PROCEDURES AND BASIC STATISTICS TO BE USED IN MAGNITUDE CONTROL OF EQUALISATION RESERVES IN FINLAND

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This report aims at describing the procedures and statistics prepared for use in calculation of the limits of the equalisation reserves of Finnish insurance companies. The report is based on work done by a committee which the Federation of Finnish Insurance Companies set up in 1962. Its purpose was to collect and work up the necessary statistics and to develop computational methods to be used in practice for the computation of equalisation reserves. In one respect the work was initiated by the Supervisory Service, which prepared new and more precise regulations concerning the determination of limits of the above reserves.

Because solvency problems of insurance companies have had great attention in recent years internationally, and the Finnish equalisation reserves are closely related to them, the theme may be sufficiently interesting to be presented once more setting out practical results. The solvency problem among other questions has been dealt with by Drs. Pesonen and Pentikäinen in former colloquia. This report can be considered as a continuation of Dr. Pesonen's paper "Magnitude Control of Technical Reserves in Finland" submitted to the Lucerne Colloquium and my paper "A Procedure to Compute Values of the Generalised Poisson Function" to the same colloquium.

Principles and practical formulae

The principles and theoretical formulae to be used in computation of the limits for the equalisation reserves have already been stated in the papers of Drs. Pesonen and Pentikäinen. Other technical reserves are not dealt with here.

The equalisation reserve is roughly defined as that part of the technical reserves which exceeds the conventional premium reserve

https://doi.org/10.1017/S0515036100008084 Published online by Cambridge University Press

and the claims reserve and which secures the company's solvability against random fluctuations of claims and fluctuation of the basic probabilities of the claims. The equalisation reserve E must be greater than a minimum $E_{\min}$ from solvability reasons and, because of taxation, should not exceed a maximum amount $E_{\max}$.

Let us denote by

- x the total amount of claims during one year on a company's own risk,
- P the corresponding net premium income,
- U the company's own capital and free reserves,
- λ the security margin in premiums and
- i the rate of interest.

If the company's size and the structure of its portfolio are assumed to remain the same, the equation

$$\Pr\{(1+i)(E_{\min}+U)+\sqrt{1+i}\,[(1+\lambda)P-x]\geq 0\}=1-\varepsilon \quad (1)$$

can serve as a definition of the minimum amount $E_{\min}$. If the reserve $E = E_{\min}$, the probability of ruin after one year is ε. [1])

The total amount of claims x is assumed to have the distribution

$$F(x) = \sum_{r=0}^{\infty} p_{\bar{n}}(r)\, S^{r*}(x), \quad (2)$$

where

$\bar{n}$ = the expected number of claims $= \Sigma \bar{n}_k$,

$p_{\bar{n}}(r)$ = the probability of r claims occurring and

$S(x) = \sum_k \frac{\bar{n}_k}{\bar{n}} S_k(x)$, where $S_k(x)$ is the distribution of one claim in branch k. The reinsurer's share is excluded except as regards Stop Loss treaties, which are taken into account in a later phase of the calculations.

The number of claims in the following is assumed to have the Poisson distribution

$$p_{\bar{n}}(r) = e^{-\bar{n}} \frac{\bar{n}^r}{r!}. \quad (3)$$

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[1]) A further restriction posed on $E_{\min}$ in Finland is

$$E_{\min} \geq \text{Max}\,\{0,\ M \dot{-} U\},$$

where M is the company's maximum net retention defined realistically.

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A Polya distribution would also have been possible, with the following methods valid mutatis mutandis.

According to the instructions of the Supervisory Service in Finland, the fluctuation of the basic probabilities of the claims is taken into account by taking

$$\bar{n}_k = (1 + q_k)n_k$$

where the number q_k is a constant estimated from experience in branch k and n_k the estimated number of claims.

