THE KINEMATICS AND AGES OF STARS IN GLIESE'S CATALOGUE

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1. Introduction

This contribution gives some results on the kinematics and ages of stars near the Sun. These results are mainly based on the catalogue of nearby stars compiled by Gliese (1957, 1969 and minor recent modifications). Table I shows the number of objects under consideration. While the old catalogue (1957) contained only stars with distances r up to 20 pc, the new edition (1969) includes many stars with slightly larger distances. In Table I, a 'system' is either a single star or a binary or a multiple system. The number of systems with known space velocities nearer than 20 pc has increased by about 30% from 1957 to 1969. The first edition of Gliese's catalogue (1957) has been analyzed in detail by Gliese (1956) and von Hoerner (1960).

TABLE I

Gliese's Catalogue of Nearby Stars

Number	Edition 1969		1957 $r \leq 20$ pc
	All	$r \leq 20$ pc	
Stars	1890	1277	1095
Systems	1529	1036	916
Systems with known space velocity	1131	770	598

Although Gliese's catalogue is the most complete collection of stars within 20 pc, this sample of stars is severely biased by selection effects and is not fully representative for all the nearby stars. Only the stars brighter than $M_v \sim +7$ are almost completely known within 20 pc. For the fainter stars, the following selection effects occur: (a) The southern sky is deficient in detected faint nearby stars; (b) Since most of the faint nearby stars are found by their high proper motions, the sample is deficient in stars with small tangential components of their space velocities (measured with respect to the Sun); (c) Due to incomplete detection, the apparent space density of faint stars decreases rapidly with increasing distance r, and this effect becomes stronger with increasing M_v. From the luminosity function derived in the following section, we predict that there are at least 3600 stars nearer than 20 pc. Only about 1300 of them have been detected up till now. Hence the majority (>65%) of stars within 20 pc are still undetected.

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2. Luminosity Function

We derive the luminosity function φ for nearby stars by counting Gliese's stars in appropriate volumes of space (all declinations and $r \leqslant 20$ pc for $M_v < 7.5$; $\delta > -30°$ and $r \leqslant 20$ pc, 10 pc, 5 pc for $M_v = 7.5$ to 9.5, 9.5 to 11.5, $\geqslant 11.5$). For $M_v > 13.5$, only lower limits for φ can be obtained. Our unit of φ is 'stars per unit magnitude interval in a complete sphere of radius $r = 20$ pc' (Table II and Figure 1). The resulting luminosity function is rather flat in the range $5 < M_v < 9$ (see also Arakelyan, 1968; Mazzitelli, 1972). From the data of Gliese's catalogue, it remains uncertain where the maximum of φ occurs, but $M_{v,\max} \geqslant 13$ is indicated. From the predicted total number of stars within 20 pc, namely at least 3600, the majority are stars on or near the main sequence (about 3300 = 91%). 27 giants (0.7%) are observed and about 300 white dwarfs (8%) are predicted for $r \leqslant 20$ pc. The local stellar number density is at least 0.11 stars pc^{-3}.

The luminosity function derived from Gliese's catalogue is in rather good agreement with the results obtained by Luyten (1968). This is remarkable, since the methods are quite different. While we directly count Gliese's stars according to their individually known absolute magnitudes, Luyten uses proper motion surveys and needs therefore additional statistical assumptions about the velocities of the stars. Luyten found the maximum of φ at $M_v \sim 13.9$ (1968) and more recently at $M_v \sim 13.6$ (1974). This means that our luminosity function may have actually reached the maximum, and that the decrease of our values of φ for $M_v > 14$ may be partly real. At the bright end, our luminosity function agrees well with the data derived by McCuskey (1966) from a larger volume of space (LF regions).

For the local stellar mass density, we find $\varrho_s = 0.046\, \mathcal{M}_\odot\, \mathrm{pc}^{-3}$ by using our data (Table III). If we use Luyten's luminosity function for $M_v \geqslant 13.5$, the value of ϱ_s increases only from 0.046 to 0.049 $\mathcal{M}_\odot\, \mathrm{pc}^{-3}$. The value of ϱ_s is much smaller than the total mass density in the solar neighbourhood, $\varrho_{\mathrm{tot}} = 0.15\, \mathcal{M}_\odot\, \mathrm{pc}^{-3}$, determined dynamically (Oort, 1965). The local density of the observed interstellar matter is about $0.02\, \mathcal{M}_\odot\, \mathrm{pc}^{-3}$. Hence the problem of the missing local mass remains as severe as it was before, if we do not invoke many faint M dwarfs of uncommonly low space velocities, as proposed by Murray and Sanduleak (1972) and Weistrop (1972).

