53. A MODEL FOR THE NUCLEUS OF ENCKE'S COMET

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Abstract. A study of the nongravitational deviations in the motion of P/Encke suggests that the nucleus of this comet can conceivably be approximated by a core-mantle model, composed of a porous solid core with embedded ices, surrounded by a thick envelope of free ices that gradually sublimates out.

1. Introduction

Recent investigations have brought more evidence on the total mass output from comets. Perhaps the most exciting and also most obvious are the recent observations of extensive hydrogen and hydroxyl atmospheres of comet 1969 IX (Tago-Sato-Kosaka) from OAO 2 and comet 1969i (Bennett) from OAO 2 and OGO 5 (Code *et al.*, 1970; Houck and Code, 1970; Bertaux and Blamont, 1970), which definitively confirm that the C_2 and CN bands, typical of the visual and photographic spectra, are produced by a very small fraction of the total output of material from the comets. In the light of this and other less direct evidence, Whipple's (1950) interpretation of the nongravitational effects in comets in terms of loss of volatile materials from the nucleus deserves full credit.

The enormous increase of information about the dynamics of comets from recent orbital calculations can also contribute to our understanding of the physical processes in, on and near the cometary nucleus, but an elaborate theory consistently interpreting the dynamical data is still missing.

2. Periodic Comet Encke

In order that we can study long-term variations in the mass output from comets, observations from as long a period of time as possible have to be considered and the results checked with the other evidence.

The most extensive information that can serve for this purpose is the list of data on the deviations of the motion of periodic comet Encke from the law of gravitation. The writer has compiled such a list for the period of almost two centuries (Sekanina, 1969a). Marsden's (1969, 1970) calculations on the motion of this comet suggest that at recent apparitions the radial component of the nongravitational acceleration may have been insignificant relative to the transverse component, or at any rate that it was not an order of magnitude larger than the transverse component. Both the analysis of the nongravitational terms in Marsden's equations of motion (Sekanina, 1972) and a theoretical study of the distribution in the gas-jet impulses over the nuclear surface of an Encke-type comet (Sekanina, 1970) support the idea that the effective direction of the nongravitational forces acting on such a comet is expected to be essentially perpendicular to the comet's radius vector.

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We shall measure the dynamical effect of the nongravitational acceleration in terms of the classical parameter κ, introduced by Encke (1829). It is defined by

$$\kappa = 600 \int_{t}^{t+P} \frac{d\mu}{dt} \, dt, \tag{1}$$

where μ is the daily mean motion of the comet. Expanding Whipple's (1950) ideas, the writer has shown that κ is generally proportional to the relative mass-loss rate of the comet per revolution (Sekanina, 1969b):

$$\kappa = C(\Delta M/M), \tag{2}$$

with the proportionality coefficient

$$C = -C_1 C_2 D (1 - e^2)^{1/2} \frac{\int_{-\pi}^{\pi} f(r) r \, dv}{\int_{-\pi}^{\pi} f(r) r^2 \, dv}. \tag{3}$$

