3. PARAMETERS OF THE EARTH'S GRAVITATIONAL FIELD

A. H. Cook

Apart from precession, the dynamical interactions of the Earth with the rest of the solar system may all be put in terms of the external gravitational potential of the Earth, which, in the form recommended by Commission 7, is written:

$$V = -\frac{GE}{r}\left[1 - \sum_n J_n \left(\frac{a_e}{r}\right)^n P_n(\cos\theta) + \text{tesseral harmonics}\right],$$

where r is the radius vector from the centre of the Earth and θ is the co-latitude.

This expression contains three sorts of parameter:

GE is a scale factor for mass,

a_e is a scale factor for length,

J_n is a form factor giving the departure of the potential from the spherically symmetrical potential, $-GE/r$.

I shall discuss the determination of these parameters from observations made on or near the surface of the Earth, that is, in the near field region in which the observations are influenced by the higher-order zonal and tesseral harmonics in the potential.

CLOSE ARTIFICIAL SATELLITES

The secular and long-periodic terms in the variations of the elements of artificial satellites arise from the zonal harmonics in the potential which may, in principle, be determined from the observed motions of node and perigee. They cannot be determined independently of other parameters, for the quantities actually derived from the observations are $(GE)^{-\frac{2}{3}}(a_e)^n J_n$; however, the uncertainties in GE and a_e are small compared with those in J_2 and quite negligible for all other harmonics.

The real problem is that there are few independent data, for although there are many satellites, all those with nearly the same inclinations yield almost equivalent observation equations; on the other hand it seems that many harmonics of order 8 or greater are of the same order of magnitude so that they cannot be found from just a few distinct observation equations. However, by choosing satellites with the greatest semi-major axes, it is possible to determine the harmonics of lowest order and it is likely that reliable values are available for J_2, J_4 and J_6 and that in particular,

$$10^6 J_2 = 1082{\cdot}65 \pm 0{\cdot}05.$$

GE can be found from the mean motion of an artificial satellite provided that it is high enough that air drag is insignificant. The semi-major axis is also required and since this must be determined geometrically from the surface of the Earth, the quantity that is actually determined is some function of GE and a_e, the form depending on the geometry. J_2 also enters since it makes a small change in the mean motion. Estimates of GE that are less dependent on a_e and J_2 have been obtained from the Ranger space probe.

SURFACE MEASUREMENTS

Working at the surface of the Earth, lengths of arcs, directions of the gravity vector and the magnitude of gravity can be measured. In relating such measurements to the parameters of the external potential there are two principal difficulties, the irregular form of the surface on which the measurements are made and the poor distribution of the data owing to the fact that measurements of arc length and of the direction of gravity can only be made on land, whereas the seas occupy about three-quarters of the surface of the Earth. The magnitude of gravity can be measured at sea but not with the same accuracy as on land.

If the surface of the Earth were an equipotential of gravity and centrifugal force and were an ellipsoid of revolution, it would be a simple matter to determine its equatorial radius from measurements of lengths of arcs over the surface and of the directions of gravity (normal to the surface) at the ends of the arcs. There is also a well-developed theory for the value of gravity in terms of the external potential and the ratio m of the centrifugal force to the Newtonian attraction at the equator. Thus

$$g = g_e \left[1 - \left(2m - \frac{3}{2} J_2 \right) \cos^2\theta + \ldots \right],$$

$$g_e = \frac{GE}{a_2} \left[1 - \mu_a + \frac{3}{2} J_2 - m + \ldots \right],$$

where g_e is the observed value of gravity (resultant of Newtonian attraction and of centrifugal force) at the equator and μ_a is the relative mass of the atmosphere. Hence J_2 may be found from the variation of g and GE from the absolute value of g_e.

Owing the the presence of arbitrary higher harmonics, an equipotential surface would not in fact be an ellipsoid of rotation, but it is a simple matter to include the effects of such harmonics in the expressions for the magnitude and direction of gravity. In principle, these effects will be averaged out if the observations are uniformly distributed over the surface of the Earth but since the observations are for the the most part restricted to land areas, the values of J_2 and of the mean absolute value of gravity derived from the surface measurements alone would be in error due to the effects of higher harmonics. However, now that some at least of these harmonics can be found from satellite observations, it is possible to apply suitable corrections to the surface observations and so improve the estimates of a_e and GE.

