SUCCESSES AND CHALLENGES OF THE THEORY OF WHITE DWARF SPECTRAL EVOLUTION

G. FONTAINE and F. WESEMAEL
Département de Physique, Université de Montréal

Of all stars in the Hertzprung-Russell diagram, white dwarfs are those for which the clues on past evolution given by photospheric abundances are probably the hardest to decipher. This is because the cooling phase of white dwarfs, a relatively uneventful phase from an evolutionary point of view, is, in contrast, a most active phase for the evolution of the chemical composition of the envelope. Indeed, it is now well established that the often puzzling variety of surface abundances observed in white dwarf stars can be traced to the simultaneous operation, in the outer layers of these stars, of a variety of physical processes which will also erase the abundances present in the photosphere at the onset of cooling.

Downward element diffusion in the intense gravitational field of the degenerate star is perhaps the mechanism which is the most closely identified with white dwarf stars. However, convective mixing, ordinary diffusion, radiative forces, winds, and accretion from the interstellar medium all are equally important processes which, at times, compete efficiently with the rapid element segregation expected in those stars. The various regions, along the cooling sequence of white dwarfs, where individual processes are expected to operate, have been summarized by Fontaine and Wesemael (1987). We illustrate here various combinations of these mechanisms which have been found in white dwarfs, and show how their competition affects the observed abundance patterns. The unity underlying these cases stems from the fact that, in many cases, progress in investigating these complicated situations has come only through the combination of evolutionary calculations with new and powerful numeriques techniques which have been developed at Montréal (Pelletier 1986; Pelletier, Fontaine, and Wesemael 1989).

1. Helium in DA Stars in the Presence of Radiative Forces

As a first illustration of the way the various processes discussed above operate in competition against each other, we present some results obtained by Vennes *et al.* (1988), on the competition between radiative forces and downward settling, as well as that between ordinary diffusion and settling, in the atmospheres and envelopes of hot DA white dwarfs. It seems quite likely that these processes are the ones which control the presence of helium in the photospheres of hydrogen-rich stars at high effective temperatures.

The results presented here follow the evolution of the chemical composition in a chemically-stratified 0.6 $M_\odot$ DA white dwarf. The outermost hydrogen-rich envelope

G. Michaud and A. Tutukov (eds.), Evolution of Stars: The Photospheric Abundance Connection, 421–434.

extends down to a fractional mass depth $q_H = \Delta M_H/M = 10^{-10}$, and is initially contaminated by a small, uniform amount of helium ($c_2 = n_{He}/(n_{He} + n_H) = 10^{-4}$).

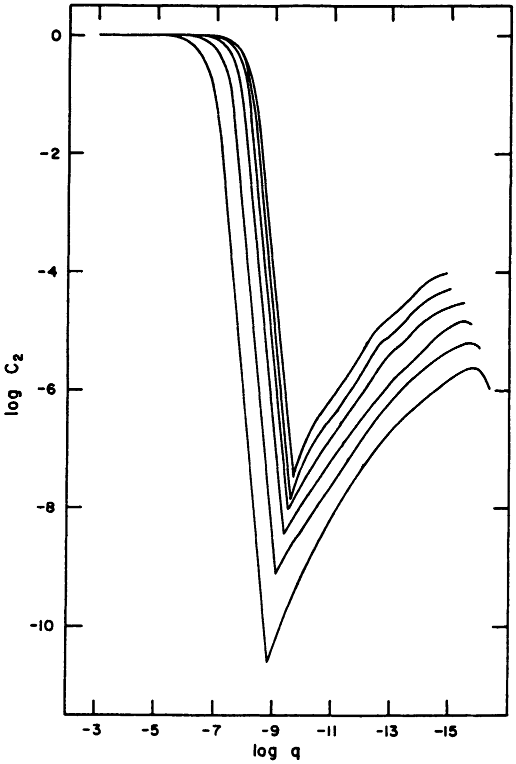

Figure 1: Helium distribution in an evolving 0.6 $M_\odot$ DA white dwarf. The curves correspond to models at 75,000 K (top), 65,000 K, 55,000 K, 45,000 K, 35,000 K, and 25,000 K (bottom).

The underlying helium-rich envelope is contaminated as well by small traces of hydrogen ($c_2 = 0.999$) which are initially uniformly distributed. Figure 1 shows the evolving helium distribution for six models along the cooling sequence, starting from 75,000 K. There are clearly two regimes to this distribution, which are associated with the competition between different physical processes in different parts of the stellar envelope: in the outer layers, the helium distribution is found to relax extremely rapidly (*i.e.*, within a few hundred years) to an equilibrium situation with the downward gravitational settling of helium counterbalanced by the upward radiative support through the bound-bound transitions. This balancing provides support for small quantities of helium in the *atmospheres* of hot DA stars, at the level of $\log c_2 \sim -5$. As the star cools, the equilibrium abundance of helium supported in the atmosphere reajusts itself to the changing physical conditions, in particular to the decrease in radiative flux which accompanies the cooling. At 25,000 K, the coolest model considered here, the photospheric helium abundance has dropped by a factor

~ 100 from its equilibrium value near $T_e = 75,000$K.