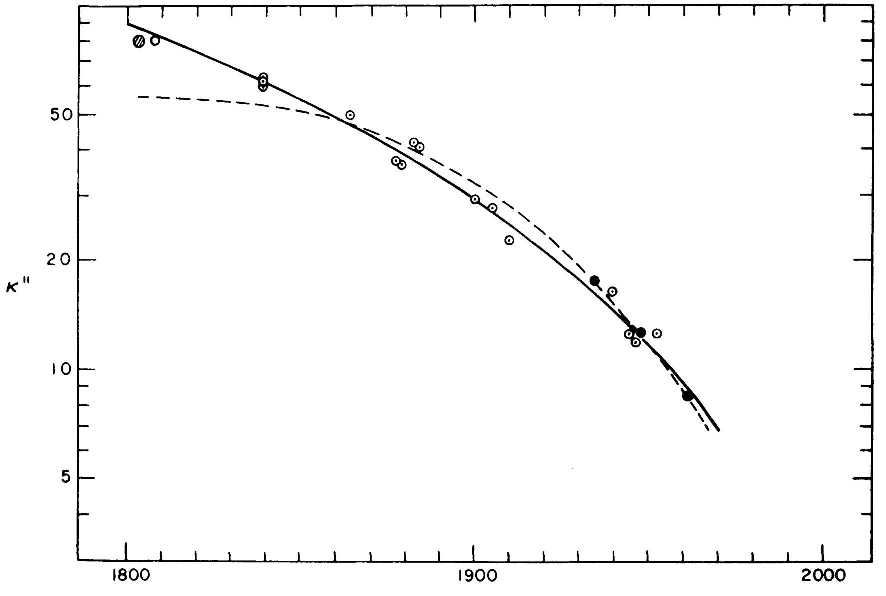

Fig. 1. Nongravitational parameter κ as a function of time for Encke's Comet. Circled points are κ values obtained in a classical way; solid circles are from Marsden's (1970) double exponential curve; the shaded circle is a provisional value, determined by the writer from Encke's 'pure elliptic revolution periods', covering the period 1786 to 1819; and the open circle is a value derived in the same way for the period 1795 to 1819. The full curve represents the best fit by means of Equation (10); the dashed curve is the Marsden double-exponential fit from 1927 to 1967 and extrapolated back.

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Here C_1 is a positive constant, C_2 the transverse component in units of the total acceleration, D the effective velocity of ejection (i.e., integrated over the nuclear surface), e the orbital eccentricity, and $f(r)$ the law of variation of the nongravitational acceleration with solar distance r. In terms of the Whipple hypothesis the sign of C is just a matter of the sense of rotation. P/Encke has a secular acceleration, so that κ is positive. The variations in the nongravitational effects of this comet since the beginning of the nineteenth century are represented in Figure 1.

3. Models of Mass Output

We assume that the comet moved originally in an orbit that prevented it from being exposed to intense solar radiation. Whatever the internal structure of the nucleus, we assume that it was uniform at a certain time t_0, when the comet was gravitationally captured (presumably after a number of close approaches to Jupiter) and forced to move in a rather stable short-period orbit with perihelion distance comparable with its present value. The comet then started losing its volatile supplies at a significant rate.

We also assume that the nucleus of P/Encke is monolithic, spherical in shape, of radius R_0 at time t_0, and rotating rather fast. Assuming that specific, simple mechanisms govern the mass transfer in the nucleus and from its surface, we determine the relative rate of mass output per revolution $\Delta M_j/M_j$ as a function of the number of revolutions j since the time t_0.

For an icy nucleus with scattered dust impurities embedded in it, the law for the rate of mass loss is simply

$$\frac{\Delta M_j}{M_j} = \frac{3\alpha}{1 - \alpha j}, \tag{4}$$

where α is the linear rate of shrinkage of the nuclear radius in units of the original radius R_0 ($\alpha \ll 1$). Because of Equation (2), in this model the secular acceleration increases with time, in sharp contrast to what is observed for P/Encke.

Thus we shall consider instead a composite model, assumed to consist of a porous matrix of solid material, with ice filling the pores. The tensile strength of the nucleus is assumed to be sufficiently high to withstand the pressure from the diffusing volatile component, so that the nucleus of radius R_0 does not shrink with time. The mass distribution at t_0 in either of the two components is assumed to be uniform throughout the nucleus, the apparent ice density being ρ_0, the total apparent density ρ_c. During the first post-capture approach of the comet to the Sun, all the ice in a surface layer of thickness $\alpha_1 R_0$ is assumed to sublimate out. This loss is in part compensated by diffusion of volatiles from deeper layers to the surface. Before the comet comes to the Sun again, it is assumed that the uniform distribution of ice inside the nucleus is entirely restored at an apparent density of

$$\rho_1 = \rho_0(1 - \alpha_1)^3 < \rho_0. \tag{5}$$

During the second approach to the Sun, the surface layer of thickness $\alpha_2 R_0$ is depleted of ice, and so on. The output rate of this model follows the law

$$\frac{\Delta M_j}{M_j} = \frac{\dfrac{1-(1-\alpha_j)^3}{(1-\alpha_j)^3} \prod_{k=1}^{j} (1-\alpha_k)^3}{\rho_c/\rho_0 - \sum_{i=1}^{j} \dfrac{1-(1-\alpha_i)^3}{(1-\alpha_i)^3} \prod_{k=1}^{i} (1-\alpha_k)^3}. \tag{6}$$