Let us denote

$$\alpha'_{ki} = \int_0^\infty x^i \, dS_k(x)$$

and

$$\alpha_{ki} = \frac{\alpha_{ki}}{(\alpha_{k1})^i}$$

For the distribution (2) we then have

$$E\{x\} = \Sigma \bar{n}_k \, \alpha_{k1} = \Sigma \bar{P}_k = \Sigma(1 + q_k)P_k$$

$$\sigma^2\{x\} = \mu_2 = \Sigma \bar{n}_k \, \alpha_{k2} = \sum \frac{\bar{P}_k^2}{\bar{n}_k} \alpha'_{k2}$$

$$\mu_3 = \Sigma \bar{n}_k \, \alpha_{k3} = \sum \frac{\bar{P}_k^3}{\bar{n}_k^2} \alpha'_{k3}.$$

If

$$\frac{\mu_3}{\sigma^3} < 2.5, \tag{4}$$

which is always the case if

$$M < 2.5 \, \sigma,$$

then, according to Kauppi and Ojantakanen (Approximations of the generalised Poisson function, ASTIN-Colloquium in Arnhem), the value of x_ε for which $F(x_\varepsilon) = 1 - \varepsilon$, is, with a reasonable accuracy,

$$x_\varepsilon \approx \Sigma \bar{P}_k + y_\varepsilon \, . \, \sigma + \frac{1}{6} \frac{\mu_3}{\sigma^2} (y_\varepsilon^2 - 1), \tag{5}$$

where y_ε is the root of the equation $\phi(y_\varepsilon) = 1 - \varepsilon$. The expression (4) holds true for almost all companies.

After a little manipulation we find from equations (1) and (5)

$$E_{\min} = \frac{1}{\sqrt{1+i}} \left\{ \Sigma(q_k - \lambda)P_k + y_\varepsilon \cdot \sigma + \frac{1}{6}\frac{\mu_3}{\sigma^2}(y_\varepsilon^2 - 1) \right\} - U. \qquad (6)$$

Inserting the values $i = 0.05$, $\lambda = 0$, $\varepsilon = 0.01$ as posed by the Supervisory Service gives

$$E_{\min} = 0.976\ \Sigma\ q_k P_k + 2.270.\ \sigma + 0.714\ \frac{\mu_3}{\sigma^2} - U. \qquad (7)$$

The maximum amount $E_{\max}$ can, in principle, be defined by

$$\Pr \left\{ (1+i)^r E_{\max} + \sum_{t=1}^{r} (1+i)^{r-t+1/2} [(1+\lambda)P - x] \geq 0;\ \overset{\text{for all values}}{1 \leq r \leqq a} \right\} = 1 - \varepsilon, \qquad (8)$$

if the company is allowed to retain the amount U.

In Finland, we also have the further restriction

$$E_{\max} \geq 2M.$$

For the values $i = 0.05$, $\lambda = 0$, $a = 5$ and $\varepsilon = 0.01$ it has been noticed that the equation

$$\Pr \left\{ 1.05^5\ E_{\max} + \sum_{t=1}^{5} (1.05)^{5-t+1/2} (P - x) \geq 0 \right\} = 0.99 \qquad (9)$$

gives the same $E_{\max}$ as equation (7) with an accuracy wholly sufficient for insurance companies. Applying equation (5) we get after a little manipulation

$$E_{\max} = 4.436\ \Sigma\ q_k P_k + 4.626.\ \sigma + 0.658 \cdot \frac{\mu_3}{\sigma^2}. \qquad (10)$$

Statistics and tables for use

The committee has collected extensive statistics to obtain the functions $S_k(x)$, the records of large claims extending over many years being brought into the experience. The resulting functions $S_k(x)$ are presented as graphs in Appendix 1. Moreover the tails of the distributions $S_k(x)$ are smoothed and continued according to a Pareto-distribution for a decade longer than they were in the empirical material. The influence of inflation has naturally been eliminated.

The functions $S_k(x)$ relate to the gross claims. The insurance companies have, however, various reinsurance contracts which must be taken into account. Therefore, working tables were computed, where the characteristics

$$M'_k = \frac{M_k^*}{\alpha_{k1}^*},$$

$$M_k^*$$

$$\alpha_{k1}^* = \int_0^{M_k^*} x \, d\, S_{M_k^*}(x)$$

$$\alpha'_{k2} = \frac{\alpha_{k2}^*}{(\alpha_{k1}^*)^2}$$

$$\alpha'_{k3} = \frac{\alpha_{k3}^*}{(\alpha_{k1}^*)^3}$$

of the distributions

$$S_{M_k^*}(x) = \begin{cases} S_k(x) & x < M_k^* \\ 1 & x \geq M_k^* \end{cases}$$

are represented. Three examples of such tables are presented in Appendix 2. The second and third columns in these are not needed in standard computations.

