A NEW APPROACH TO EVALUATE PLANETARY PERTURBATIONS ON A CLOUD OF DUST IN LOW ECCENTRICITY HELIOCENTRIC ORBITS

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ABSTRACT. Dynamical perturbations on *ensembles* of particles in heliocentric orbits of low eccentricity are integrated over time. The dust is perturbed by radiation pressure, Poynting-Robertson drag, their corpuscular counterparts, and by gravitation due to any number of planets. A dust cloud is represented by a set of centroids and orbital dispersions (about the centroids). Gravitational perturbations on the centroid are derived from a single matrix, valid for any planet, in the appropriate frame of reference. After transformation of the time derivatives to a common coordinate system, the perturbation rates are summed up and integrated. The time dependence of the planets' orbital elements are evaluated inside the time integral.

1. INTRODUCTION

It is time-consuming and costly to estimate overall perturbations on a dust cloud by computing trajectories of individual dust particles. The efficiency is increased through the use of centroids to represent sets of orbits. Perturbations due to each planet are computed separately. Geometries are chosen such that the planet's perturbations on a centroid may be expressed by a *single quantity* depending only on the angle that the plane containing the centroid makes to the planet's orbital plane and the ratio of heliocentric distances. A comparatively simple interpolation-formula allows the matrix representing the perturbations to be stored online. Properly scaled, a single matrix may represent perturbations by any planet in the appropriate geometry.

2. COMPUTATIONAL PROCEDURE

2.1 The approach

Perturbations on *ensembles* of particles are evaluated as shown in the flow diagram, Figure 1. The ensemble is divided into a set of fictitious orbits, each a centroid for a set of dust trajectories. Each

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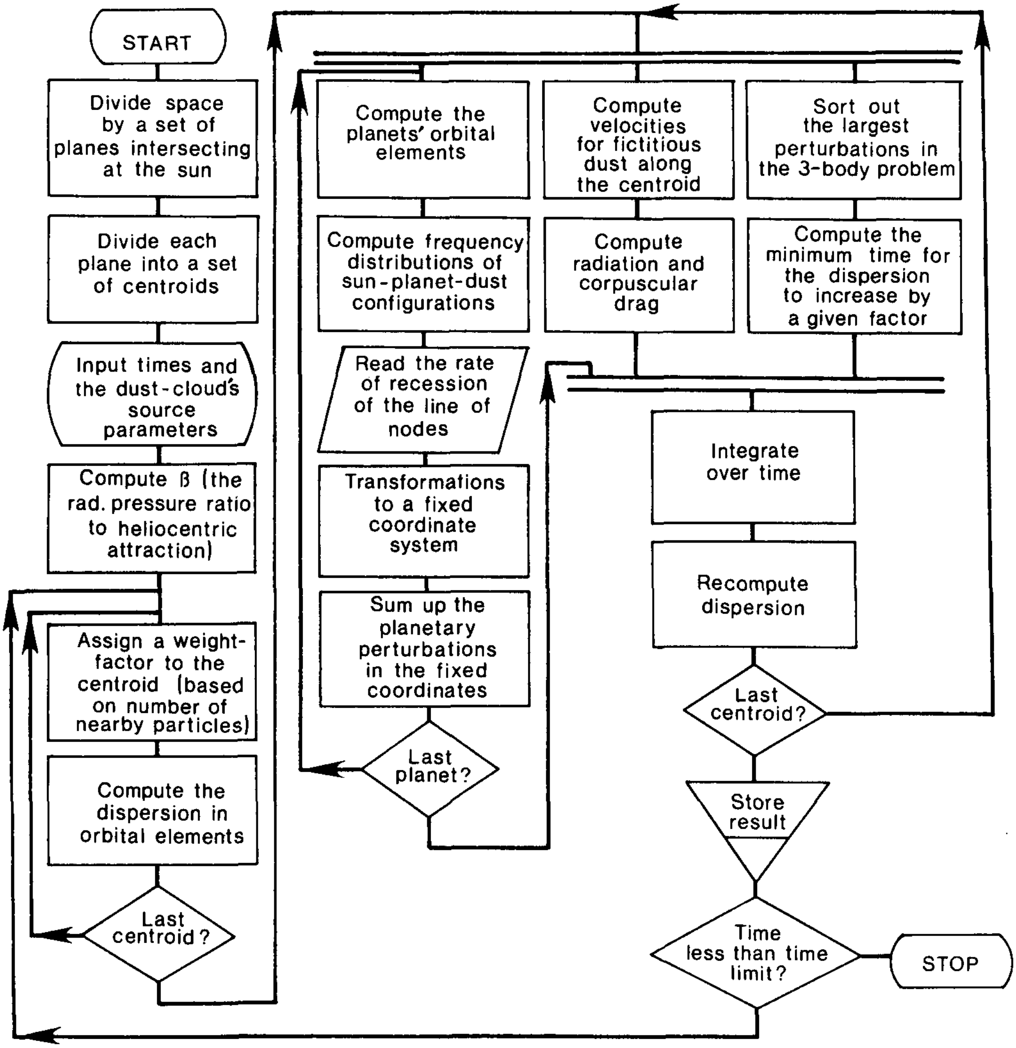

