheta_{\alpha+1}$ and ϑ_α is the largest Suslin cardinal.

The above equivalence can be shown using the material of Chapter 9.1 of [68]. We note that it follows from [68, Theorem 9.12] that LSA implies $\neg\mathsf{AD}_\mathbb{R}$.

1.1.2. The minimal model of LSA. Suppose V is a model of LSA. Let κ be the largest Suslin cardinal and suppose $A \subseteq \mathbb{R}$ has Wadge rank κ. It then follows that $L(A,\mathbb{R}) \vDash \mathsf{LSA}$. Keeping this fact in mind, we make the following definition.

DEFINITION 1.1.4. Suppose T is a first order theory extending AD^+. We say that M is *a minimal model* of T if

1. M is transitive and $M \vDash T$,
2. $\mathbb{R}, Ord \subseteq M$, and
3. for every N that is a (definable) class of M and contains all the reals and ordinals, either $N = M$ or $N \vDash \neg T$.

It follows that all minimal models of LSA have the form $L(A,\mathbb{R})$. A natural question is whether there is a unique minimal model of LSA. We will show (see the proof of Theorem 10.3.1) that in fact there is a unique minimal model of LSA which is naturally *the* minimal model of LSA. Woodin's proof of the existence of divergent models of AD^+ also shows that not all extensions of AD^+ have a unique minimal model (see [7, Theorem 6.1]).

The minimal model of LSA may not actually be big. For example, if N is a transitive model of AD^+ that contains the minimal model M of LSA and has a Suslin cardinal $> \Theta^M$ then $\Theta^M < \vartheta_0^N$. In particular, every set of reals in M is ordinal definable from a real in N. Motivated by this fact, we make the following definition.

DEFINITION 1.1.5. Suppose M is a transitive model containing all the reals and ordinals and such that $M \vDash \mathsf{AD}^+ + V = L(\wp(\mathbb{R}))$. We say M is *full* if for all transitive N such that

1. $M \subseteq N$ and
2. $N \vDash$ "$\mathsf{AD}^+ + V = L(\wp(\mathbb{R}))$",

Θ^M is a member of the Solovay sequence of N.

The following interesting problem seems central to our understanding of the models of AD^+ that we build from large cardinals or from other hypotheses.

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PROBLEM 1.1.6. Do large cardinals or forcing axioms such as PFA imply that there is a full model of LSA?

In particular, whether the models of determinacy obtained as derived models of V contain full models of LSA *or not is a major open problem of the area.* Here we make the following conjecture which is motivated by Woodin's *Sealing Theorem* (see [19]). Below uB stands for the set of universally Baire sets and for a generic g, $\mathsf{uB}_g = (\mathsf{uB})^{V[g]}$ and $\mathbb{R}_g = \mathbb{R}^{V[g]}$.

CONJECTURE 1.1.7. *Suppose κ is a supercompact cardinal and there is a proper class of Woodin cardinals. Let $g \subseteq Coll(\omega, 2^{2^\kappa})$ be generic. Then in $L(\mathsf{uB}_g, \mathbb{R}_g)$, for each $\xi < \Theta$ there is α such that $\vartheta_\alpha \in (\xi, \Theta)$ and ϑ_α is the largest Suslin cardinal below $\vartheta_{\alpha+1}$.*

Thus, in the set up of the conjecture, $L(\mathsf{uB}_g, \mathbb{R}_g)$ has full models of LSA that are cofinal in its Wadge hierarchy. The following is what is known on Conjecture 1.1.7. Woodin (unpublished) has shown that $L(\mathsf{uB}_g, \mathbb{R}_g) \vDash$ "$\mathsf{AD}_{\mathbb{R}} + \Theta$ is a regular cardinal". Sandra Müller and the first author recently showed that $L(\mathsf{uB}_g, \mathbb{R}_g)$ can be represented as a derived model of some iterate of V. They also found a stationary-tower-free proof of Woodin's Sealing Theorem. These results are unpublished. [56] presents a stationary-tower-free proof of the derived model theorem.

The question of whether the Cramer-Woodin model of LSA from [5, Theorem 65] is a full model of LSA or not seems not only interesting but also important for understanding the relationship between large cardinals and models of AD^+.

1.1.3. The content of this book. In this book, we establish three kinds of results that can be stated without mentioning the technology developed to prove them. The first set of results deals with the minimal model of LSA. Assume V is the minimal model of LSA. Then the following hold.