The tables are used in the following way. First, the insurances in force are divided into branches (classes) k. Then, the net premium incomes P_k and the expected numbers of claims n_k are determined according to special instructions. The third task is to determine realistically the maximum net retention M_k in branch k. Then

$$\alpha_{k1} = \frac{P_k}{n_k}$$

and

$$M'_k = \frac{M_k}{\alpha_{k1}}.$$

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The numbers α'_{k2} and α'_{k3} are derived by interpolation from the working tables and formulae (7) and (10) applied.

The above method automatically takes into account the effect of inflation and quota share reinsurance. It has also been considered a sufficient way of taking the various reinsurance methods (excess of loss, excedent, mixed methods) into account.

Some exceptions

One exception to the above is Stop Loss reinsurance, which has its own rules.

Certain branches have no working table. They are divided into relatively homogeneous classes i, and then P_{ki} and M_{ki} determined. The number n_{ki} is defined as

$$n_{ki} = \frac{P_{ki}}{M_{ki}},$$

after which the numbers

$$\alpha'_{k2i} = 1$$

and

$$\alpha'_{k3i} = 1$$

are used. The method is based on Pesonen's paper "On the calculation of the generalised Poisson function", par. 7, 5th Astin-Colloquium.

There are some situations in which the above mentioned methods are inapplicable. Then case by case estimation methods and perhaps Monte Carlo methods and computers are used. Prepared computer programs also make it possible to deal with exceptional situations.

Conclusions

I have tried to give a picture of the rules and statistics intended as standard in computing limits for the equalisation reserves in Finnish insurance companies. These reserves are compulsory and intended to secure the companies' solvability in case of exceptional high total losses.

The rules and statistics seem to meet the problem well. They are quite simple and easy to use in all our companies, even if required every year. Yet they seem to be nearly as accurate as it is possible in practice, even for parameter values other than those posed by the Finnish Supervisory Service.

