USE OF THE OVERLAP METHOD FOR THE CONNEXION OF PROPER MOTIONS TO GALAXIES PROBLEMS OF THE ZONE WITHOUT NEBULAE

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1: INTRODUCTION

There are two methods for connecting the proper motions to the galaxies. The one used under the direction of A. N. Deutsch at Pulkovo, and the other method at the Lick Observatory.

Only the method used by the Lick Observatory allows the use of overlapping plates. It is the only one which will be dealt with in this first part.

2: LICK METHOD

The method has been described in detail by C. D. Shane and S. Vasilevskis (I.A.U. VIII, 1952, p. 794) and S. Vasilevskis (A. J. 62 p. 113 and 127, 1957).

We use an objective grating giving first order spectra with $\Delta m = 4$ magnitudes. In each field, a two hour exposure (system I) shows nebulae easily measurable around magnitude 16, and with a similar intensity, the first order spectra of stars of magnitude about 12. A second one minute exposure, (system II), a little shifted enables us to measure the central image of the 12th magnitude stars and the first order spectra of the bright stars.

The 6° x 6° plates have an overlap of 1° with adjacent plates. For measures done with electronic devices there are random errors, ε_1 of about 0".20 on the nebulae, and ε_2 of about 0".10 on the stars, first order spectra or central images.

We choose in each field fifty nebulae, (those most easily measured), and fifty stars of magnitude 12 distributed regularly over the plate.

The comparison between the measures of the nebulae on the old and on the new plates gives the corresponding formulas without any proper motion between the coordinates of the two periods for system I. It is enough to establish a linear connection because the exposures are taken with the same astrograph, the same field center and in conditions as similar as possible.

$$(1) \quad \Delta(x) = Ax + By + C \qquad \Delta(y) = Dx + Ey + F$$

Generally, all the field corrections disappear, and this has been checked in a few cases.

Applying the formulae (1) to the coordinates of the 12th magnitude stars between the two periods enables one to calculate the absolute

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proper motions of these stars.

The comparison between the measures made on the central image of system II and those made on the first order images of system I for the 50 stars of 12th magnitude, shows a linear connection between the differences of systems I and II at the two periods.

$$(2) \quad \Delta(x) = ax + by + c \qquad \Delta(y) = dx + ey + f.$$

Applying formulas 1 and 2 to the measures of the bright stars on system II enables one to calculate the absolute proper motion of the bright stars.

3: ERRORS IN THE LICK FORMULAS

One can calculate the errors $\mathcal{E}(1)$ and $\mathcal{E}(\mathrm{II})$ in the $\Delta(x)$ and $\Delta(y)$ by applying formulas (1) and (2). Considering the number of stars and nebulae used, considering that we pass from system I to system II through stars measured as first order spectra in system I, therefore twice, for a star of coordinates x and y, we have:

$$\mathcal{E}^2(\mathrm{I}) = 2\mathcal{E}_1^{\,2}\left[\frac{1}{50} + \frac{x^2 + y^2}{\Sigma_x^{\,2}}\right]$$

$$\mathcal{E}^2(\mathrm{II}) = 3\mathcal{E}_2^{\,2}\left[\frac{1}{50} + \frac{x^2 + y^2}{\Sigma_x^{\,2}}\right]$$

We can easily calculate average values of $\mathcal{E}^2(\mathrm{I})$ and $\mathcal{E}^2(\mathrm{II})$ in any area because the areal repartition of the stars and nebulae is homogeneous.

On the whole area of a plate we have for instance:

$$\mathcal{E}^2(\mathrm{I}) = 0.12\,\mathcal{E}_1^{\,2} \qquad \mathcal{E}^2(\mathrm{II}) = 0.18\,\mathcal{E}_2^{\,2}$$

We reduced from it the standard errors of the proper motions over a period of 20 years. These errors resulting from the formulas are systematic on the plates.

For the 12th magnitude stars, in system I, only $\mathcal{E}(\mathrm{I})$ will be used. For the bright stars measured in system II, $\mathcal{E}(\mathrm{I})$ and $\mathcal{E}(\mathrm{II})$ will be used.

The errors are maximal at the edge of the plates. But, in the overlap zones we have several evaluations of the proper motions about which the systematic errors are independent. Finally, Table I gives the errors to be expected after averaging.