(A) (Theorem 7.2.2) HOD $\vDash$ GCH.
(B) (Theorem 10.2.1) The Mouse Set Conjecture holds.

The second set of results contains a single result which shows the consistency of LSA relative to large cardinals. We will show the following.

(C) (Corollary 10.3.1) Suppose the theory ZFC + "there is a Woodin cardinal that is a limit of Woodin cardinals" is consistent. Then so is LSA.

The third type of result establishes the existence of the minimal model of LSA assuming combinatorial principles or forcing axioms. The following belongs to this group.

(D) (Corollary 12.0.22) Assume PFA. Then the minimal model of LSA exists.

The precursors of these results already exist in print. The first author demonstrated versions of (A), (B), and (C) for the theory $\mathsf{AD}_{\mathbb{R}} +$ "Θ is a regular cardinal". The second author proved the version of (D) for the same theory. The interested reader may consult [29], [31] and [65]. The reason to prove such results is to demonstrate that the underlying technical theory is robust and can be used in a wide range of situations.

Recently the authors of [2] showed that the theory $\mathsf{CH}+$"there is an ω_1-dense ideal on ω_1" implies that the minimal model of $\mathsf{AD}_{\mathbb{R}}+$"$\Theta$ is a regular cardinal" exists. This, along with an earlier result of Woodin, show that these two theories are equiconsistent. This solved part of Problem 12 of [68]. Whether there is a natural hypothesis asserting the existence of an ideal on a small cardinal that is equiconsistent with LSA is an interesting problem. In particular, letting M' be the minimal model of LSA, κ be the largest Suslin cardinal of M' and $M = L(\Gamma,\mathbb{R})$ where $\Gamma = \{A \in \wp(\mathbb{R}) \cap M' : w(A) < \kappa\}$[3], the model $M[G*H]$ where $G*H \subseteq Coll(\omega_1,\mathbb{R}) * Add(1,\omega_2)$ is M-generic has not be studied at all. The model $M[G*H]$ where $G*H \subseteq \mathbb{P}_{max} * Add(1,\omega_3)$ has been investigated in [4], but not much is known beyond [4][4].

1.1.4. The necessity of the short-tree-strategy mice. We do not know how to prove (B)-(D) using the methods of [63], and whether this is possible or not is a very interesting question[5]. The main issue seems to be the absence of an analysis of the LSA stages of the Solovay sequence using the least-branch hierarchy. The main technical concept we use to analyze such levels is the notion of a *short-tree-strategy mice*, which is developed in Chapter 3. Thus, the question is whether it is necessary to develop this theory in order to prove results like (B)-(D).

The main issue is the following. Assume AD^+. Suppose $\vartheta_{\alpha+1} < \Theta$ and ϑ_α is the largest Suslin cardinal below $\vartheta_{\alpha+1}$. Then if $(\mathcal{P},\Sigma)$ is the hod pair generating the pointclass $\Gamma_1 = \{A \subseteq \mathbb{R} : w(A) < \vartheta_{\alpha+1}\}$ then letting δ be the largest Woodin cardinal of $\mathcal{P}$, $((\mathcal{P}|\delta)^\#, \Sigma^{stc})$ is the pair generating the pointclass $\Gamma_0 = \{A \subseteq \mathbb{R} : w(A) < \vartheta_\alpha\}$. If one's goal is to show that assuming $\mathsf{AD}_{\mathbb{R}} + \mathsf{DC} + V = L(\wp(\mathbb{R}))$, $\mathrm{HOD} \vDash \mathsf{GCH}$ then it maybe possible to *skip* Γ_0 and build the generator of Γ_1. The problem with skipping Γ_0 and moving to Γ_1 is exactly the fact that it is then unclear how to prove theorems like (A)-(D). What one would have liked is some sort of hybrid method that does not skip Γ_0 but also incorporates ideas from [63] to avoid the theory of short-tree-strategy mice. It seems to us that this may not be possible.

Suppose then we decide not to skip over Γ_0, and suppose we have succeeded in building a generator $((\mathcal{P}|\delta)^\#, \Sigma^{stc})$ for Γ_0. At this stage, we do not know what $(\mathcal{P},\Sigma)$ must be and can only see $((\mathcal{P}|\delta)^\#, \Sigma^{stc})$. Set then $\mathcal{Q} = (\mathcal{P}|\delta)^\#$ and $\Lambda = \Sigma^{stc}$. What we need to show next is that we can extend $\mathcal{Q}$ to $\mathcal{P}$ in such a way that the following hold[6]:

1. δ is the largest cardinal of $\mathcal{P}$ and $H_\delta^{\mathcal{P}} = \mathcal{Q}|\delta$,
2. for all $A \subseteq \delta$, $A \in \mathcal{P}$ if and only if A is ordinal definable from $(\mathcal{Q},\Lambda)$,
3. $\mathcal{P} \vDash$ "δ is a Woodin cardinal".