Appendix I

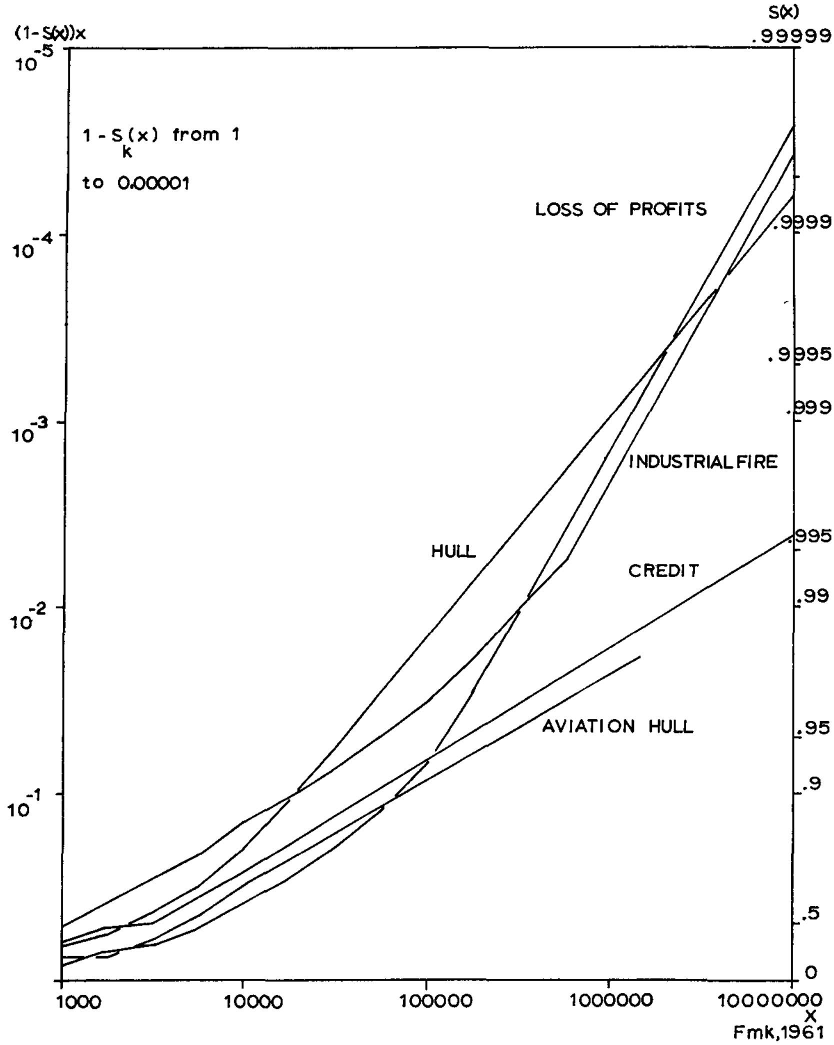

Fig. 1.

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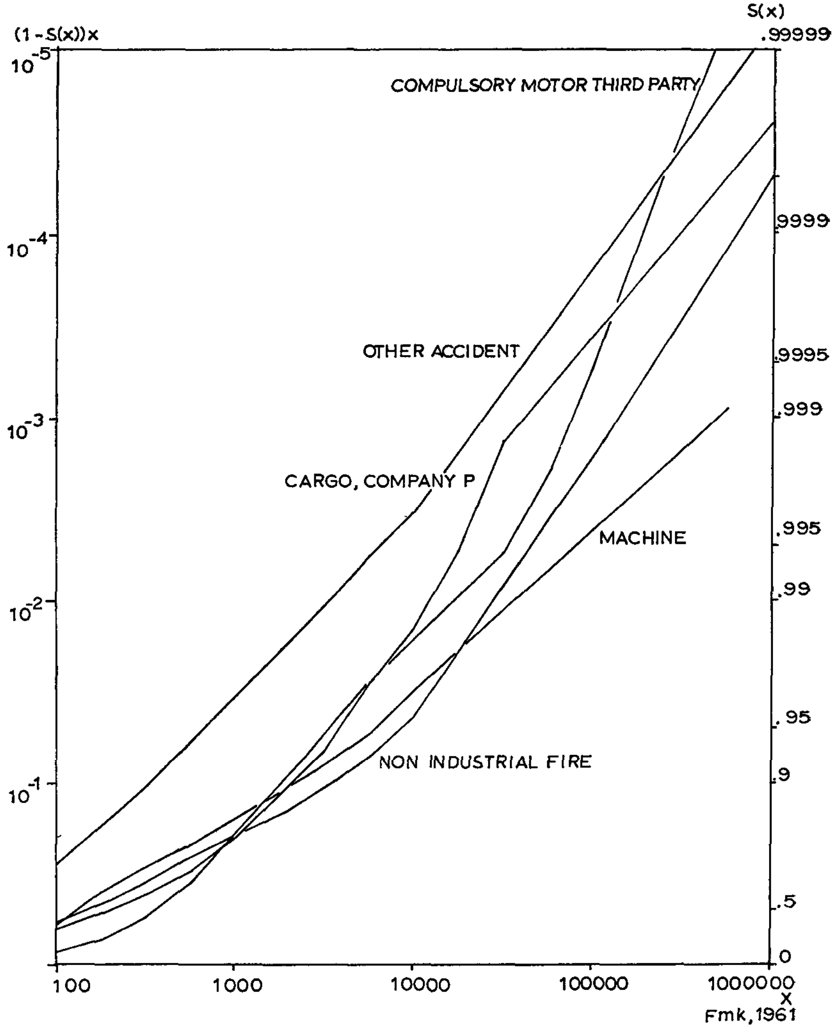

Fig. 2.