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TABLE I

Standard errors of the proper motions over 20 years in seconds of arc per century

Zones	Area in square degrees	Number of evaluations	1. Random errors Stars of 12th 1	2	2. Systematic errors Bright stars 1	2
Central	16	1	0.50	0.275	0.50	0.32
Edge	16	2	0.35	0.267	0.35	0.31
Corner	4	4	0.25	0.232	0.25	0.27
Weighted		1.77	0.42	0.264	0.42	0.31

We find that the systematic errors are almost equal to the random errors.

4: RISKS OF SYSTEMATIC ERRORS IN THE LICK METHOD

All the preceding evaluations infer that the results are not perturbed by anomalies contrary to the admitted hypothesis. We are going to examine the risks that may exist.

1) Within system I, there is a risk of systematic errors between the images of the nebulae and the images of 12th magnitude stars in first order spectra. This, owing to the different shape of the images, which is unavoidable, but also owing to the two hours time of exposure which involves differential shifting on the plate, due to the refraction, shifting that cannot be balanced by guiding and that can reach 1". This spreading adding to the spectral spreading or to the spreading of nebulae might very well give systematic errors from one part of the field to another.

2) When passing from system I to system II, we use the 12th magnitude stars with their first order spectra in system I and the central images in system II. Between the long exposure of system I and the short exposure of system II there are unavoidable systematic deformations due to differential refraction. This is why Vasilevskis determines complete linear formulas, formulas (2), instead of shifting in x and y to pass from system I to system II.

Can these formulas suitably represent the passage from one system to the other? Probably, because the stars used to determine these formulas are rather numerous and chosen with a similar magnitude, and because these formulas are fairly general.

3) For the passage in system II from the 12th magnitude stars to the bright stars, the exposure times are short, the guiding mistakes will be very small, but the possibility of important magnitude effects remains. These risks are increased by the fact that the measures are done on the one hand on central images, on the other hand on the first order spectra.

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As a conclusion, the Lick method is probably correct, but shows risks of systematic errors, the checking of which is advisable. The checking done on a few examples is not enough in the case of very big work of this kind. This is clearly shown by the studies done on the A.G.K. 2 - A.G.K. 3. Thus it is useful to perform systematic checking on all fields in order to avoid the systematic errors to some aberrant exposures which are unavoidable in such an undertaking. We shall show how we can perform such checking by using overlapping plates.

5: USE OF THE OVERLAP METHOD

The overlaps of plates in the Lick program are not very wide, but a more careful study shows that even in this case the use of the overlap method must be efficient. We are going to examine what we could obtain.

The information to be used are:

1) For the nebulae the differences Δx_0 of measures between the two exposures on a field. We have equations with error $\mathcal{E}_1\sqrt{2}$

$$Ax + By + C = \Delta x_0$$

2) For the 12th magnitude stars measured in system I on two first-order spectra, if $\delta(x)$ is the unknown shifting due to the proper motion, Δx_1 the difference of the measures between the two exposures, we have equations with error $\mathcal{E}_2$

$$Ax + By + C + \delta(x) = x_1$$

3) For the 12th magnitude stars measured as central images in system II, if Δx_2 is the difference of the measures between the two exposures, we have equations with error $\mathcal{E}_2\sqrt{2}$.

$$(A + a)x + (B + b)y + C + c + \delta(x) = \Delta x_2$$

4) For the bright stars measured as first-order spectra, in system II, if Δx_3 is the difference of the measures between the two exposures, we have equations with error $\mathcal{E}_2$

$$(A + a)x + (B + b)y + C + c + \delta(x) = \Delta x_3$$

To study the systematic errors to be expected in the results, we must determine the errors of the constants A, B, C and A+a, B+b, C+c that enable us to calculate the absolute proper motions of the 12th magnitude stars and of the bright stars.

The unknowns that enter in our equations are the constants A, B, C, A+a, B+b, C+c, for each pair of plates and the $\delta(x)$ and $\delta(y)$ due to the proper motions for each star.