If $\alpha_1 = \alpha_2 = \cdots = \alpha = \text{const}$, Equation (6) simplifies to

$$\frac{\Delta M_j}{M_j} = \frac{\exp \gamma - 1}{1 + [(\rho_c/\rho_0) - 1] \exp(\gamma j)}, \tag{7}$$

where

$$\gamma = \ln \frac{1}{(1-\alpha)^3} > 0. \tag{8}$$

Equation (7) indicates that the nongravitational acceleration decreases with time, in qualitative agreement with what is observed for P/Encke.

Since the apparent ice density decreases with time, the thickness of the surface layer to be depleted each revolution may increase with time. Accepting the relation

$$\alpha_j \rho_{j-1}^{1/3} = \text{const}, \tag{9}$$

Equation (6) becomes

$$\frac{\Delta M_j}{M_j} = \frac{3\alpha(1-\alpha j)^2}{\rho_c/\rho_0 - 1 + (1-\alpha j)^3}, \tag{10}$$

where $\alpha = \alpha_1$, $\rho_c/\rho_0 \gg 1$. Equation (10) suggests that κ decreases with time if $\rho_c/\rho_0 \geqq 1.5$, but that it first increases and then decreases if $\rho_c/\rho_0 < 1.5$. Of course, only the former case is physically acceptable, and we again get qualitative agreement between the model and the observations.

The qualitative difference between the two versions of the composite model is that in the latter case the nucleus becomes entirely depleted of volatiles after completing $(1/\alpha)$ revolutions about the Sun since the time of capture (finite lifetime), whereas in the former version the nucleus has an infinite lifetime.

4. Failure of the Two Versions of the Composite Model. Core-Mantle Model. Conclusions

Applying the two versions of the composite model to P/Encke, we find that they fit the data almost equally well (mean residual in κ $\pm 2''.10$ for the finite version, $\pm 2''.19$ for the infinite), but the mass output rates differ by an order of magnitude (Table I), the ones resulting from the infinite version being much too high.

The finite version would be promising if it did not yield unacceptable values for the time of capture in terms of the density ratio ρ_c/ρ_0 (these two quantities cannot be separated from each other). It follows that the comet should have been captured

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2600 yr ago if 99% of its mass were due to ice, 1000 yr ago if 88% were ice, and only 500 yr ago if 53% were ice. In fact, from the investigation by Whipple and Hamid (1952) of the Taurid meteor stream, it follows that P/Encke must have been captured at least 5000 yr ago; and from physical considerations we can estimate that ice, trapped in a porous but compact structure of the nonvolatile matrix of the composite nucleus, could not make up more than, say, 45% of the total mass.

TABLE I

Relative output rates of P/Encke computed from the composite model

Time	$\Delta M/M$ (percent per revolution) finite version	infinite version
1800	0.34	6.4
1833	0.26	5.2
1867	0.18	3.8
1900	0.12	2.3
1933	0.069	1.3
1967	0.031	0.66

This discrepancy suggests that the composite model is quantitatively unacceptable and that ice must have played a much more important role in the history of Encke's Comet than the above model indicates. This has been the main reason for our suggesting a combination of the composite model (finite version) and the icy model into a core-mantle model.

The proposed core-mantle model has a solid, compact though porous core with embedded ices. The core is surrounded by a thick shell, or envelope, of free ices. It is physically reasonable to assume that the shell ices are contaminated by loosely distributed dust particles. After the comet has been captured and forced to orbit the Sun in a short-period path, the outer icy shell shrinks gradually owing to sublimation, as in the pure icy model. This process continues until the shell is completely sublimated out and the underlying solid core is exposed to solar rays. Free sublimation is then replaced by activated diffusion, and from this time on the model can be treated as a finite composite model. The mass output in the core phase is therefore given by Equation (10), where j is the number of revolutions from the time the core was dismantled. It can be shown that the mass-loss rate in the mantle phase is also described by an expression of the same form, provided that α is replaced by α_e, the rate of shrinkage of the envelope (in units of the initial nuclear radius R_0 per revolution), and that $\rho_c/\rho_0 - 1$ is replaced by $(R_c/R_0)^3(\rho_c/\rho_e - 1)$, where R_c is the radius of the core and ρ_e is the *apparent* density of the envelope. The process of dismantling of the core should be accompanied by a significant decrease in the output of volatiles due to the expected redistribution of the solar radiation effects on the new type of surface (with more energy needed for heating low-conductivity solid materials of the core). Quantitatively, the output deficit can be characterized by a coefficient Γ ($0<\Gamma<1$), defined by