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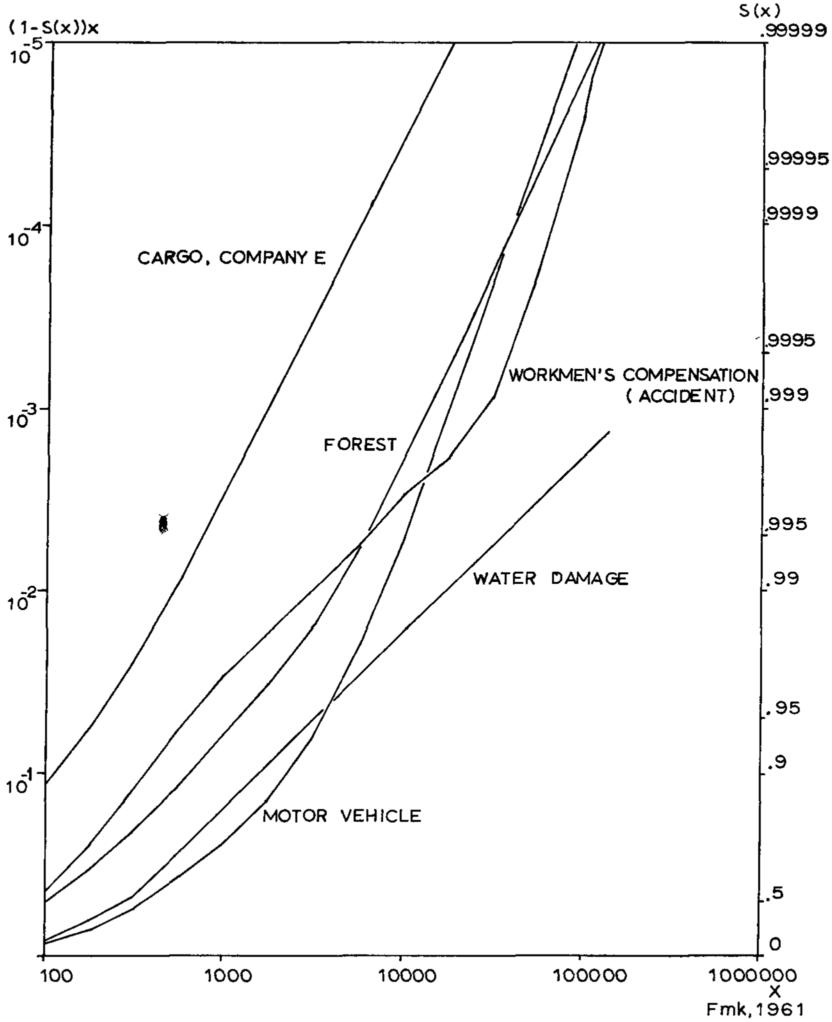

Fig. 3.

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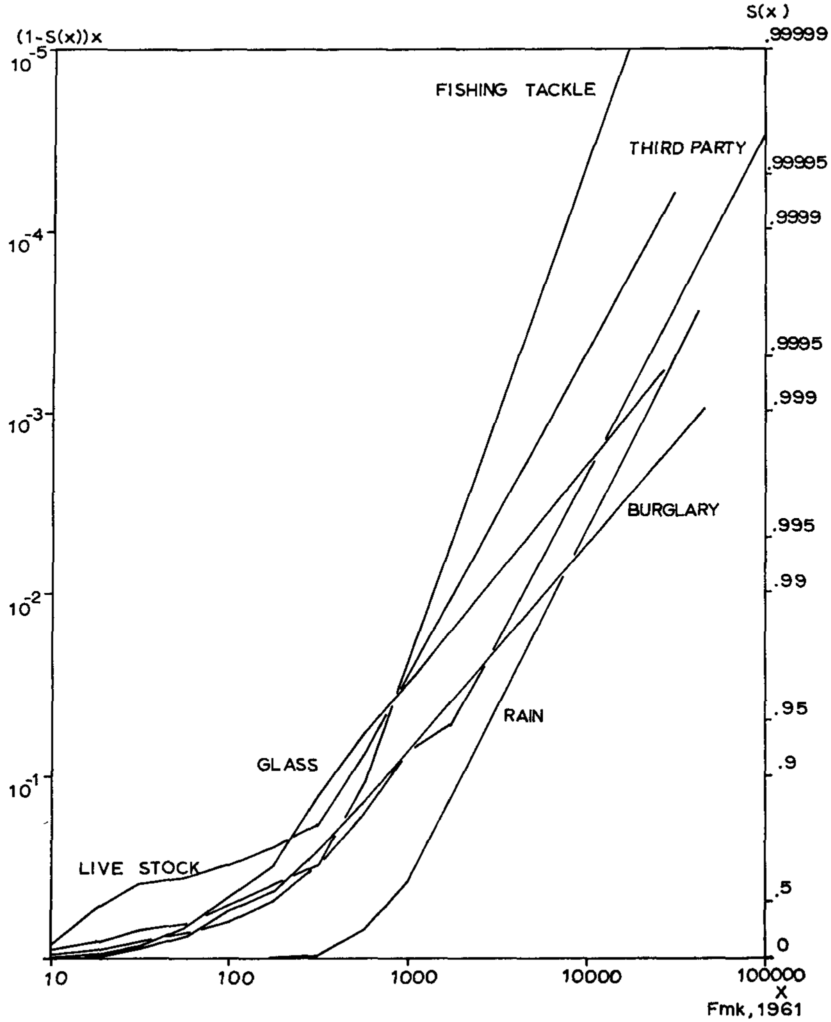