Figure 1. Flow chart diagram illustrating a methodology to evaluate the combined effects of planetary, electro-magnetic radiation and corpuscular perturbations on the shape of a cloud of interplanetary dust.

centroid is assigned a "weight" depending on the number of dust particles that it represents, and the dispersion about each centroid is computed. Mean secular perturbations are determined from the frequency distribution of sun-planet-dust configurations. This is repeated for each planet and the perturbations are summed up with other velocity-dependent disturbances, determined in a similar fashion from the distribution in orbital velocities. This procedure corresponds to a smearing out of the planets' masses along their orbits. A dispersion in orbital elements develops for actual trajectories due to differences in the sequence in which the

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sun-planet-dust configurations occur. This effect is evaluated for each perturbation separately, and the dispersions are combined.

2.2 The time integral

For computational convenience, the dust cloud may be divided by a set of planes intersecting at the sun. Each plane contains a set of fictitious orbits, centroids for a set of dust trajectories. The number of particles represented by a given centroid and the distribution in orbital parameter space are calculated from the starting conditions of the cloud. The rate of dispersion in orbital parameter-space is estimated as described below, and the perturbations on the centroid are integrated over a time interval in which the dispersion remains comparable to that of the original distribution. After the integration, a new set of orbits is assigned to each centroid and the process is repeated until the desired time period is covered. The planets' orbital elements may be estimated using the method of Brouwer and Van Woerkom (1950). The integration step is reduced in cases where a planet's orbit is found to change significantly over a single step.

2.3 Perturbations on the centroid

The expression for the average rate of recession $\overline{d(\Omega)/dt}$ of the line of nodes (defining the plane of the centroid) relative to the planet's orbit involves an integral that may be resolved into Jacobi's incomplete elliptic integrals of the first and second kinds. A matrix is generated in which each element is a solution to the integral at given tilt and radii of the centroid orbit. An expression of the form

$$\overline{\frac{d\Omega}{dt}} \approx \frac{K}{\sqrt{1-\beta}} \; \frac{r(1+\cos i)}{(1+r^2-2r\cos^{2/3} i)^{5/4}}$$

was found to be an efficient interpolation formula, where K is a negative constant, β is the radiation force in units of the sun's gravitational attraction and i is the angle that the plane containing the centroid makes to the planet's orbital plane. The radius r of the centroid orbit is in units of the planet's heliocentric distance. The planetary mass and effects of radiation pressure intervene outside the integral. The matrix may therefore be used to evaluate the perturbation by <u>any</u> planet relative to its own orbit. It is a simple matter to transform the perturbations to a common frame of reference in which all perturbations may be summed up.

The change in orbital radii resulting from Poynting-Robertson drag is easily computed from Robertson's (1937) expressions. Similar formulae represent the corpuscular drag.

2.4 Dispersion about the centroid

The dispersion about the centroid is first computed in the restricted three-dimensional three-body problem, which is extended to incorporate radiation and corpuscular forces. This is repeated for each planet, and the dispersions are combined. An orthogonal coordinate system is introduced centered on the dust particle and is frozen into an inertial frame of reference at the moment in which the perturbations generating the dispersion are evaluated. The accelerations in this system are the sum of two three-dimensional matrices. With the approximation that the sun and all the planets revolve around the Sun-Jupiter center of mass, the first matrix represents the perturbations by the Sun-Jupiter system including radiation and corpuscular forces as a function of one of these body's coordinates. The second matrix represents perturbations by any other planet. The resulting accelerations translate into perturbations of the orbital elements, when fed into Gauss equations. Distributions about the centroid are calculated based on a frequency distribution of sequences of sun-planet-dust configurations for each planet separately.

3. SUMMARY

A method is described whereby the combined perturbations by planetary gravitation, solar-radiation and corpuscular forces on a dust cloud may be evaluated. The method takes advantage of similarities in the numerical solutions for perturbations by the planets depending on the geometry. In its present form, the method is applicable to nearly circular orbits. A generalization to orbits of arbitrary eccentricity is planned.

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