Since we know for many stars in Gliese's catalogue both the absolute magnitude and the space velocity individually, we can investigate whether the luminosity function is correlated with the kinematical behaviour and hence with the ages of the stars.

TABLE II

Luminosity function

M_v	−1	0	1	2	3	4	5	6	7	8	9
φ	1	4	14	24	43	78	108	121	102	132	159
M_v		10	11	12	13	14	15	16	17	18	19
φ		245	341	512	597	>341		>213		$\geqslant 16$	

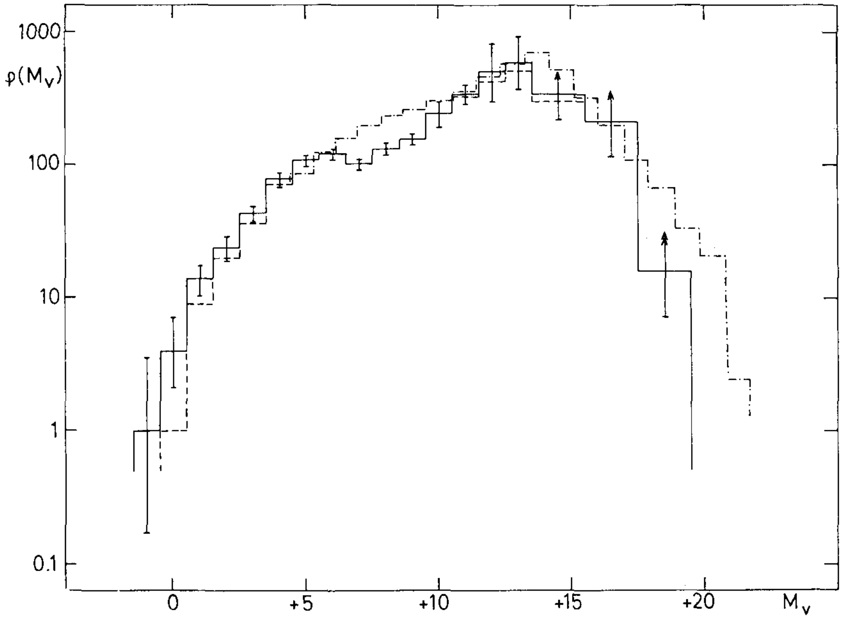

Fig. 1. Luminosity function $\varphi(M_v)$. Full line: total φ; dashed line: φ for stars on or near the main sequence; dashed-dotted line: Luyten's φ (1968).

TABLE III

Stellar mass density near the Sun

ϱ [$\mathcal{M}_\odot$ pc^{-3}]			Based on
Stars on or near the main sequence:			
$M_v < 9.5$	0.020	0.038	776 stars within various volumes depending on M_v
$9.5 \leqslant M_v < 13.5$	0.014		
$13.5 \leqslant M_v$	0.004		
Giants		0.001	27 stars within 20 pc
White dwarfs		0.007	5 stars within 5 pc
Sum		0.046	

The luminosity of stars brighter than $M_v \sim 5$ is affected by evolutionary effects which are difficult to eliminate. Hence we shall consider stars fainter than $M_v \sim 5$. For those stars, the luminosity function is essentially determined by the spectrum of stellar masses at the time of formation. In Table IV, we have grouped the stars according to their space motions (W is the velocity perpendicular to the galactic plane, S is the total space velocity, both referred to the adopted circular velocity). Because of the strong correlation between the space velocities and the ages of the stars, these groups have rather different mean ages. The quantity shown in Table IV is the number of stars as a function of M_v and $|W|$ or S, normalized in the range $5.5 \leqslant M_v <$

TABLE IV

Relative luminosity functions for kinematical groups

M_v	$\lvert W \rvert$ [km s^{-1}]				Total space velocity S			
	0–5	5–10	10–20	>20	0–20	20–35	35–50	>50
+ 5	1.6	0.8	0.8	0.9	1.9	0.8	1.4	0.7
6	0.9	1.0	1.1	1.1	0.7	1.1	1.1	1.1
7	1.1	1.0	0.9	0.9	1.2	0.9	0.9	0.9
8	0.9	1.1	1.0	0.8	1.1	0.9	1.3	0.7
9	0.9	0.8	0.8	0.8	0.8	1.0	0.9	0.7
10	0.9	1.0	0.9	1.2	0.5	0.6	0.9	1.7
+11	0.8	0.9	0.9	1.2	0.6	1.0	1.5	0.7

<7.5 and given relative to the total luminosity function at each value of M_v. If the luminosity function does not vary with age, then we expect the value 1.0 everywhere in Table IV. Except in the line for $M_v = 5$, which may be slightly biased by evolutionary effects, there does not occur any statistically significant variation of the luminosity function among the different kinematical groups. We conclude that the distribution of stellar masses at birth does not significantly depend on the time of formation, at least when averaged over longer periods of time and larger regions in space in the covered range of masses.