Deeper in the envelope, where radiative support is negligible, downward gravitational settling competes with ordinary diffusion driven by the composition gradient associated with the hydrogen–helium interface. In contrast to the situation near the surface, however, an equilibrium situation is not achieved over the cooling timescale associated with these evolutionary models, and hydrogen and helium are still separating at depth.

For very small amounts of hydrogen, the separation process would, of course, be completed very quickly. It is currently suggested that hot DA white dwarfs may have such thin layers of hydrogen which has, in fact, separated out of underlying helium-dominated regions. These layers must be thin enough ($\Delta M/M \sim 10^{-15} - 10^{-13}$) so that spectral observations can probe completely through them and be sensitive to the underlying helium. This *stratified atmosphere model* provides a most plausible explanation for the bulk of the soft X-ray observations of hot white dwarfs which require an additional opacity source (provided here by helium) besides that provided by pure hydrogen. Vennes *et al.* (1988) found that the amount of helium supported by radiative levitation in hot DA white dwarfs is quite insufficient to provide the needed opacity source, and suggested the stratified atmosphere model as an alternative. Examples of actual stratified model atmosphere calculations for hot DA white dwarfs are given by Vennes, Fontaine, and Wesemael (1989). It is to be noted that the amounts of hydrogen which are implied by this model are *much smaller* (up to 11 orders of magnitude) than the predictions of standard evolution theory. This must be seen as a real challenge to the latter.

2. Heavy Elements in Hot White Dwarfs: Radiative Levitation and Winds

Elements heavier than helium are sometimes observed in the spectra of hot white dwarfs. It is believed that selective radiative forces have a key role to play in this phenomenon. The most detailed calculations of radiative forces in a white dwarf context are those of Chayer *et al.* (1989) and Chayer, Fontaine, and Wesemael (1991). Some of their results are summarized in the Figure 2, which shows the expected equilibrium abundance of C, N, O, Si, and Fe at the photosphere of an evolving 0.6 $M_\odot$ white dwarf as a function of the effective temperature.

Combined with the current detection limits, these results indicate that radiative levitation can be of importance in DA white dwarfs with $T_e \gtrsim$ 20,000 K and non-DA stars with $T_e \gtrsim$ 30,000 K. Thus, given a source of heavy elements (of primordial origin or otherwise), radiative levitation can qualitatively account for the presence of metallic "pollutants" in the atmospheres of hot white dwarfs.

In this context, a nice success of the radiative support theory is the explanation put forward by Vennes *et al.* (1989) to account for the puzzling EUV spectrum of the 55,000 K DA white dwarf Feige 24. A convincing fit to the observed spectrum is obtained by assuming that there are *many* metals in small individual amounts present in the atmosphere of Feige 24. It is the collective effects of all these abundances that provide the opacity source which regulates the emergent flux in the EUV region. The abundances of metals used in the model are in fairly good agreement with those predicted by radiative levitation.

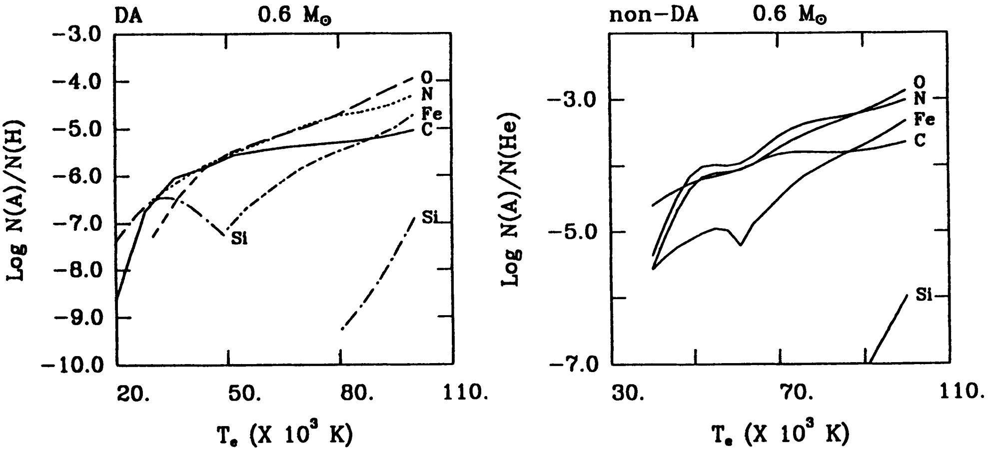

Figure 2: Abundances of C, N, O, Si, and Fe supported by radiative forces at the photosphere of a DA (left panel) and non-DA (right panel) 0.6 $M_\odot$ white dwarf as a function of effective temperature.