[3] $w(A)$ is the Wadge rank of A.

[4] But see also [21].

[5] [63] does show that $\mathcal{H} \vDash \mathsf{GCH}$ but only assuming HPC.

[6] Below $H_\delta^{\mathcal{P}}$ is the set of all $X \in \mathcal{P}$ whose hereditary cardinality is $< \delta$.

The main issue seems to be with proving clause 2. It is a version of MSC for Λ, and the only way we know how to prove it is by building a Λ-mouse over $\mathcal{Q}$ whose derived model contains the set $\{(x,y) \in \mathbb{R}^2 : x$ is ordinal definable from y and $(\mathcal{Q},\Lambda)\}$. This requires a certain level of uniformity: $\mathcal{Q}$ and what we build on the top of $\mathcal{Q}$ have to be the same kind of objects, as otherwise the construction over $\mathcal{Q}$ can project across δ violating clause 3 above.

1.1.5. Some historical remarks on the large cardinal structure of hod mice. The large cardinal structure of hod mice has been somewhat of a mystery. While originally it seemed hod mice must have very limited large cardinal structure, nowadays the prevailing belief is that they in fact can have any large cardinal whatsoever[7]. First we make the following definition.

DEFINITION 1.1.8. Θ_{reg} is the theory $\mathsf{ZF} + \mathsf{AD}_{\mathbb{R}} +$ "Θ is a regular cardinal".

Prior to [29], the theory Θ_{reg} was believed to be beyond the short extender region and was believed to be at the complexity level of supercompact cardinals. Because Woodin was able to force strong combinatorial statements over a model of Θ_{reg} that would normally require large cardinals at the level of supercompact cardinals or beyond[8], the aforementioned belief seemed to be very reasonable.

The main goal of [29], which is based on the first author's PhD thesis, was to analyze HOD of the minimal model of Θ_{reg}[9]. While any model of determinacy has a rich large cardinal structure below its Θ[10], V_Θ^{HOD} of the minimal model of Θ_{reg} is very simple in the following sense (see Theorem 1.1.9).

Suppose $V \vDash \mathsf{AD}^+$. The Solovay pointclasses are exactly the stages of the Wadge hierarchy where a "new" *non-definable from below* set appears. For α such that $\vartheta_\alpha \leq \Theta$ let $\mathsf{SP}_\alpha = \{B \subseteq \mathbb{R} : w(B) < \vartheta_\alpha\}$. If $\vartheta_\alpha < \Theta$ and $A \subseteq \mathbb{R}$ has Wadge rank ϑ_α then A is not ordinal definable from any set of reals $B \in \mathsf{SP}_\alpha$ and moreover, every set in $\mathsf{SP}_{\alpha+1}$ is ordinal definable from A and a real. Thus, in a sense, once we perceive a set of reals of Wadge rank ϑ_α, we know everything about $\mathsf{SP}_{\alpha+1}$. Putting it differently,

(†) in the Wadge hierarchy, nothing of any interest happens among sets whose Wadge rank belongs to the interval $(\vartheta_\alpha, \vartheta_{\alpha+1})$.

In general, † is not true. All sorts of structures: Suslin cardinals, large cardinals with complicated partition properties etc, exist in that Wadge interval. However, the hod mice that are below the theory Θ_{reg} cannot have regular limits of Woodin cardinals, and moreover, the Woodin cardinals and their limits of such a hod mouse exactly correspond to the Solovay sequence[11] in the following sense.

[7] At least in the short-extender region.

[8] E.g., $\mathsf{MM}^{++}(c)$ (see [68]) and $\mathsf{CH}+$ "there is an ω_1-dense ideal on ω_1" (see [2]).

[9] Prior [29], it was not know that there is a unique minimal model of Θ_{reg}.

[10] E.g., Θ is a limit of strong partition cardinals, see [13].

[11] By a theorem of Woodin, each $\vartheta_{\alpha+1}$ is a Woodin cardinal of HOD. See [18].

