

How well can the evolution of the scale factor be reconstructed by the current data?

Sandro D. P. Vitenti¹ and Mariana Penna-Lima²

¹GRACO, Institut d'Astrophysique de Paris,
UMR7095 CNRS, 98 bis boulevard Arago, 75014 Paris, France
email: dias@iap.fr

²Divisão de Astrofísica, Instituto Nacional de Pesquisas Espaciais,
Av. dos Astronautas 1758, 12227-010, São José dos Campos, Brazil
email: mariana.lima@inpe.br

Abstract. Distance measurements are currently the most powerful tool to study the expansion history of the universe without assuming its matter content nor any theory of gravitation. In general, the reconstruction of the scale factor derivatives, such as the deceleration parameter $q(z)$, is computed using two different methods: fixing the functional form of $q(z)$, which yields potentially biased estimates, or approximating $q(z)$ by a piecewise n th-order polynomial function, whose variance is large. In this work, we address these two methods by reconstructing $q(z)$ assuming only an isotropic and homogeneous universe. For this, we approximate $q(z)$ by a piecewise cubic spline function and, then, we add to the likelihood function a penalty factor, with scatter given by σ_{rel} . This factor allows us to vary continuously between the full n knots spline, $\sigma_{rel} \rightarrow \infty$, and a single linear function, $\sigma_{rel} \rightarrow 0$. We estimate the coefficients of $q(z)$ using the Monte Carlo approach, where the realizations are generated considering Λ CDM as a fiducial model. We apply this procedure in two different cases and assuming four values of σ_{rel} to find the best balance between variance and bias. First, we use only the Supernova Legacy Survey 3-year (SNLS3) sample and, in the second analysis, we combine the type Ia supernova (SNeIa) likelihood with those of baryonic acoustic oscillations (BAO) and Hubble function measurements. In both cases we fit simultaneously $q(z)$ and 4 nuisance parameters of the supernovae, namely, the magnitudes \mathcal{M}_1 and \mathcal{M}_2 and the light curve parameters α and β .

Keywords. cosmology: miscellaneous, cosmology: observations, cosmology: theory

1. Introduction

The current observational data are still not able to decide between many different models proposed to explain the recent accelerated expansion of the universe. These models include, among others, modified gravitational theories and the addition of an exotic cosmological fluid, in the context of general relativity. As an alternative study, we can make a pure kinematical description of this recent phase of the universe, avoiding the choice of a gravitational theory and matter content. In this description one still models the universe as a metric manifold and uses the cosmological principle restricting the metric to the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. In this case, the deceleration function is given by

$$q(z) = \frac{(1+z)}{H(z)} \frac{dH(z)}{dz} - 1, \quad (1.1)$$

where z is the redshift and $H(z)$ is the Hubble function.

Different approaches have been used in the literature in order to reconstruct some kinematic quantities such as the luminosity distance $D_L(z)$, $H(z)$, $q(z)$ and the jerk

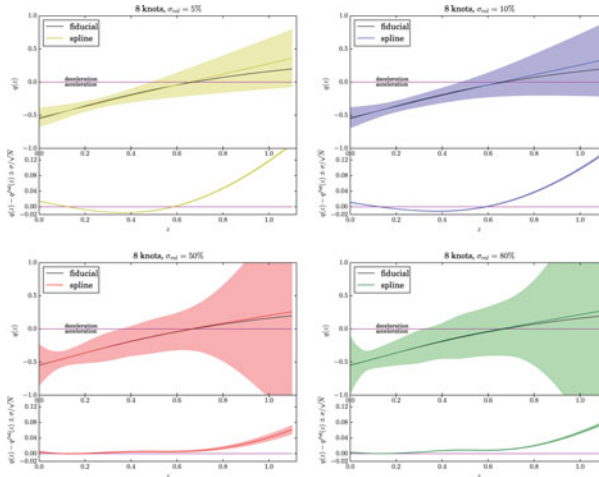


Figure 1. The top part of each panel shows the reconstructed curve of $q(z)$ (spline), computed using the mean values of the estimators, and its 1σ error bar. Each panel displays the result obtained for a given σ_{rel} . The bottom part shows the bias, $q(z) - q^{fid}(z)$, and the 1σ error bar of the mean curve, i.e., σ/\sqrt{N} .

function. One of these methods assumes *a priori* a functional form of the scale factor $a(t)$ ($1 + z = a_0/a$), or equivalently $D_L(z)$, $H(z)$ and so on (Riess *et al.* 2004, Visser 2004, Rapetti *et al.* 2007, Lu *et al.* 2011, Shafieloo 2012). This parametric reconstruction is strongly model dependent, so, besides having small variances the results present large biases.