There are problems, however, with the simple picture of equilibrium between settling and radiative levitation in hot white dwarfs. Indeed, there are many known objects which show *no* detectable traces of heavy elements in their spectra. Moreover, a detailed look at the available observations indicates that the observed abundance patterns in these stars which show metal lines are not consistent with the abundances predicted by the theory (see, *e.g.*, Chayer *et al.* 1987). It is suspected that weak winds are responsible for this state of affair (Chayer *et al.* 1987, Chayer, Fontaine, and Wesemael 1989). Indeed, weak winds ($\sim 10^{-21}$ $M_\odot$ yr^{-1}) can play havoc with the abundances predicted by simple radiative support theory. To understand fully the observed abundance patterns in hot white dwarfs, we believe that it is necessary to pursue detailed time-dependent calculations of the diffusion process in presence of radiative forces and stellar winds. Such calculations are currently underway.

3. The Settling of Heavy Elements in Cool Stars

It has been recognized since Schatzman (1958) that the intense pressure gradient in the outer layers of white dwarfs would cause trace elements heavier than the dominant constituent to settle rapidly out of the atmosphere of those stars. This process is so efficient in white dwarf stars that — were this mechanism the only one operating in the outer layers — no heavy elements would be detectable in the atmospheres of either hydrogen-rich or helium-rich white dwarfs. Of course nature is not that simple, and metals are observed both in hot stars, where, as we have seen, selective radiative support and perhaps winds counteract gravitational settling, and in cool white dwarfs, where accretion from the interstellar medium is suspected to replenish the metal supply (see below).

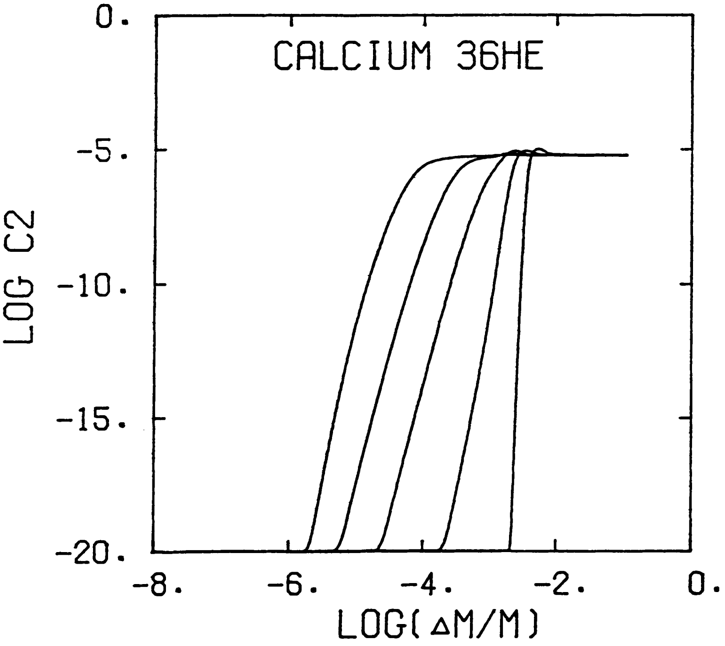

Figure 3: Settling of calcium in an evolutionary sequence of helium-rich envelopes for a 0.6 $M_\odot$ white dwarf. The calcium abundance profiles shown are, from left to right, for $T_e = 25{,}000$ K, 20,000 K, 15,000 K, 10,000K, and 5000 K. Note the calcium accumulation apparent near log $q = -3$.

Until recently, all investigations of the settling of heavy elements had concentrated on estimates of the diffusion timescales, generally at the bottom of the convection zone, without providing any information about the time evolution of the element distribution. With the advent of new numerical techniques, which allow one to follow the evolution of the surface abundance over more than 25 orders of magnitude without significant loss of accuracy or crippling numerical instabilities, progress in this specific area has been made possible. Figure 3 presents sample results of calculations (Dupuis *et al.* 1991*a*) which consider the interplay of evolution with diffusion for a representative element (calcium). Shown is the calcium distribution as a function of fractional mass in the envelope at five effective temperatures chosen along the cooling sequence. At each temperature, the initial element distribution was assumed to be that calculated with the previous value of T_e. That distribution was then allowed to evolve for a time equal to the difference in cooling age between these two temperatures. The photospheric abundances encountered in these calculations are all many orders of magnitude below detection levels; quite clearly, this can only corroborate the need for mechanisms competing with downward settling in cool stars which exhibit metal lines.

Of particular interest in Figure 3 is the presence of tiny bumps in the calcium abundance profile near $\log q = -3$. These bumps appear in regions where the downward diffusion of calcium becomes negligible because of its complete pressure ionization, and because of the strong degeneracy of the background plasma. Calcium diffusing from shallower layers will accumulate in that region. Note also that, over the cooling timescale of the 5000 K model (roughly 6×10^9 yr), settling leaves much of the stellar mass ($\log q \gtrsim -2.4$) essentially unaffected. This may have interesting implications for the overall evolution of the star because heavy elements will contribute to the conductive opacity in the core material. The added opacity source (neglected in *all* evolutionary sequences calculated so far) will increase the cooling time scale of the white dwarf. It remains to be seen whether or not this effect is significant. The question is currently under scrutiny.