3. Age Groups in the Colour-Magnitude Diagram

The ages of Gliese's stars may be estimated either from the position in the colour-magnitude diagram or from the intensity of the Ca II emission (Section 4). If we wish to determine individual ages by fitting theoretical isochrones, we can use only those absolute magnitudes which are derived from accurate trigonometric parallaxes, since Gliese's photometric or spectroscopic parallaxes already assume M_v. Unfortunately, the number of evolved stars with accurate trigonometrical values for M_v is too small for a sound statistical treatment. This is obvious from Figure 2, which shows the colour-magnitude diagram for the stars of Gliese's catalogue with values of M_v based on trigonometric parallaxes having a probable error not larger than 10%. Therefore, we form groups in the colour-magnitude diagram and use the average age of each group for our kinematical studies. The groups are shown in Figure 2. Groups 1–5 and 6a–d are stars on or near the main sequence in various intervals of $B-V$. The mean age τ is assumed to be roughly half the lifetime of a main sequence star at the mean $B-V$. Group 7 consists of old giants.

Figure 3 shows the velocity distribution of Gliese's stars on or near the main sequence with $B-V<0.50$ (spectral types $\leqslant$F6) and $r \leqslant 20$ pc. The U-axis points towards the galactic center and the V-axis in the direction of galactic rotation. The space velocities are corrected for the solar motion, $U_0 = +9$ km s^{-1} and $V_0 = +12$ km s^{-1} (Delhaye, 1965). The most prominent feature in Figure 3 is the vertex deviation.

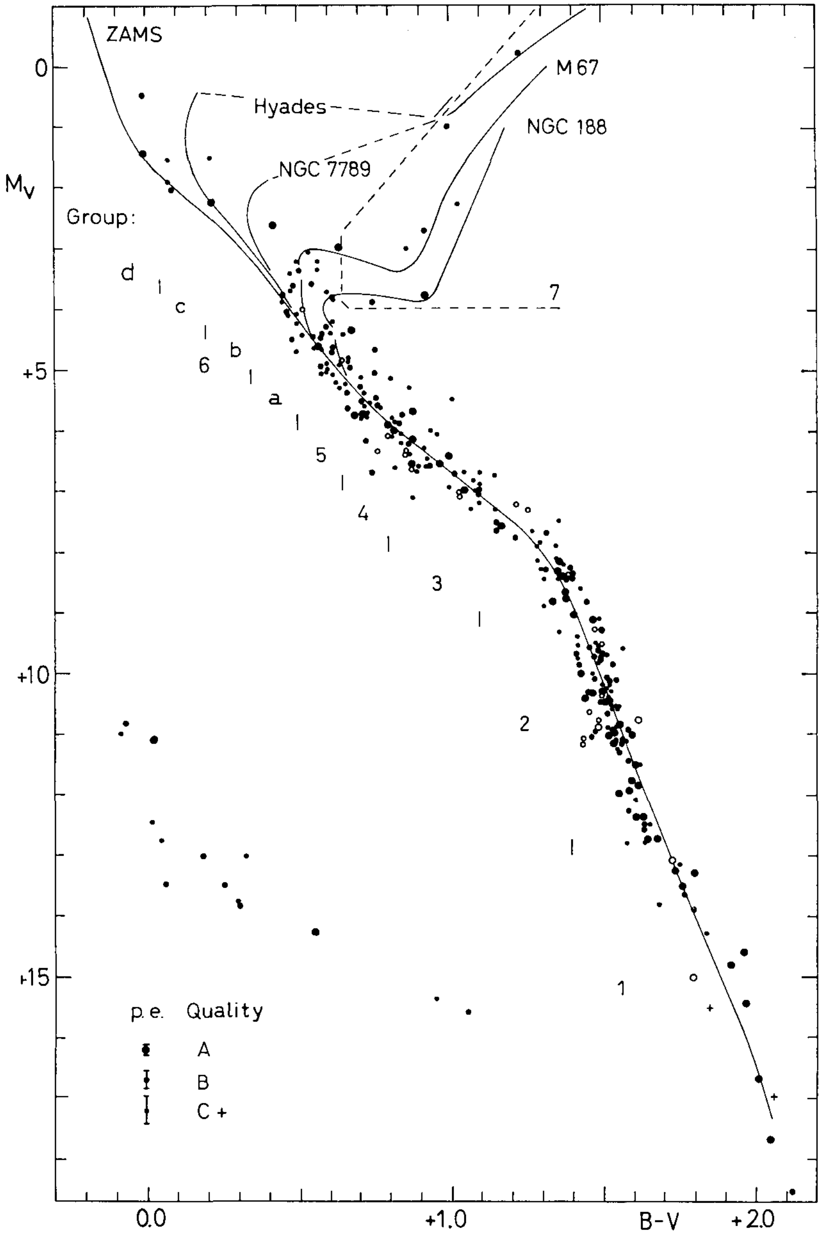

Fig. 2. Colour-magnitude diagram for Gliese's stars with accurate trigonometric parallaxes