At sea, observations are made on an equipotential surface, but on land of course, they are not. It is well known that results equivalent to those that would be obtained on an equipotential surface are derived if the observed values of gravity are increased by the free-air correction which takes account of the difference of potential between sea-level and the point at which the observations were made. There is a similar correction for observations of the direction of gravity. But this procedure leads to the practical difficulty that because the height of the ground varies relatively rapidly, it is difficult to obtain representative values of free-air anomalies without very many closely spaced observations. For this reason, and because so much of the sea area is rather inaccessible, it is very unlikely that J_2 will ever be determined from surface measurements as accurately as it can be from close satellites and at the present time the uncertainty of the latter determinations is at least 20 times less than that of the former. Thus J_2 is no longer found from surface gravity measurements. a_e and the mean absolute value of gravity must continue to be found from surface observations but significant improvements have occurred in the last 20 years in two respects, for arc length measurements have been extended into widespread networks that give much better coverage than before and, as a result of satellite observations, corrections can be applied for some of the higher harmonics. Both improvements ameliorate the difficulties arising from the poor distribution of data, and it seems that the uncertainty of a_e found from arc length measurements is a few parts in a million. Determination of the mean absolute value of gravity, and hence of GE involves further difficulties connected with the determination of the absolute value of gravity at a few laboratories equipped for such measurements and with the connection of such measurements to the worldwide network of measurements of differences of gravity. These problems, essentially technical ones, are not fully solved yet, but good progress is being made and while at the present time the absolute value of g_e probably has an uncertainty of about five parts in a million, there is a good chance that it will be reduced to about one part in a million in a few years' time.

To summarize, the main difficulties in the way of determining parameters of the external potential of the Earth from surface measurements arise from the presence of the higher harmonics and the fact that, on account of the poor distribution of observations and consequent inadequate averaging, the higher harmonics lead to errors in the estimates of J_2, a_e and g_e. None the less, in the last 20 years, very considerable advances have occurred in our knowledge of these quantities, mainly on account of observations of artificial satellites, of the extension of networks of geodetic survey and on account of the great development of gravity measurements at sea. Coupled with radar measurements of the distance of the Moon and new determinations of the mass of the Moon from space probes, we now have a set of data on the external potential of the Earth that appears to be consistent to within a few parts in a million.

4. MASSES OF THE PRINCIPAL PLANETS

G. M. Clemence

INTRODUCTION

The Working Group, appointed by the Executive Committee of the Union to consider revision of the conventional system of astronomical constants, decided not to recommend any changes in the conventional values of the masses of the principal planets. The remarks that follow are intended to explain why I think the decision of the Working Group was wisely taken, and to indicate some work that should be done before recommending a revised set of planetary masses.

I begin by attempting to estimate the most probable value of the mass of each planet separately, using observational evidence that is as much as possible independent of any assumptions about the masses of the others. Then I consider the problems that are encountered in attempting to combine the separate determinations into a consistent system.

In view of the quantity and diversity of the observational material, a substantial portion of which is known to be affected by systematic errors of obscure origin, the task is a difficult one. I do not think that another person, working independently, would be likely to arrive at the same numerical values as are given here. Therefore, although I have been obliged to give numbers, I do not strongly defend any of them. I hope only that they are sufficiently exact to justify the general conclusions.

MASSES OF THE PLANETS

After the name of each planet Table 1 gives the conventional value of the ratio of the mass of the Sun to the mass of the planet, including atmosphere and satellites. The conventional value is, in general, the value used in the planetary theories that serve as the basis of the national and international ephemerides. The only exception is the value for Jupiter, which in the theories of the four inner planets is 1047·35; the discrepancy is completely trivial, since the relevant perturbations consist of only four significant figures. Below the name, numbered serially, are the results of the principal determinations of the mass-ratio with the probable errors assigned by the authors, authors and dates, and indications of the observational data.

As a rule, I have excluded the older determinations that were based on observational material included in later determinations. The principal exceptions are as follows:

The masses of Venus derived by Spencer Jones and by Morgan and Scott both include Greenwich observations of the Sun 1900–23, but the material common to the two determinations

https://doi.org/10.1017/S0251107X00030157 Published online by Cambridge University Press