that is

$$\frac{\log v}{v} < \frac{2}{\sqrt{v}} - \frac{2}{v} \rightarrow 0 \text{ as } v \rightarrow \infty .$$

Put $v = x^{\alpha}$, $\alpha > 0$. Then also

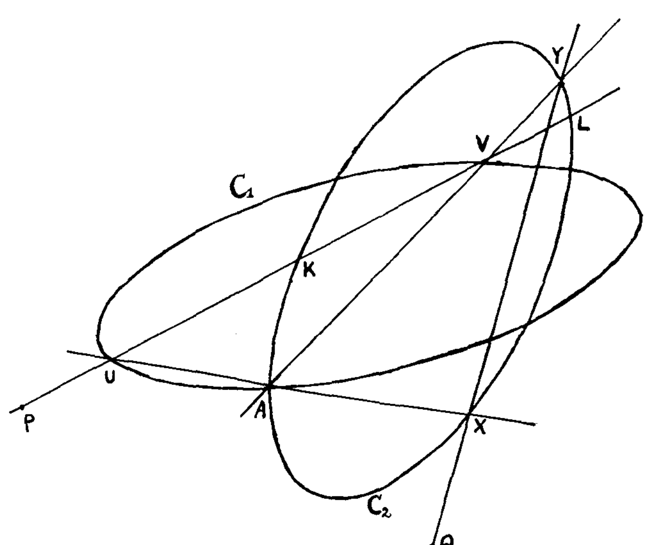
$$\frac{\log x}{x^{\alpha}} \rightarrow 0 \text{ as } x \rightarrow \infty .$$

This shows that $\log x$ tends to infinity $\left(\text{with } \Sigma \frac{1}{n}\right)$ more slowly than any positive power of x, as x tends to infinity.

A Synthetic Derivation of the Class of the Φ Conic

By H. E. Daniels.

The theorem that a line cutting a pair of conics in four harmonically separated points envelopes a conic, called the Φ conic, is a familiar result which admits of a simple proof by analytical methods. A synthetic proof, however, if we exclude the use of (2, 2) correspondences, is rather elusive. I have not been able to find such a proof in any book, and the only one published as far as I am aware is that set as a question in the 1934 Mathematical Tripos, due to Mr F. P. White. The proof written out below is rather more direct and may therefore be worth recording.



We are given the conics C_1 and C_2, and A is any one of their intersections. Take any point P in the plane. Then lines through P cut C_1 in pairs of points U, V which form an involution on C_1. These pairs are projected from A by a pencil of lines in involution whose pairs meet C_2 in the pairs of points X, Y of an involution. Their joins XY must therefore pass through a fixed point Q.

Clearly, each line PUV can only lead to the unique line QXY, and each QXY can only be arrived at from a unique PUV; in other words, the lines PUV and QXY are pairs of a (1, 1) correspondence.

Consider now lines through P and Q conjugate with respect to the conic C_2. They are seen to be pairs of another (1, 1) correspondence; for given any line α through P, its pole with respect to C_2, and therefore the line β joining Q to the pole, is uniquely determined, and starting from β we come back uniquely to α.

Now in general there are two lines through P for which the corresponding lines of the pencil through Q in the two (1, 1) correspondences coincide; that is, there are two lines through P such that $(KLXY)$ is harmonic, in which case by projection from A, $(KLUV)$ is harmonic. Thus through a general point in the plane there pass two lines only which cut the two conics harmonically. It follows that such lines envelope a conic.

A Useful Expansion in Applications of Determinants

By A. C. AITKEN.

The expansion, in the form of a series, of the quotient of two determinants, in which the numerator determinant differs from the denominator in one column only, is moderately well known. It was found in 1825 by F. Schweins (see Muir's *History*, Vol. 1, pp. 171-2), and has many times since been rediscovered or proved *ad hoc* for particular applications.

There is a special case of this expansion, so useful that it seems worth while to present it on its own account. It is, for example,

$$|a_1 b_2 c_3 d_4| \div |b_2 c_3 d_4| = a_1 - \frac{a_2 \, . \, b_1}{1 \, . \, b_2} - \frac{|a_2 b_3| \, |b_1 c_2|}{b_2 \, |b_2 c_3|} - \frac{|a_2 b_3 c_4| \, |b_1 c_2 d_3|}{|b_2 c_3| \, |b_2 c_3 d_4|}, \quad (1)$$

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