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We have schematized the problem by separating the calculations in α and δ, and assuming that nebulae and stars were all spread in equal number at the centers of each square degrees of the 6° x 6° plates. We have assumed that there were only 36 nebulae and 36 12th magnitude stars per plate instead of 50. At the end we shall make the necessary corrections in order to pass from 36 to 50 by multiplying all the systematic errors we meet by $\sqrt{\frac{36}{50}}$

By using all the equations, we form without any difficulties, a big matrix giving unknowns by the method of least squares. We obtain directly, by elimination of the unknowns $\delta(x)$ and $\delta(y)$, the matrix giving only the constants of the pairs of plates. We can then resolve by succesive approximations as we have done for the constants of A.G.K. 2 - A.G.K. 3.

The errors E' of the results given by the formulas have been calculated in linear form in terms of the errors of measurement on each plate, E (0i) in the positions of each nebulae, E (1i) and E (2i) in the positions of the 12th magnitude stars in the systems I and II, and E (3i) in the positions of the bright stars.

$$E' = \sum_i \beta(ki)\, E(Ki) \qquad \text{K from 0 to 3}$$

hence

$$\overline{E'^2} = \overline{E_0^2}\,\Sigma_0\beta^2 + \overline{E_1^2}\,\Sigma_1\beta^2 + \overline{E_2^2}\,\Sigma_2\beta^2 + \overline{E_3^2}\,\Sigma_3\beta^2$$

The calculation has been done by iteration, it is rather long. In the next paper the results are given in Table I.

The comparison between the Table I of this paper and of the next paper, shows that, although the overlapping of plates is rather small, it is better to use the overlapping to decrease the systematic errors E' to be feared; the latter then become smaller than the random errors.

The advantages are a little greater if one measures many bright stars. One comes then to form a bright star system which is linked to the nebulae.

This can be shown otherwise by computing from the sums β^2 and the densities in nebulae and stars used the extent of the influence of each kind of measure-error on the results. This calculation supposed a linear decrease of the dependences with the distances to ρ as in paper (1) page 322. This is only one indication for because of the very small overlapping the decrease of the dependences with the distances is rather irregular.

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TABLE II

Extent of dependences

	Lick Measures						Measures with all A.G. stars					
	Formulas I			Formulas II			Formulas I			Formulas II		
	$\Sigma\beta^2$	σ	$\rho°$	$\Sigma\beta^2$	σ	$\rho°$	$\Sigma\beta^2$	σ	$\rho°$	$\Sigma\beta^2$	σ	$\rho°$
Nebulae	0.0105	1.4	5.6	0.010	1.4	5.8	0.009	1.4	6.0	0.008	1.4	6.2
Stars 12th (I)	0.0270	1.4	3.4	0.017	1.4	4.5	0.026	1.4	3.4	0.013	1.4	5.1
Stars 12th (II)	0.0035	1.4	10	0.020	1.4	4.1	0.007	1.4	7	0.006	1.4	7.6
Bright stars	0.0043	0.77	12	0.026	0.77	4.9	0.003	5.1	5.5	0.002	5.1	6.8

6: CHECKING OF THE VALIDITY OF THE HYPOTHESES

The use of overlaps will probably be of greater interest for the checking of the validity of the formulas than for the expected improvement of the results.

If on any plate, the same formulas are not available in system I for the nebulae and for the 12th magnitude stars, we shall assume that the calculated proper motion of the nebulae on this plate are systematically different from zero. These differences will be clearer if the dependences in terms of the measures of the nebulae go further. It is advantageous in this respect to measure all the stars of the A.G.

If there are any difficulties when passing from system I to system II through the 12th magnitude stars, it will be noted when comparing the proper motions calculated for the same stars in the two systems. If we note too big differences, we can determine this by comparing the proper motions obtained for the same stars but on different plates. And decide whether it is the representation of system I or system II, which demands more complex formulas.

The difficulties within system II will probably be due to magnitude effects in the bright stars, but we can study them and correct them, if necessary, as for A.G.K. 2 - A.G.K. 3, by introducing a magnitude parameter in the formulae concerning system II. To settle the magnitude parameters, it will probably be necessary to introduce the information we have on the relative proper motions of bright stars.

7: APPLYING THE RESULTS TO THE STUDY OF THE A.G.K. 2 - A.G.K. 3

On the one hand the study of the quality of the A.G.K. 2 - A.G.K. 3 has been done. On the other hand, we have just seen what can be expected of the Lick program.