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THEOREM 1.1.9 ([29] and Theorem 7.2.2). *In the minimal model of* Θ_{reg}*, and in fact of* LSA*,* δ *is a Woodin cardinal of* HOD *or a limit of Woodin cardinals of* HOD *if and only if* δ *is a member of the Solovay sequence.*

Theorem 1.1.9 implies that HOD of the minimal model of Θ_{reg} has no Woodin cardinals in the interval $(\vartheta_\alpha, \vartheta_{\alpha+1})$, and in this sense, † is true below Θ_{reg}[12]. Therefore, to represent V_Θ^{HOD} of the minimal model of Θ_{reg} as a hod mouse, we do not need to understand exactly what happens between $(\vartheta_\alpha, \vartheta_{\alpha+1})$ in V as none of what happens there makes HOD look complicated in that interval[13].

The world of determinacy might have been a simpler place if † was always true, but [29] shows that the theory Θ_{reg} is much weaker than a Woodin cardinal that is a limit of Woodin cardinals. LSA, the main topic of this manuscript, is the next natural determinacy theory that is consistency wise stronger than Θ_{reg}, and while the hod mice of this manuscript do have inaccessible limit of Woodin cardinals, Theorem 1.1.9 is still true. This once again implies that the large cardinal structure of hod mice at the level of LSA is limited and in fact, in such hod mice

(‡) there is no Woodin cardinal δ and a $\kappa < \delta$ such that κ is δ-strong.

Moreover, prior to the current work, it was believed that ‡ and Theorem 1.1.9 are just consequences of AD^+. This belief was based on various arguments due to Woodin that showed that if δ is a member of the Solovay sequence then there cannot be $\kappa < \delta$ whose Mitchell order was much bigger than δ. However, Theorem 10.3.1 shows that LSA is weaker than a Woodin cardinal that is a limit of Woodin cardinals, and further unpublished work of the first author showed that the large cardinal structure of hod mice, at least in the short extender region, may not be limited. In particular, neither † nor Theorem 1.1.9 are consequences of AD^+. The first author then made the following conjecture.

CONJECTURE 1.1.10. *Assume* $\mathsf{AD}^+ + V = L(\wp(\mathbb{R}))$*. Define the sequence* $(\eta_\alpha : \alpha \leq \Omega)$ *as follows:*

1. $\eta_0 = \vartheta_0$.
2. *Assuming* $\eta_\alpha < \Theta$ *and setting* $\kappa = (\eta_\alpha^+)^{\mathrm{HOD}}$*,* $\eta_{\alpha+1}$ *is the supremum of all* β *such that there is an ordinal definable surjection* $f : \wp_{\omega_1}(\kappa) \to \beta$.[14]
3. *For a limit ordinal* ξ*,* $\eta_\xi = \sup_{\alpha<\xi} \eta_\alpha$.

Then δ *is a Woodin cardinal or a limit of Woodin cardinals of* HOD *if and only if* $\delta = \eta_\alpha$ *for some* α.

Using the methods of [63], Steel verified Conjecture 1.1.10 assuming HPC + NLE (see [63, Theorem 11.5.7]). More recently, the first author, using ideas from [63], constructed a hod mouse that has a Woodin cardinal that is a limit of Woodin

[12]It is a well-known fact from inner model theory dating back to [22] that iteration strategies of mice or hod mice acquire complexity only because of Woodin cardinals.

[13]This was the original motivation of the so-called "layering" used both in [29] and in this book.

[14]Recall that $\wp_{\omega_1}(\kappa)$ is the set of countable subsets of κ.

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cardinals. This result confirms the belief that hod mice may have a complicated large cardinal structure.

1.1.6. Organization. Chapters 2-8 develop the basic theory of hod mice for AD^+ models up to the minimal model of LSA; a consequence of this analysis is (A). The last four chapters focus on applications. Chapter 11 proves that $\square_{\kappa,2}$ holds in HOD of AD^+ models up to the minimal model of LSA for all HOD-cardinals κ. Our main use of this chapter is Chapter 12, where a proof of (D) is given. Chapter 9 develops the basic theory of *condensing sets*, which is needed in constructions of hod mice in various situations. Chapter 10 uses the material developed in the previous chapters to prove (B) and (C). The last chapter (Chapter 12) proves (D) by constructing a hybrid version of K^c. This chapter uses methods developed in the previous chapters, [36], and [65].

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Finally, as it must be clear to anyone flipping through the pages of this manuscript, the inspiration behind this work comes from seminal contributions to descriptive inner model theory made by John Steel and Hugh Woodin. We thank them for the monumental work they have done during the past four decades.

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