No obvious streams or moving groups (Eggen, 1965) show up in Figure 3. In Table V, we present the results for the velocity dispersions σ and for the asymmetric drift of the stars in our groups. We include the McCormick K+M dwarfs contained in Gliese's catalogue, since they should be representative for the majority of the common

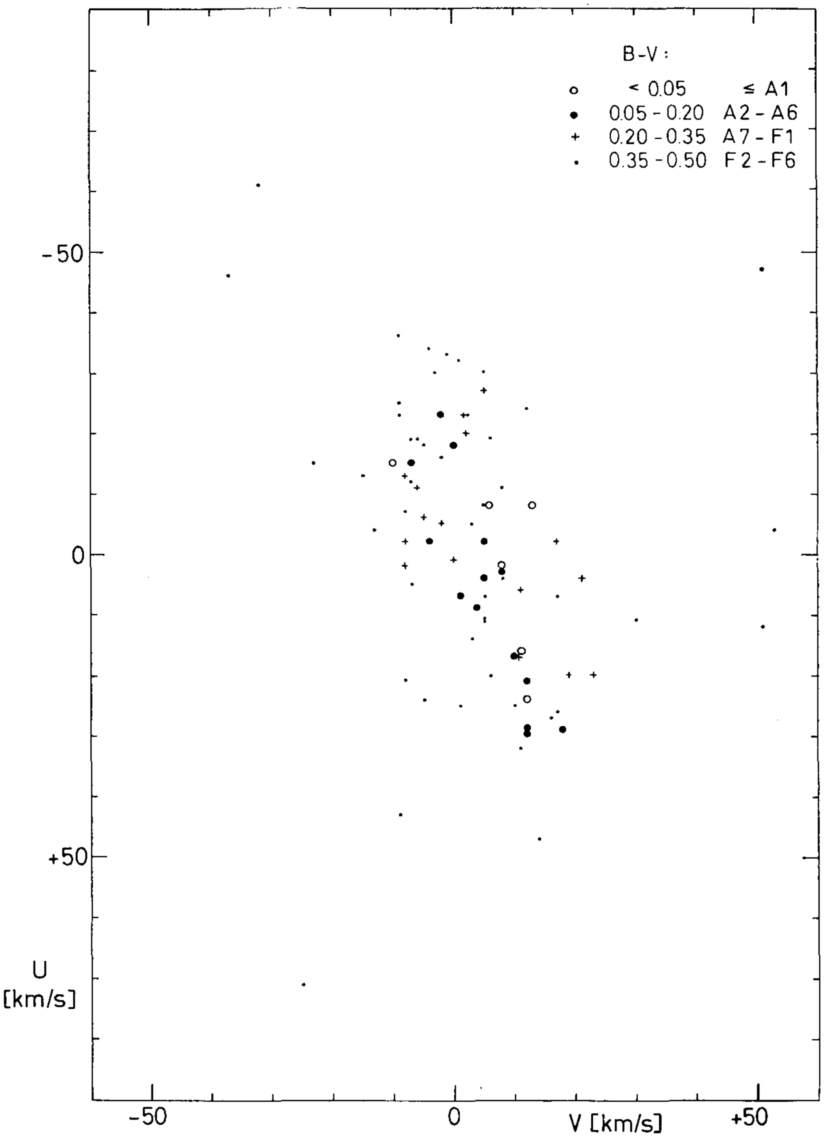

Fig. 3. Velocity distribution of Gliese's stars with $B-V<0.50$ and $r \leqslant 20$ pc.

nearby stars (see Section 6). The quantity $V_{\odot}$, listed in Table V, is the difference between the V-component of the Sun and of the mean motion of the stars in the group. Both the observed velocity dispersions and the asymmetric drift increase rather monotonically with the mean age of a group. The kinematical behaviour of the old giants and of the white dwarfs (see also Gliese, 1971) is quite similar to that of the McCormick K+M dwarfs. From Figure 4, we conclude that the relative increase of the velocity dispersions σ with age τ is roughly the same in the com-