4. The Presence of Metals in Cool White Dwarfs: Competition between Settling and Accretion

As was emphasized above, the surface abundances expected under the sole presence of downward gravitational settling in white dwarf stars are vanishingly small. Some mechanism must be present to replenish the surface reservoir of metals in those objects, and accretion from the interstellar medium is thought to be the most likely such mechanism in cool stars. In an undoubtedly oversimplified model, white dwarfs are generally considered as spending the vast majority of their cooling time traveling in a tenuous medium, with accretion proceeding at an extremely low background rate. Periodically, however, the star will encounter a region of enhanced density (which we call here cloud, but which probably looks more like a patch), from which accretion can proceed, for a shorter time, at a higher rate. In the simulations described here (Dupuis *et al.* 1991*b*), a typical encounter was assumed to last roughly 10^6 yr, during which the white dwarf would accrete at a rate of $5 \times 10^{-15} M_\odot$ yr^{-1}. These encounters are separated by roughly 5×10^7 yr, during which the star accretes at a low background rate of $5 \times 10^{-19} M_\odot$ yr^{-1}. We assume further, for want of a better knowledge of the accretion process, that heavy elements are accreted in solar proportions. Because the encounters are widely separated in time, each can be considered as an independent event, in the sense that successive encounters leave no cumulative effects on the observable surface abundances.

Figure 4 shows the results of our numerical simulations of these events. In this example, the accreting star is considered to be a 10,000 K, helium-rich white dwarf. The bottom part of the figure shows the time dependence of the metal abundance in the course of a cloud encounter for both calcium and silicon. The first phase, for times less than $\sim 2 \times 10^7$ yr, is essentially a transient phase, where the memory of the initial conditions (here a solar abundance in both cases) is erased by downward gravitational settling. A steady state situation is then reached, whereby an equilibrium abundance is maintained in the atmosphere under the combined action of an accretion flux at the top of the photosphere and a diffusion flux at the bottom of the convection zone. Because the accretion rate in the tenuous phase is rather small, the equilibrium abundance in the photosphere is of the order of 10^{-10} for silicon, and even lower for calcium. After 5×10^7 yr, the white dwarf encounters a cloud, and accretion now briefly proceeds at the much larger rate, chosen to mimick accretion near the Bondi–Hoyle rate. The photospheric abundance increases sharply, but with the accretion pulse being relatively short-lived (10^6 yr), the abundance never reaches a steady-state value associated with this large accretion rate, but rather decays back to the background equilibrium value. The slope of this decay varies from element to element, as it is a function of the diffusion velocity at the bottom of the convection zone. Also indicated on Figure 4 is the abundance associated with a Ca K line of 5 Å equivalent width, a sort of limit of visibility for that ion. Clearly, the K line will be visible in those stars during only a small fraction of the cooling time, and the best time to 'catch' a white dwarf with metal lines is during, or at most a few diffusion times after, an accretion episode.

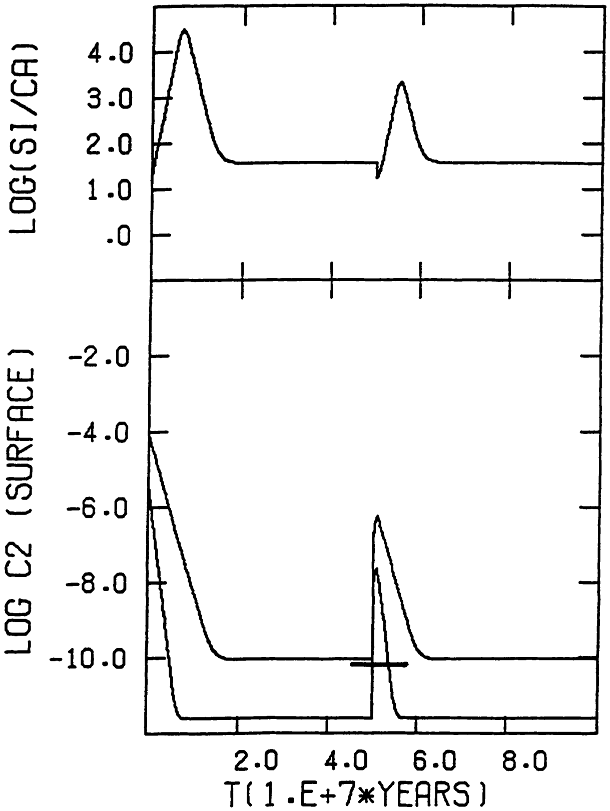

Figure 4: Evolution of the surface abundance of calcium and silicon in a 10,000 K helium-rich white dwarf undergoing an accretion episode. The top panel shows the evolution of the silicon-to-calcium abundance ratio, while the bottom panel shows the evolution of the silicon-to-helium (top curve) and calcium-to-helium (bottom curve) abundance ratios. The accretion rates are 5×10^{-19} $M_\odot$ yr^{-1} in the low state and 5×10^{-15} $M_\odot$ yr^{-1} in the high state.