In order to minimize the assumptions on the fitted curve, a second approach is to approximate the kinematic quantity by a piecewise n th-order polynomial function. This “non-parametric” reconstruction is dominated by the over-fitting feature, but it provides small biased estimators (see Daly *et al.* 2008 and Lazkoz *et al.* 2012).

2. Methodology

In this work, we describe $q(z)$ by a piecewise cubic polynomial function, also known as *cubic spline*. Imposing continuous second derivatives, the only free parameters of $q(z)$ are its values at the knots, i.e., $q(z_i) = q_i$, where i runs from 0 to $n - 1$ (being n the number of knots). We then address both parametric and non-parametric methods including a penalty factor $P_i(\sigma)$ in the likelihood L , namely,

$$F = -2 \ln(L) + \sum_{i=2}^{n-1} P_i(\sigma), \tag{2.1}$$

where the penalty factor is given by

$$P_i(\sigma) = ((\bar{q}_i - q_i)/(\sigma_{abs} + \bar{q}_i \sigma_{rel}))^2, \tag{2.2}$$

$\bar{q}_i = (q_{i-1} + q_{i+1})/2$ and $\sigma_{abs} = 10^{-5}$. Varying the value of σ_{rel} , see Figs. 1 and 2, we are able to recover a full n knots spline (over-fitting dominated), for large σ_{rel} , and a single linear function in the entire redshift interval, for small σ_{rel} , where q_i 's are better constrained but they can be biased (if the assumed functional form significantly differs from the true one).

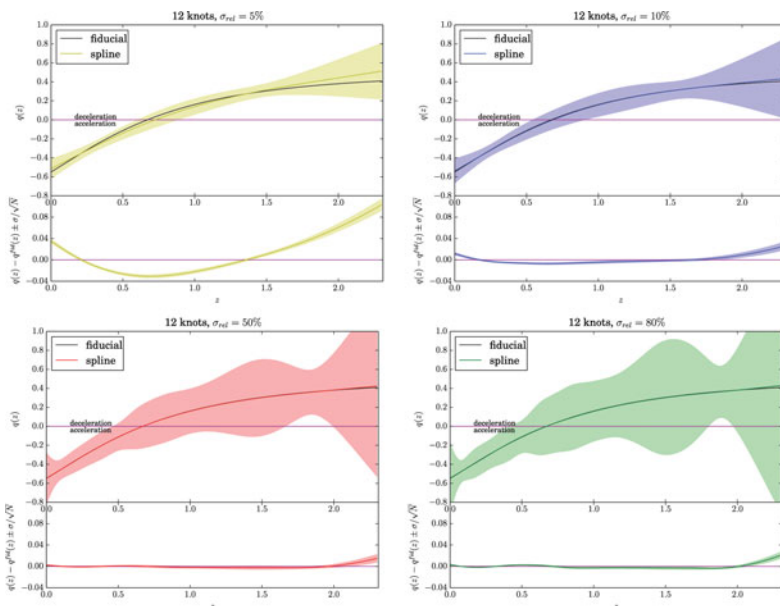


Figure 2. Equivalent to the caption of Fig. 1. These results were obtained considering Eq. 3.1 and 12 knots.

We use the SNeIa sample from SNLS3 (Conley *et al.* 2011 and Sullivan *et al.* 2011) and, thus, the likelihood is build as

$$-2 \ln(L_{SNIa}) = \Delta \vec{m}^T C_{SNIa}^{-1} \Delta \vec{m} + \ln(\det(C_{SNIa})), \tag{2.3}$$

where $C_{SNIa} = C_{stat}(\alpha, \beta) + C_{sys}$ is the data covariance and

$$\Delta m_i = 5 \log_{10}(D_L(z_i^{cmb}, z_i^{hel})) - \alpha(s_i - 1) + \beta C_i + M_{hi} + 5 \log_{10}(c/H_0) + 25 - m_{Bi}. \tag{2.4}$$

The SNIa phenomenological model contains four parameters α , β , M_1 and M_2 , where the first two are related to the stretch-luminosity and colour-luminosity, respectively, and M_1 and M_2 are absolute magnitudes.