$$\lim_{j\to j_c+} (\Delta M_j)_{\text{core}} = (1-\Gamma) \lim_{j\to j_c-} (\Delta M_j)_{\text{mantle}}, \tag{11}$$

where j_c is the time of core dismantling. In practice the dismantling process and the associated discontinuity in output occur rather smoothly. In physical terms this means that a layer of the nucleus was exposed, in which the tensile strength of the solid matrix, generally increasing from the surface toward the center, reaches a critical point; at this point the matrix is just able to withstand pressure from activated ices without losing any more substantial amounts of solid particles.

The application of the core-mantle model to P/Encke leads to the following suggestions about its evolution:

(1) The comet appears to be currently in the core phase. The averaged ejection velocity comes out 0.18 km s^{-1} (if $C_2 \simeq 1$, see Section 2), which suggests that for a thermal velocity of 0.6 km s^{-1} the anisotropy coefficient is as high as 0.3. The comet entered this phase about the year 1800 if the embedded ices made up originally 10% of the total mass of the core, about the year 1700 if they made up 20%, and about 1600 if 35%. The comet is expected to be entirely depleted of volatile substances in about 60 to 70 yr, or some 20 revolutions from now. This conclusion is in fairly good agreement with the photometric death date of the comet predicted by Whipple and Douglas-Hamilton (1966).

(2) Extrapolating the suggested evolution of the comet backward to its mantle phase, we find that a shell several meters in thickness would have been blown off during each revolution. We note that a sphere of pure water ice moving in the orbit of Encke's Comet would shrink at a rate of 2 to 3 m per revolution; contamination by highly absorbing meteoric particles would raise the rate to 4 to 5 m per revolution, while a sphere of unpacked water snow would diminish at a rate of 6 to 8 m per revolution. The corresponding brightness decrease with time would have been very slight, not more than 0.2 magnitude per century. The comet was probably not brighter than absolute magnitude 6 some 1000 yr ago.

(3) In contrast, the fading of the comet in its core phase is expected to be considerably greater, and progressively increasing. Assuming proportionality between the total gas output and the loss of constituents providing the C_2 bands that are dominant in the comet's visual spectrum, the expected luminosity deficit should currently amount to about 3 magnitudes per century, or 0.1 magnitude per revolution. This is in good agreement with the available photometric data (e.g., Sekanina, 1964; Whipple and Douglas-Hamilton, 1966).

(4) The estimate of the original dimensions of the nucleus of Encke's Comet depends much on the expected time of capture. If capture took place about 5000 yr ago (as indicated from association with the Taurids), the radius of the original nucleus would have been about 10 km; if it happened 20 000 yr ago, the radius would have been more like 25 to 30 km.

(5) Capture 5000 yr ago is likely because of its dynamical implications. The non-gravitational acceleration has produced a secular reduction in the comet's aphelion

distance. We can estimate that the aphelion distance amounted to about 4.7 AU 5000 yr ago, and that it would have been 5.2 AU 20 000 yr ago. In either case, the comet's aphelion might have been already inside Jupiter's orbit, but not until 5000 yr ago did the minimum distance between the two orbits become large enough to ensure the stability of that of the comet; before then, expulsion of the comet into a distant orbit would have been possible. Further, some 5000 yr ago the comet should have been near 3:1 resonance with Jupiter, and that might have helped to keep the comet out of the reach of Jupiter, although the comet had a large enough nongravitational acceleration eventually to escape from a possible libration. The Taurids are perhaps a product of the comet's early violent evolution in the innermost part of the solar system.

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