The top of Figure 4 shows the *relative* abundance ratio between silicon and calcium. Its variations can be understood as follows: during the low-accretion phase, the equilibrium abundance ratio is fixed by the relative diffusion timescale of calcium and silicon, as well as by the abundance ratio in the accreting material, which is considered to be solar here. At the onset of the pulse, the Si/Ca assumes very briefly its solar value, as accretion completely overwhelms the settling process. Eventually, however, the effects of settling are felt, and the Si/Ca ratio first increases, until a steady-state is reached for calcium at the bottom of the convection zone. From there on, the Si/Ca ratio will decrease, as silicon is still being depleted by settling, until that element, in turn, reaches its equilibrium value set by the balance between diffusion below the convection zone and accretion. The Si/Ca ratio remains fixed from there on.

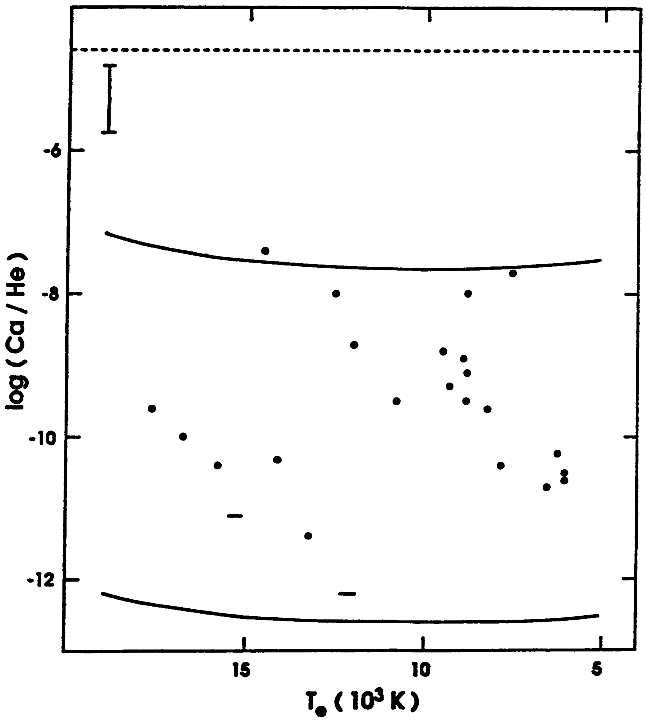

Figure 5: Relative abundance of calcium with respect to helium as a function of effective temperature in cool, helium-rich white dwarfs. Small horizontal bars represent upper limits. The typical uncertainty is also indicated in the left, upper corner. The horizontal dashed line is the solar Ca/He ratio, while the two solid lines define the range of expected abundances on the basis of the two-phase model described in the text. For these curves, rates of 5×10^{-15} and $5 \times 10^{-20} M_\odot\ yr^{-1}$ have been adopted.

The predictions of this two-phase accretion model are in good agreement with determinations of the metal abundances in several cool, helium-rich white dwarfs. This agreement is particularly encouraging, since the parameters used to describe the two-phase model of the interstellar medium are based on considerations which are completely unrelated to white dwarf stars. As an illustration, Figure 5 contrasts the observed value of the Ca/He abundance ratio in cool, helium-rich white dwarfs with the range allowed by our model. The curved lines are the steady-state values calculated for our low and high accretion rates, respectively. These lines thus represent the range of abundances allowed in our model, and it is gratifying that the bulk of the observations is found to lie between those lines. Furthermore, the two-phase model is also in fairly good agreement with the *relative* abundance ratios observed in cool helium-rich stars. Using all metal abundance ratios available in cool white dwarfs, Dupuis, Fontaine, and Wesemael (1991) have found that a consistent picture of the accretion process for *all* cases is obtained if one assumes that calcium is overabundant by a factor of ~ 3, and silicon underabundant by a factor of ~ 7, in the accreting material.

Much progress remains to be made in our understanding of the accretion process in white dwarf stars. For example, to what extent is fractionation in the accreting material important in modifying the relative abundances (*i.e.*, the Si/Ca ratio) as seemed to be required by the model of Dupuis, Fontaine, and Wesemael (1991)? Or what is the screening mechanism that prevents the accretion of hydrogen while allowing that of heavy elements? What role do grains play in the accretion process? While our understanding remains incomplete, it is also clear that the two-phase model discussed here appears successful in explaining the general trends observed in helium-rich white dwarfs, and has now been set on much firmer footing. This bodes well for future investigations of this problem.

