BOOK REVIEWS

Mathematical Methods in Risk Theory by HANS BÜHLMANN, Springer-Verlag, Berlin, Heidelberg, New York 1970. pp. XII + 210.

This is the third text book on the subject of risk theory to be published in English in the past 18 months, a sufficient indication of the growth of knowledge in this branch of applied probability theory. However, in spite of this significant burst of activity some significant gaps remain.

Modern developments in risk theory have largely stemmed from problems arising in the transaction of non-life insurance. A great deal of work has been done on the analytical aspects of both direct and reinsurance business but the continued adverse results being shown throughout the world suggest that the underlying models are not yet on an adequate basis for convincing those concerned with commercial decisions.

The introductory text book by Beard, Pesonen and Pentikäinen was designed to provide a fairly comprehensive coverage of practical applications of risk theoretical methods without delving too deeply into the more detailed mathematical aspects. Seal's book, Stochastic Theory of a Risk Business provided a very comprehensive survey of practically all contributions to the subject, written largely from the viewpoint of the mathematical statistician rather than the practising actuary. Professor Bühlmann's book now examines a number of the basic concepts underlying risk theory, including the very interesting developments in Credibility Theory which stemmed from the contact in ASTIN between members of the Casualty Actuarial Society and European actuaries. The sorely pressed manager responsible for Motor business, or excess loss reinsurance, will find little comfort in this latest addition to the literature, but he will find a systematic and illuminating treatment of fundamental concepts.

A considerable debt is owed to Professor Bühlmann in making available his deep understanding of the subject. Any scientific discipline must be soundly based if it is to be of any value for practical application. Much of the fundamental work in risk theory has been distributed over actuarial and other scientific publications and the task of the teacher has been considerably eased by having a modern and convenient source of this basic work. The students of today will be enabled to tackle the practical problems with greater confidence.

The book falls into two parts, Part I treating the theoretical model and Part II the consequences of this model. Chapter 1 is a resumé of the necessary elements of probability theory, without using measure theory. The distribution of claim amounts is used as a basis for introducing random variables, the reader being taken rapidly through continuous and discrete distributions, characteristic functions, generating functions, expectations, covariance and other concepts.

In Chapter 2 the author introduces the stochastic process and defines the compound Poisson process (using Feller's terminology). Treating the claim number process as a stochastic process he shows how the transition probabilities can be found from Kolmogorov's forward system. From this he considers

the claim interoccurrence time which leads directly to operational time and the homogeneous claim number process. A limited discussion on contagion is followed by the basic forms for the accumulated claim distribution.

The concept of the Collective is introduced in Chapter 3 as a generalisation of the risk processes dealt with in Chapter 2. This brings in the meaning of structure function and to the weighted Poisson and negative binomial distributions with a brief reference to infinitely divisible distributions and their importance in the theory.

Chapter 4 is concerned with premium calculation and the author distinguishes four principles, namely (a) the expected value (b) the standard deviation (c) the variance and (d) the zero utility, which are met in practice. He carefully distinguishes a risk premium and a collective premium and compares the results using the four principles. The argument then develops naturally into a detailed analysis of the credibility premium, i.e. the technique of using actual experience to modify a previously determined risk premium. This is a very valuable presentation of the very recent developments in this area.

The retention and reserve problems are dealt with in Chapter 5. For proportional reinsurance he derives the well known rule that the proportionality factors for risk depend on the loadings in the premiums. However, the discussion is quite deep and involves consideration of the conceptual difference between risk and collective premiums. The absolute retention problem cannot be solved without introducing some formulation of the business aims of the reinsurer and is dealt with in Chapter 6. The treatment of reserves is brief as the author is concerned only with that required to cover fluctuations in the risk operations.

Chapter 6 occupies about one third of the book and is a most valuable and interesting survey of the current status of risk theory as applied to an insurance operation. The heading, "The Insurance Carrier's Stability Criteria" is self-explanatory and the author points out three criteria which have been studied, i.e. (i) probability of ruin (ii) dividend policy and (iii) utility, to which may be attached the names (i) Lundberg and Cramér (ii) de Finetti (iii) Borch. The ruin probability is first derived for finite and infinite time and for discrete and continuous intervals and from this the author is able to find the parameter to complete the net retention and stability problems.

The second criterion may be briefly described as optimising by seeking the largest possible expected discounted sum of dividends given the initial free reserves. The problem is essentially selecting from the various random walks that characterise the movement of the free reserves and the author sets out the analysis in the discrete and continuous cases.

The third criterion is a development in that a utility function is introduced to discriminate between the various random walks of the free reserves. This has been studied by Borch and opens the way to consideration of problems involving market considerations, e.g. reinsurance exchanges.

The book concludes with a short appendix on the generalised Riemann-Stieltjes integral and specialised bibliography of 67 items.

Most sections of the book conclude with a few pertinent exercises but no solutions are provided so that the student working on his own is at some disadvantage. But this is a small inconvenience compared with the benefit of having an up to date and readable text for which we are greatly indebted to Professor Bühlmann.

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