For the secular proper motions the situation is as follows:

In A.G.K. 2 - A.G.K. 3 we have 8 stars per square degree with random errors $0''.85$ and local systematic errors of amplitude $0''.4$ reduced by a factor $\sqrt{2}$ by the mean in a square of $2° \times 2°$.

In the Lick program, we obtain 1 star by square degree with random error $0''.4$ and local systematic errors of amplitude $0''.2$ reduced by a factor $\sqrt{2}$ by the mean on a square of $3° \times 3°$.

In the study of systematic errors of A.G.K. 2 - A.G.K. 3, we obtain by comparison in squares $2° \times 2°$, $3° \times 3°$ and $4° \times 4°$ standard errors respectively of $0''.5$; $0''.35$ and $0''.27$.

It is not possible to study the local systematic errors of A.G.K. 2 - A.G.K. 3, but only the mean systematic errors on big surfaces.

With more stars in the Lick program, for example half the stars of A.G.K. 2 - A.G.K. 3, this study will be possible.

8: ADVANTAGES BY BROADER OVERLAPPING

The use of overlapping in the Lick program is advantageous, but not very important, because the overlaps are too small.

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With overlapping giving two observations on each object, as in A.G.K. 2 - A.G.K. 3, the systematic errors to be feared will be much smaller and will improve the checking of validity of formulas. The work involved in taking the plates and making the measurements will be only 1.2 times.

In a future program an investigation will be made for comparing the efficiency of broader overlapping and of larger numbers of objects measured.

Some results are given in the next paper.

THE ZONE WITHOUT NEBULAE

This zone near the galactic plane has a width varying between 20° and 25°.

To obtain absolute proper motions in this zone we can only transfer the information obtained from nebulae from the neighbouring zones.

We want to survey the feasibility of different astrometric methods with the present plates.

9: OVERLAPPING OF LICK PLATES

In the next paper some calculations show that these overlaps are much too small to obtain correct solutions for the plates without nebulae. However since we have these plates, we shall try to make use of them.

The constants of the plates, except the additive constants, depend on differences between physical conditions of the taking and measuring the plates at the two epochs.

1) Temperature of astrograph, focussing and differntial refraction.

2) Temperature and orientation during plate measures.

It is possible to calculate the influence of these factors on the constants.

After a solution for many plates in the zones with nebulae, we try to establish rules for estimation of values of constants or connection between constants, using the values of physical factors.

Introduction of this information in the matrices of overlaps with correct weight, will improve the solutions for the zone without nebulae.

The improvement will be noteworthy only if the accuracy of these estimates is of the same order as in the case of solutions with nebulae.

This method is a little analogous to the method recorded by S.V.M. Clube and R.H. Stoy for the southern hemisphere at Tampa, in 1968, and to some considerations of mine (Ann. Strasb. VI, 1964, p. 107).

10: USE OF A.G.K. 2 - A.G.K. 3

First, we must study the systematic errors of proper motions in the zone with nebulae using the results of the Lick and Pulkovo programs.

If these systematic errors show a sufficient continuity, it will be possible to interpolate in the zone without nebulae. This possibility

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is not sure because the same continuity should exist in the system A.G.K. 2.R. - A.G.K. 3.R., with an accuracy of about 0."2 in the secular proper motions. This is possible, but can be decided only by the facts in the future.

11: USE OF A SPECIAL A.G.K. 2 - A.G.K. 3

If the preceding method is not good enough, the reason will probably be that the continuity in the system A.G.K. 2.R. - A.G.K. 3.R. is insufficient.

In this case, it is possible to establish one A.G.K. 2 - A.G.K. 3 special with a very small weight for the reference coordinates. This system will be essentially a photographic system connecting the plates by overlapping. Perhaps this system will be better than the normal system for the continuity in proper motions. This work is ready and rather easy in case of need.

It will be possible to check in the zone with nebulae if the continuity of systematic corrections is better for the special A.G.K. 2 - A.G.K. 3, or for the normal A.G.K. 2 - A.G.K. 3. This is difficult to foresee, but must again be decided by the facts.

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(1) P. Lacroute, Highlights of Astronomy, I. A. U., 1967, p. 319 - 337.

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