5. Dredge-Up of Carbon in Cool Non-DA Stars

A significant exception to the pattern of abundances explained above is the behavior of carbon in the envelopes of helium-rich white dwarfs. Not only is carbon present in quantities which far exceed those predicted by the simple accretion–diffusion model explored above, but the maximal carbon abundance in cool helium-rich objects is observed near the effective temperature where the helium convection zone is the deepest in those stars, near 10,000 K–13,000 K (see Fig. 7 below). This situation led to the suggestion that — in contrast to the other metals observed in white dwarfs — carbon had an intrinsic origin, *i.e.*, the observed carbon originated in the deep envelope, and was brought up to the surface through convective dredge-up (Koester *et al.* 1982, Fontaine *et al.* 1984).

The most thorough calculations of this dredge-up process are those of Pelletier *et al.* (1986). The basic physical situation is summarized in Figure 6, where we show the evolving structure of a cooling, helium-rich white dwarf. The main feature of the outermost, helium-rich envelope is the presence of an extensive convection zone, which extends all the way to the photosphere. This confirms that the observed surface composition of these stars can be affected by processes which occur near the base of the convection zone. The convection zone, itself, reaches its maximum depth near 11,000 K in these calculations, and recedes only slightly toward the surface with further cooling. Also shown are contours of constant carbon abundance ($c_2 = n_C/(n_{He} + n_C)$). In the first phases of cooling (*i.e.*, above log $T_e = 4.40$), the carbon abundance profile is still affected by the arbitrary initial conditions imposed on the calculations. At cooler temperatures, however, the calculations demonstrate clearly the upward diffusion of carbon as the helium and carbon distributions evolve toward an equilibrium situation. This upward diffusion is interrupted when the convection zone abruptly digs into the carbon-enriched layers, and this is the first opportunity for carbon to be dredged-up to the surface in observable quantities. At still cooler effective temperatures, below log $T_e = 4.0$, carbon actually diffuses back into the core due to some complex ionization effects (Pelletier *et al.* 1986).

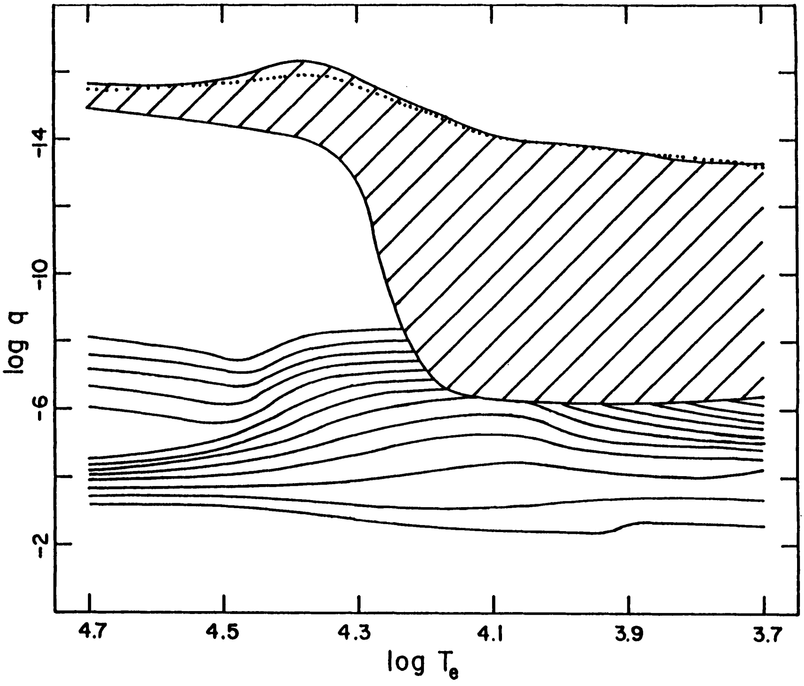

Figure 6: Mass fraction *vs* effective temperature for an evolutionary sequence for a 0.6 $M_\odot$ helium-rich white dwarf, log $q_{He} = -3.5$, and ML1 convection. The shaded area gives the location and profile of the helium convection zone. The dotted line corresponds to the location of the photosphere. The continuous lines give contours of constant carbon abundance with, respectively from top to bottom, $c_2 = 10^{-10}, 10^{-9}$, $10^{-8}, 10^{-7}$, $10^{-6}, 10^{-5}$, $10^{-4}, 10^{-3}$, $10^{-2}, 10^{-1}$, 0.5,0.9, and 0.99.

The expected carbon enrichment is shown in Figure 7, together with observed abundances and upper limits culled from a variety of sources. References to the observational material are given in Pelletier *et al.* (1986). The main features of the curves shown in Figure 7 are intimately related to the behavior of both the convection zone and the carbon abundance profile discussed above. Hence, the steep rise in the expected carbon abundance on the hot side of the peak is a mirror image of the steep descent of the convection zone in the deeper envelope layers shown in Figure 6. The maximal depth of the convection zone, reached between 10,000 K and 13,000 K, leads to the predicted maximum in the carbon abundance shown in Figure 7, while the smooth decrease at still lower effective temperatures is a direct consequence of the ionization-driven downward diffusion of carbon discussed above.