We use data from Baryon Acoustic Oscillation (BAO) surveys such as WiggleZ, SDSS and 6dFGRS as described in Hinshaw *et al.* (2013). The BAO likelihood is given by

$$-2 \ln(L_{BAO}) = \Delta \vec{d}_v^T C_{BAO}^{-1} \Delta \vec{d}_v, \tag{2.5}$$

where C_{BAO} is the BAO constant covariance matrix and

$$\Delta d_{vi} = d_{vi} - \frac{r_{rec}}{D_v(z_i)}. \tag{2.6}$$

The BAO depends on the comoving sound horizon at recombination r_{rec} and the geometric estimate of the effective distance $D_v(z_i)$. The last is calculated directly rewriting the distance as an integral of $q(z)$, however, r_{rec} have to be treated as an additional parameter since our model does not define r_{rec} .

The Hubble function likelihood is build as

$$-2 \ln(L_{Hubble}) = \Delta \vec{H}^T C_{Hubble}^{-1} \Delta \vec{H}, \tag{2.7}$$

where C_{Hubble} is the diagonal constant covariance matrix and $\Delta H_i = H_i - H(z_i)$. This likelihood includes the $H(z)$ measurements which were obtained independently of BAO.

The data is summarize, for example, in Zheng *et al.* (2014). We also include the H_0 ($z = 0$) measurement from Riess *et al.* (2011).

We apply the Monte Carlo (MC) approach to obtain the estimators of q_i 's, α , β , M_1 and M_2 . For this, we first define a fiducial model from which N random realizations will be generated. In particular, we assume a flat universe ($k = 0$) and a Λ CDM cosmological model with $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. For each realization, the best-fitting values of the parameters are obtained by minimizing Eq. (2.1). At each step, i.e., for each resample, the arithmetic mean and the variance of the estimators are computed. This loop ends when the required precision is achieved. In this work, the number of realizations needed varies between 10^4 and 2×10^5 .

3. Results and Concluding Remarks

We first apply the methodology considering a spline of $q(z)$ with 8 knots within the redshift interval [0.01, 1.4]. We perform this analysis using only the SNLS3 data, i.e., its covariance matrix to generate the realizations, and Eqs. (2.5) and (2.1) to obtain the means and variances of the 9 q_i estimators, α , β , M_1 and M_2 . We consider 4 different values of $\sigma_{rel} = 0.05, 0.1, 0.5$ and 0.8 . In Fig. 1 we show the reconstructed $q(z)$ function, computed using the mean of the estimators, and its 1σ error bar.

In this second case we consider 12 knots within the redshift interval [0, 2.3]. We perform this analysis using the joint likelihood:

$$-2 \ln(L) = -2 \ln(L_{SNIa}) - 2 \ln(L_{BAO}) - 2 \ln(L_{Hubble}). \quad (3.1)$$

Fig. 2 displays the results for those 4 different σ_{rel} values.

It is worth emphasizing that the parameters α , β , M_1 , M_2 and H_0 are recovered in all cases, i.e. ML estimators for these quantities have negligible biases.

In both figures we plotted the threshold line, $q(z) = 0$, which indicates the deceleration/acceleration regions. In the 8 cases studied, we obtain the indication of an accelerated expansion with more than 1σ confidence level in the interval $0 \leq z \lesssim 0.4$.

In this work, we used Λ CDM as the fiducial model, in which $q(z)$ is almost linear. Therefore, in order to obtain a more conservative reconstruction of $q(z)$, it is necessary to apply this study for fiducial models with different functional forms of $q(z)$. Once we get suitable number of knots and σ_{rel} , which work well for all fiducial models, we can apply the method using real data and recover a conservative estimate of the recent expansion history of the universe (Vitenti & Penna-Lima, in prep.).

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