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Size Dependent Dynamics of Cantilever Beams Immersed in Viscous Fluid

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The dynamic behavior of a cantilever beam can be dramatically affected by the properties of the fluid in which it is immersed, as illustrated by numerous studies. The modeling for the frequency response in vacuum can be obtained easily for many cantilever beams of practical interest, whereas the analysis of the effect of immersion in fluid poses a formidable challenge. To alleviate this problem, Sader presented a rigorous theoretical model for the frequency response of rectangular microcantilever beams that are undergoing flexural vibrations and immersed in viscous fluids[1], which is of particular relevance to applications of the atomic force microscopy (AFM). A comparison of this model with detailed experimental measurements on AFM cantilever beams was subsequently performed, which demonstrated the validity and accuracy of the theoretical model for cantilever beams immersed in both gas and liquid[2]. However, rapid growing application of dynamic-AFM and AFM in nanomechanical measurements of soft biological matter in physiological buffer solution has resulted in extreme importance of study on dynamics of a cantilever beam with ultra small geometry in viscous fluid where size depend effect is obvious[3-6]. Unfortunately, Sader's model is technically limited to continuum mechanics, as a result, failed in interpreting for the size effect. This gap in the literature is particularly significant in application to AFM. Consequently, we present a simple model for flexural vibration of rectangular microcantilever beams considering the size effect below, in which the couple stress theory (Cosserat theory) is introduced to the dynamic deflection function, which will be of value for users and designers of dynamic AFM et al.

After detailed theoretical analysis of the vibration of a rectangular microcantilever beam immersed in viscous fluid and excited by an arbitrary driving force, we report the obtained governing equation for its dynamic deflection function $w(x,t)$ to be $EI\left[1+\frac{12(1+v^2)}{1+v}\frac{l_b^2}{h^2}\right]\frac{\partial^4 w}{\partial x^4}+\rho A\left(1+\frac{\pi\rho_f b^2}{4\rho A}\Gamma(\omega)\right)\frac{\partial^2 w}{\partial t^2}=F_{drive}$, where E and v are the young's modulus and the poison ratio, h and l_b are the thickness and material length scale parameter of the cantilever beam. ρ and ρ_f are the density of the cantilever beam and the fluid, respectively. L is the total length and A is the area of the cross section. $\Gamma(\omega)$ is the hydrodynamic function and F_{drive} is the driving force that excites. Here, we define the normalized resonate frequency as $\omega_{norm}=\omega/\omega_0$, where ω and ω_0 are the resonate frequencies of a cantilever beam in fluid with and without consideration of the size effect. ω_{norm} has been derived to be equal to $\left[1+\frac{12(1+v^2)}{1+v}\frac{l_b^2}{h^2}\right]^{1/2}$. We numerically analyzed the effect of size on frequency response of a rectangular silicon cantilever beam; see Fig.1. The numerical results have shown an agreement with the experiments, which indicates that the resonate frequency of a rectangular cantilever beam immersed in viscous fluid depends on its size effectively when the thickness is near or smaller than the material length scale parameter. We emphasis here that the purpose of the numerical analysis is to validate the effect of size on frequency response of a microcantilever beam, the future work on the exactly comparative work with experimental results is now going on, which will be reported later. Another thing we'd like to emphasis is that the couple stress theory has been introduced to the deriving of the dynamic deflection function of a micro/nano cantilever beam immersed in viscous fluid above, which is first reported.

Methods for prediction for dynamic behavior of long beam-like micro components immersed in viscous fluid, considering the size effect, could be easily derived based on the presented modeling, which is of great value to users and designers of micro-electro-mechanical systems (MEMS).