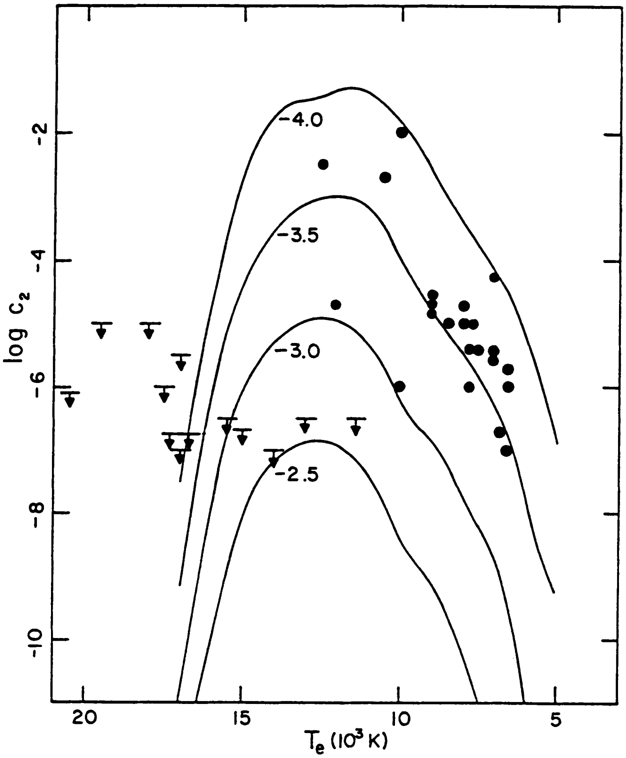

Figure 7: Predicted and observed carbon abundance in cool helium-rich white dwarfs as a function of effective temperature. The continuous curve corresponds to the theoretical predictions for four evolutionary sequences which differ by the assumed value of log q_{He}, which is indicated. The filled circles and arrows represent measurements and upper limits obtained from spectroscopic analyses of DQ and cool DB stars, respectively.

One interesting aspect of Figure 7 is that it allows us to measure the thickness of the helium envelope in those helium-rich stars which exhibit carbon in their atmospheres. Clearly, the amount of carbon pollution is a sensitive fonction of the helium envelope mass: in massive helium envelopes, the transition region between helium and carbon is located much deeper than the base of the convection zone, and the resulting carbon pollution is minimal. Of course, there is a sensitivity as well to the adopted treatment of the convective efficiency and to the stellar mass, which both determine the extent of the convection zone in those objects. A more efficient convective energy transport, or a lower stellar mass, leads to a deeper convective envelope, and thus to a more extensive carbon pollution of the observable layers. All in all, models with fairly thin helium envelopes, in the range $-4.0 \lesssim \log q_{He} \lesssim -3.5$, appear to yield the best fit to the observations. These estimates are about 100 times smaller than the expected amounts of helium that are supposed to be left in white dwarfs according to stellar evolution theory.

6. Convective Mixing in Cool DA Stars

As a DA star cools down below 20,000 K, a convection zone develops in its superficial layers because of the recombination of hydrogen (Koester 1976, Vauclair

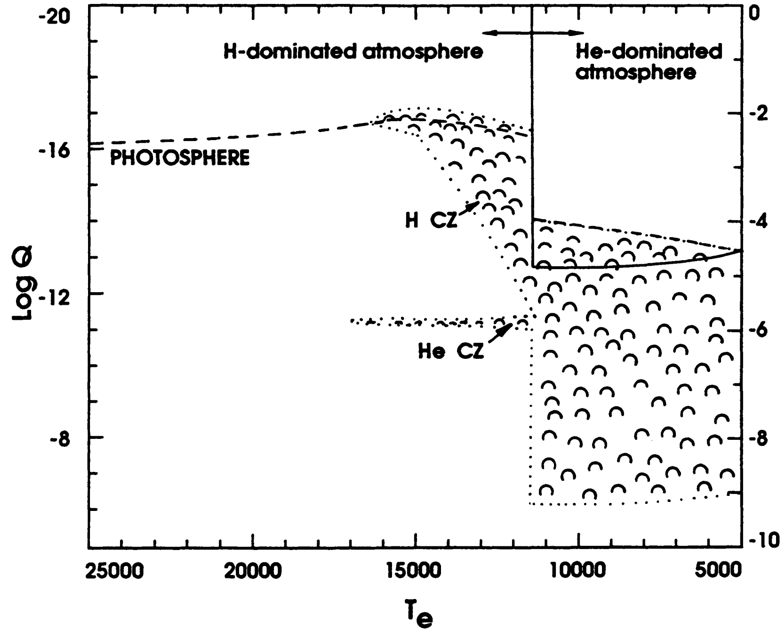

Figure 8: Structure of the outer layers of a DA star as a function of effective temperature. The variable q is, again, the fractional mass depth, $q = \Delta M/M$. This particular model, at 0.6 $M_\odot$, has a hydrogen layer mass $q_H = 10^{-11}$ overlying a deep helium envelope. The hydrogen convection zone first develops near $T_e \sim 16{,}500$K, and merges with the underlying helium convection zone at 11,400 K. The dashed line indicates the position of the photosphere, while the continuous line in the upper-right region of the diagram indicates the value of the hydrogen surface abundance, $n_H/(n_{He} + n_H)$), which is to be read on the right-hand-side scale.