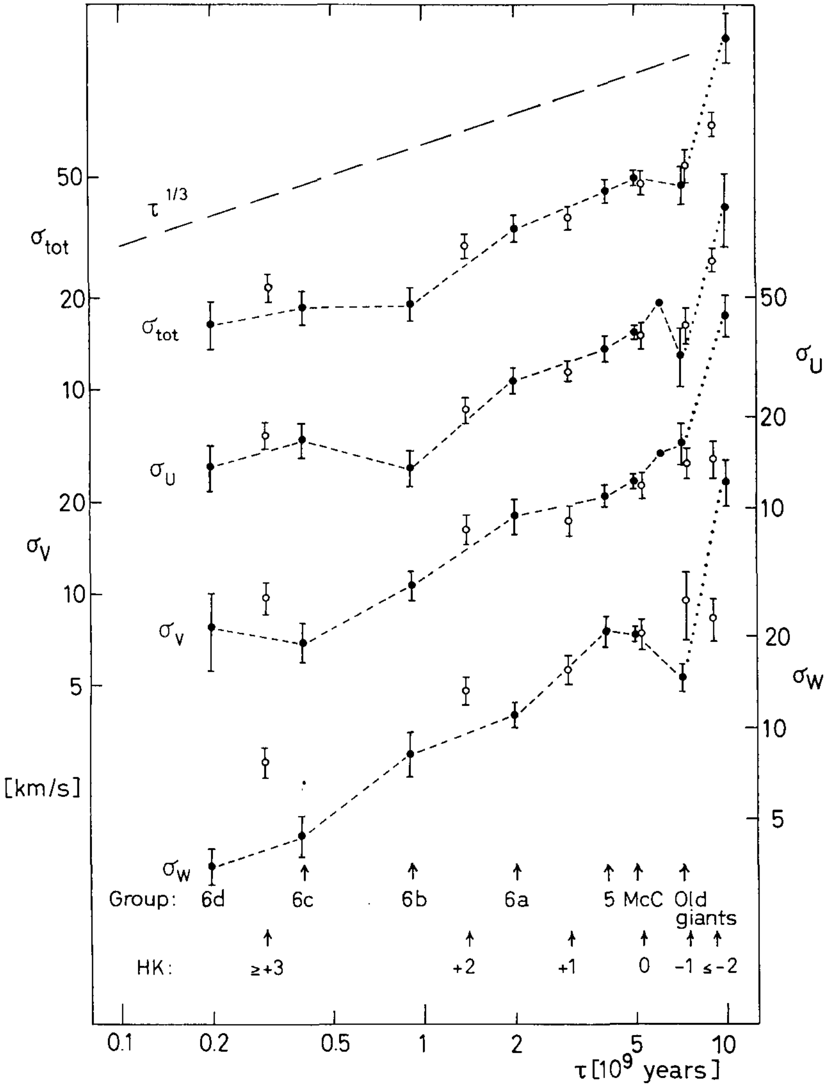

Fig. 4. Velocity dispersions as a function of age.

ponents parallel to the galactic plane, U and V, and perpendicular to the plane, W. The accumulating effect of encounters of the field stars with large complexes of stars and interstellar matter would lead to $\sigma \propto \tau^{1/3}$ (Spitzer and Schwarzschild, 1953), which is in quite good agreement with the observed behaviour (Figure 4). But other mechanisms may also explain the increase of σ with τ (Barbanis and Woltjer, 1967). In numerical experiments, we have studied the increase of σ_U and σ_V for stars born with small velocity dispersions, by the gravitational field of a stationary density wave of Lin's type. Preliminary results indicate that at least a significant fraction of the increase in σ_U and σ_V can be quantitatively explained by such a mechanism.

TABLE V

Velocity dispersion for groups in the colour-magnitude diagram

Group	N	σ_{tot}	σ_U	σ_V	σ_W	$V_\odot$	Mean age $\langle\tau\rangle$ [10^9 yr]
		[km s^{-1}]					
6d	6[a]	16	14	8	4	+5	0.2
6c	14[a]	19	17	7	4	+7	0.4
6b	16[a]	19	14	11	8	+7	0.9
6a	47[a]	34	27	18	11	+10	2
5	76[a]	44	34	21	21	+17	4
4	84[a]	48	34	27	20	+27	5
3	126[a]	49	36	28	17	+19	5
McCormick K+M dwarfs	317	50	39	23	20	+19	5
Old giants	25[b]	47	32	31	15	+31	7
White dwarfs	13[b]	50	42	22	18	+15	
Subdwarfs	8[b]	145	101	82	65	+104	10

[a] $r \leqslant 20$ pc, without classified subdwarfs.
[b] $r \leqslant 20$ pc.