References

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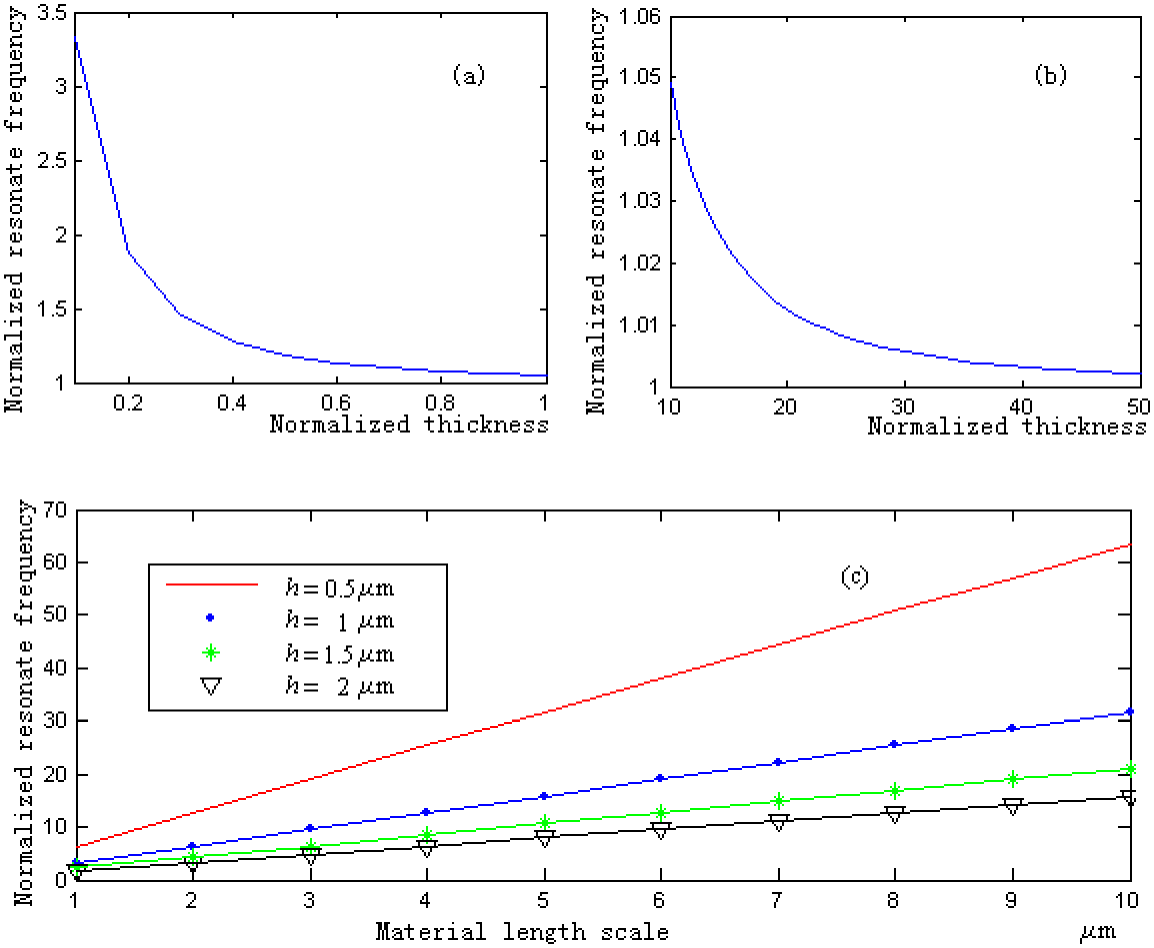

Fig.1 Effect of size on normalized resonate frequency of a rectangular micro cantilever beam; normalized resonate frequency is defined as ω/ω_0, where ω is the resonate frequency in fluid and ω_0 is the resonate frequency in vacuum. (a), (b) Effect of normalized thickness h/l_b, where h and l_b are the thickness and material length scale parameter of the cantilever beam, on normalized resonate frequency; (c) effect of material length scale parameter on the normalized resonate frequency. It is evident that the resonate frequency depends on the size effectively when the thickness is near or smaller than the material length scale parameter.