Fig. 4.

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Appendix 2

Examples of working tables

Credit insurance

M'_k	M^*_k Fmk, 1961	α^*_{k1} Fmk, 1961	α'_{k2}	α'_{k3}
1,7	2512	1477	1,6	2
1,8	3981	2213	1,7	2
2,0	6310	3152	1,8	3
2,3	10000	4279	2,0	4
2,8	15850	5621	2,2	5
3,5	25120	7223	2,6	7
4,4	39810	9147	3,1	11
5,5	63100	11459	3,8	18
7,0	100000	14235	4,6	28
9,0	158500	17570	5,8	46
11,6	251200	21574	7,3	75
15,1	398100	26384	9,3	123
19,6	631000	32164	12,0	205
25,6	1000000	39103	15,4	345
33,4	1585000	47441	20,0	583
43,7	2512000	57453	25,9	990
57,3	3981000	69478	33,7	1689
75,2	6310000	83926	44,0	2891
98,7	10000000	101275	57,5	4963
129,8	15850000	122119	75,3	8542
170,7	25120000	147150	98,7	14730
224,6	39810000	177211	129,5	25440

Industrial fire insurance

M'_k	M^*_k Fmk, 1961	α^*_{k1} Fmk, 1961	α'_{k2}	α'_{k3}
1,6	1413	893	1,4	2
1,8	2239	1212	1,5	2
2,2	3548	1618	1,7	3
2,7	5623	2118	2,0	4
3,3	8913	2703	2,4	6
4,1	14130	3406	2,9	10
5,2	22390	4273	3,6	16
6,7	35480	5307	4,4	25
8,6	56230	6510	5,5	41
11,3	89130	7897	6,9	67
15,0	141300	9433	8,8	111
20,4	223900	10991	11,1	185
28,3	354800	12529	14,1	318
40,1	562300	14024	18,1	561
58,5	891300	15244	22,7	956

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M'_k	M^*_k Fmk, 1961	α^*_{k1} Fmk, 1961	α'_{k2}	α'_{k3}
87,5	1413000	16141	27,9	1618
133,4	2239000	16778	33,9	2731
205,9	3548000	17233	40,7	4650
320,3	5623000	17556	48,5	7999
501,1	8913000	17786	57,5	13882
787,2	14130000	17950	67,8	24293
1239,4	22390000	18065	79,5	42601
1955,0	35480000	18148	92,9	75299
3088,4	56230000	18207	108,1	133292
4884,1	89130000	18249	125,4	237270
7729,3	141300000	18281	146,0	432189

Compulsory motor third party insurance

M'_k	M^*_k Fmk, 1961	α^*_{k1} Fmk, 1961	α'_{k2}	α'_{k3}
1,8	708	400	1,4	2
2,3	1122	494	1,6	3
3,0	1778	583	1,9	4
4,2	2818	665	2,3	7
6,0	4467	741	2,8	12
8,7	7080	815	3,6	22
12,6	11220	890	4,7	43
18,4	17780	968	6,4	87
26,8	28180	1050	8,8	178
39,7	44670	1126	11,9	349
60,1	70800	1178	15,1	609
92,9	112200	1208	17,9	962
145,5	177800	1222	20,1	1372
229,3	281800	1229	21,8	1892
362,6	446700	1232	23,3	2547
573,7	708000	1234	24,4	3364
908,5	1122000	1235	25,3	4388
1439,7	1778000	1235	26,1	5671
2279,9	2818000	1236	26,6	7164