4. Age Classification According to the Ca II Emission

It has long been known that the different kinematical behaviour of dwarfs with and without emission lines (dMe and dM stars) indicates an age difference for these groups. However, significant progress has been recently achieved by the observational work of O. C. Wilson. Wilson classifies the Ca II emission intensity at the H and K lines, using a visual scale ranging from about +8 (very strong emission) to −5 (extremely weak or no emission). Wilson's estimates (Wilson and Woolley, 1970) of the Ca II emission intensity, HK, are available for many McCormick K+M dwarfs in Gliese's catalogue, which should be representative of the common nearby stars (Figure 5). In Table VI, we give the velocity dispersions for these stars as a function of their Ca II emission intensity HK. The monotonic increase in σ with decreasing HK proves that the Ca II emission intensity is strongly correlated with the age of a star. Hence, the Ca II emission intensity may be used as a rather accurate indicator for the ages of the unevolved late-type dwarfs.

It is, however, difficult to convert the relative ages, obtained from Wilson's HK estimates, into absolute ages. Since the basic astrophysical mechanism for the decay of the Ca II emission is not accurately known, we may use either the relation between σ and τ from Section 3 to obtain a 'kinematical' calibration of HK(τ), or we may use the cumulative frequency of stars as a function of HK and an assumed rate of star formation for estimating τ. In Table VII we derive the mean ages of the various HK groups by assuming a constant rate of star formation over 10^{10} yr in a cylinder perpendicular to the galactic plane. The relative number of stars in such a cylinder is approximately obtained by multiplying the number N of stars in a volume near the plane by $|W|$ (see also Section 6). Although the absolute ages obtained should

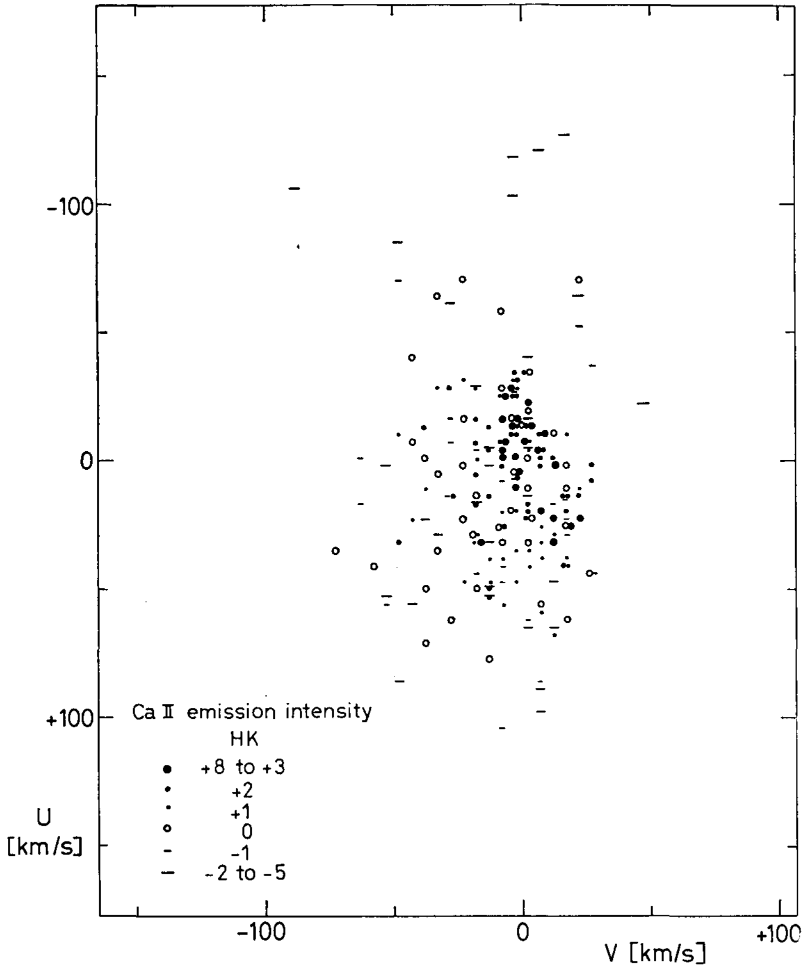

Fig. 5. Velocity distributions of McCormick K + M dwarfs in Gliese's catalogue with Wilson's classification of the Ca II emission intensity.

TABLE VI

Velocity dispersion of McCormick K+M dwarfs as a function of Ca II emission intensity

HK	N	σ_{tot}	σ_U	σ_V	σ_W	$V_\odot$
				[km s^{-1}]		
+8 to +3	23	22	18	10	8	+11
+2	40	30	21	16	13	+15
+1	36	37	29	17	15	+18
0	41	48	38	23	20	+25
−1	24	55	40	27	26	+21
−2 to −5	31	75	66	27	23	+25
Hα emission	18	34	25	19	14	+18

TABLE VII

Age estimates for McCormick K+M dwarfs assuming a constant rate of formation

Ca II emission intensity HK	N	$\langle \lvert W \rvert \rangle$ [km s^{-1}]	$N \cdot \langle \lvert W \rvert \rangle$	$\langle \tau \rangle$ [10^9 yr]
+8 to +3	23=12%	7	158= 6%	0.3
+2	40 21%	11	420 16%	1.4
+1	36 18%	13	464 17%	3.0
0	41 21%	16	665 25%	5.2
−1	24 12%	17	399 15%	7.2
−2 to −5	31 16%	18	547 21%	9.0

TABLE VIII

Vertex deviation ψ

for stars with $\sqrt{U^2+V^2} \leqslant 40$ km s^{-1} and $\lvert W \rvert \leqslant 15$ km s^{-1}

Group	N	ψ	m.e.	McCormick K+M dwarfs			
				HK	N	ψ	m.e.
On or near the main sequence:				+8 to +3	21	+23°	± 6°
				+2	30	+35	20
6d	6	+23°	±13°	+1	17	+ 2	6
6c	14	+20	4	0	19	+37	54
6b	15	+29	13	−1	10	+37	16
6a	34	+18	5	−2 to −5	4	(+57	40)
5	45	+22	6				
4	36	+ 4	14	Hα emission	12	+18	6
3	68	+ 5	6				
McCormick K+M dwarfs	159	+17	5	McCormick K+M dwarfs			
				$\lvert W \rvert$	N	ψ	m.e.
				0−5	67	+16°	6°
Other stars:				5−10	52	+18	10
Old giants	11	−70	26	10−20	58	+13	12
				>20	30	+14	16
White dwarfs	3	(+4	4)	All	159	+17	5
				All, but with weight $\lvert W \rvert$		+20	

be very crude, they fit astonishingly well with the relation between σ and τ derived in Section 3 (Figure 4). Furthermore, for the Hyades, the observed Ca II emission intensity, HK = +3 (Wilson, 1964, 1966), would lead to an age similar to the evolutionary cluster age of about 0.9×10^9 yr. The ages derived in Table VII indicate that the total decay time of the Ca II emission seems to be quite long ($\gtrsim 10^{10}$ yr) for K and M dwarfs.

5. Vertex Deviation

The question how the vertex deviation depends on the age of the stars, has recently been studied from an observational point of view by Woolley (1970), Wilson and Woolley (1970), Woolley *et al.* (1971), and Mayor (1972). They all find that the vertex deviation ψ is large for very young stars, small for stars of intermediate ages and practically zero for old stars. Unfortunately, however, the method used by these authors is biased in favour of such a result: They consider stars with an orbital excentricity e smaller than some chosen value, $e \leqslant 0.15$ for example. The region $e \leqslant 0.15$ in the velocity space is, however, roughly an ellipse with a major axis pointing exactly towards the galactic center. For a group of young stars, with a very small velocity dispersion, the bias towards $\psi \sim 0$ is insignificant. But for old stars, with a large velocity dispersion, the sampling volume in the velocity space defined by $e \leqslant 0.15$ favours small values of ψ, even if the real vertex deviation should be as large as for young stars. In order to avoid such a bias, but still to restrict the sample to low-velocity stars, we have chosen a circular limit in the UV-plane. The resulting vertex deviations ψ for our groups, using $(U^2 + V^2)^{1/2} \leqslant 40$ km s^{-1} and $|W| \leqslant 15$ km s^{-1}, are given in Table VIII. The large mean errors of ψ which occur are mainly due to the circular sampling volume in the UV-plane. While the young stars show a significant vertex deviation of about $+20°$, it is not obvious whether the vertex deviation for low-velocity stars decreases with increasing age. The old stars with high peculiar space velocities (not listed in Table VIII) do not show a significant vertex deviation.

6. Representative Common Nearby Stars

For many dynamical purposes, such as the construction of dynamical models of the Galaxy or the discussion of the gravitational stability of the Galaxy, one is interested in the total velocity distribution of all nearby stars. As pointed out in the introduction, the sample of all Gliese stars is not adequate for that purpose because of selection effects. However, the subgroup of the McCormick K+M dwarfs in Gliese's catalogue should be a rather representative sample for the velocity distribution of all nearby stars. The McCormick stars are free from the kinematical selection effects, since they are discovered on objective prism plates (Vyssotsky, 1963), and they should be a representative mixture of stellar ages. The velocity dispersions and the asymmetrical drift of the McCormick K+M dwarfs in Gliese's catalogue are listed in Table V. The velocity distribution of these stars, not reproduced here, shows the well-known asymmetry in the velocity component V, and a pronounced vertex deviation for the low-velocity stars.

The solar neighbourhood is a sampling volume situated close to the galactic plane. While the velocity distribution of the McCormick stars is representative for nearby stars, now at $z \sim 0$, this local distribution is not representative for all stars in a cylinder perpendicular to the galactic plane, because the velocity distribution varies with the distance z from the plane (Vandervoort, 1970). This can be shown by grouping the

TABLE IX

Velocity dispersion of McCormick K+M dwarfs grouped according to $|W|$

$\|W\|$	N	σ_U	σ_V	$V_\odot$	Mean height $\langle\|z\|\rangle$ [pc]
		[km s^{-1}]			
0–5	86	31	19	+15	28
5–10	66	31	20	+17	55
10–20	91	37	21	+20	100
>20	74	52	31	+25	300
All	317	39	23	+19	
All, but each star weighted by $\|W\|$		48	29	+23	200

nearby stars according to their vertical space velocity, $|W|$. Table IX shows the remarkable increase of the velocity dispersions parallel to the galactic plane, σ_U and σ_V, with $|W|$. We have also indicated the mean height $\langle|z|\rangle$, averaged over an oscillation period in z, of stars which pass now through the plane with a vertical velocity $|W|$. The increase of σ_U and σ_V with $|W|$ is an effect of the different ages of the stars. Stars with higher vertical velocities $|W|$ are on the average older and move therefore with higher velocity dispersions in U and V, since σ_U, σ_V grow together with σ_W (Section 3).

How can we derive a velocity distribution which is representative for all stars in a cylinder perpendicular to the galactic plane? Let us assume that the z-motions of the stars are well-mixed (stationary state) and decoupled from the motions parallel to the plane, and that the oscillation period T_z does not depend on the amplitude of the z-motion (approximately fulfilled for $|z| < 500$ pc). The probability p of finding a star in a small element Δz at $z=0$ is given by the time for crossing Δz, T_c, divided by T_z. Since T_c is inversely proportional to $|W|$ and $T_z \sim$ const., we get $p \propto |W|^{-1}$. In order to derive a velocity distribution representative for the cylinder, we have to weight each star in our sampling volume at $z=0$ by a weight p^{-1} or just by $|W|$. The weight $|W|$ is a conservative lower limit in so far that T_z actually increases slowly with $|W|$ (see also Woolley *et al.*, 1971). In the last line of Table IX, we give the resulting velocity dispersions representative for the cylinder. The velocity dispersions σ_U and σ_V integrated over z, are higher than those at $z=0$ by about 25%. The z-averaged value $\sigma_U = 48$ km s^{-1}, compared with 39 km s^{-1} at $z=0$, has some interesting implications: the stability of our Galaxy against a local gravitational collapse is improved, but the propagation of a density wave of Lin's type may be rendered more difficult. These topics are treated in detail by Toomre in his contribution to this Joint Discussion.

Acknowledgements

This contribution rests heavily on the results of a broader statistical analysis of Gliese's catalogue which has been carried out by Mr H. Jahreiss (1974) and myself. We would like to thank Dr W. Gliese very much for many stimulating discussions and helpful comments.

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DISCUSSION

Buscombe: The claim of completeness of bright southern stars is incorrect. A 5th mag. F dwarf, γ Cir B, is the closest bright member of an optical double. In my opinion, the dependence on trigonometrical parallaxes to delineate groups of stars has been over-emphasized.

Wielen: The completeness of bright stars within 20 pc is meant in a statistical sense. Of course, due to changes in the adopted values of the parallaxes, there will be always some borderline cases for any chosen limit in r. In the present investigation, we have used the 'resulting parallaxes' given in Gliese's catalogue which are a combination of trigonometric, spectroscopic and photometric distance determinations. However, only the trigonometric parallaxes should be used for deriving individual ages of evolved stars from isochrones, since the spectroscopic and photometric parallaxes derived by Gliese are based on *assumed* absolute magnitudes.

Irwin: Do you implicitly assume, when you calculate incompleteness factors for stars from 10 to 20 pc distant, that the stars are uniformly distributed in space? Or do you allow for any thinning out of the stars in the z direction?

Wielen: For deriving the luminosity function, I assume that the stars would be uniformly distributed within $r \leqslant 20$ pc, if there were no selection effects. I do not use incompleteness factors. The luminosity function for fainter stars is derived from smaller volumes for which the data indicate a uniform distribution within the chosen limit of r.

Murray: The kinematics of the stars in Wilson's classes $+8$ to $+3$ are very similar to those of the stars in the north galactic cap which I discussed. Is there any selection effect operating against the inclusion of these stars in Gliese's catalogue?

Wielen: The McCormick K + M dwarfs in Gliese's catalogue for which Wilson gives Ca II emission intensities, should be a representative sample of dwarfs in the spectral range covered (mainly from K8 to M2). Considering Gliese's catalogue *as a whole*, the stars in Wilson's classes $+8$ to $+3$ should be underrepresented, because they have small space velocities and are therefore affected by the kinematical selection effect.