and Reisse 1977). This evolution is represented in Figure 8, where we show the evolving structure of a 0.6 $M_\odot$, hydrogen-line (DA) white dwarf below 25,000 K. The hydrogen envelope has an assumed fractional mass of 10^{-11}. A helium convection zone first develops near the transition region near 17,000 K, while the recombination of hydrogen leads to the appearance of a hydrogen convection zone, higher in the atmosphere, near 16,500 K. The hydrogen convection zone digs in deeper into the envelope, until it touches and merges with the helium convection zone. The temperature at which this occurs, called the mixing temperature, is a function of the hydrogen layer mass and of the adopted efficiency of convective mixing (Tassoul, Fontaine, and Winget 1990). For the particular model studied here, mixing occurs near 11,400 K, and the post-mixing surface composition is completely dominated by helium.

Detailed calculations of the mixing process carried out by Forestini (1991) similar to the case presented here, but exploring a wide variety of parameters, confirm the early suspicion that DA white dwarfs with thin hydrogen layers turn into stars with atmospheres completely dominated by helium. For example, a 0.6 $M_\odot$ DA star with $q_H = 10^{-14}, 10^{-12}$, and 10^{-10} mixes at $T_e \approx$ 13,200 K, 12,580 K, and 10,170 K, and the post-mixing surface abundance is log $(n_H/(n_{He} + n_H)) \approx -7.2, -5.2$, and -3.2, respectively. Such helium-dominated atmospheres should show featureless DC spectra.

Some recent results by Bergeron *et al.* (1990) cast serious doubt on this picture. In a spectroscopic study of cool DA stars below 14,000 K, these authors succeeded in determining helium abundances in hydrogen-line stars despite the fact that helium is spectroscopically invisible at these temperatures. The result of their investigation is that most of hydrogen-line stars they analyzed *did show* some evidence of helium in their atmospheres, with — in a few cases — helium actually being the dominant atmospheric constituent! For the bulk of their sample, $n_{He}/n_H \sim$ 0.10–0.30. It seems quite clear that the helium observed in these objects is the result of convective mixing of the thin, overlying hydrogen layer with the helium envelope. However, the observed helium abundance appears much lower than the abundances predicted by the mixing scenario discussed above, to the point that the mixed stars do in fact preserve their DA character rather than turn into DC stars, as expected. Either our understanding of the efficiency of convective mixing in the envelopes of white dwarf stars is seriously flawed, or some other mechanism — perhaps accretion of hydrogen from the interstellar medium — is at work in these stars.

7. Concluding Remarks

This brief survey of topics underscores the rich variety of physical processes encountered in white dwarf stars. Clearly, much success has been achieved in outlining which processes are important in specific situations. However, a thorough understanding of the spectral evolution remains an elusive goal which will require additional effort. Even at this stage, however, our understanding of the chemical composition of white dwarf stars has already greatly contributed to constraining, and sometimes calling into question, important results of stellar evolution. It seems clear that ongoing investigations of the remaining dilemnas in this field will continue to contribute in this direction.

We gratefully acknowledge the essential contribution of our collaborators, P. Bergeron, P. Chayer, J. Dupuis, M. Forestini, C. Pelletier, and S. Vennes, to the work summarized here. This work was supported in part by the NSERC Canada and by the Fund FCAR (Québec). G. Fontaine was also partially supported by a Killam Fellowship.

REFERENCES

Bergeron, P., Wesemael, F., Fontaine, G., and Liebert, J. 1990, *Ap.J. (Letters),* **351**, L21.

Chayer, P., Fontaine, G., and Wesemael, F. 1989, in *I.A.U. Coll. 114, White Dwarfs*, ed. G. Wegner (New York: Springer Verlag), p. 253.

——. 1991, in *Proc. 7th European Workshop on White Dwarfs*, ed. G. Vauclair and E.M. Sion, NATO ASI Series, in press.

Chayer, P., Fontaine, G., Wesemael, F., and Michaud, G. 1987, in *I.A.U. Coll. 95, The Second Conference on Faint Blue Stars*, ed. A.G.D. Philip, D.S. Hayes, and J. Liebert (Schenectady: Davis), p. 653.

Dupuis, J., Fontaine, G., Pelletier, C., and Wesemael, F. 1991*a*, *Ap.J.*, in press.

——. 1991*b*, *Ap.J.*, in press.

Dupuis, J., Fontaine, G., and Wesemael, F. 1991, *Ap.J.*, in press.

Fontaine, G., Villeneuve, B., Wesemael, F., and Wegner, G. 1984, *Ap.J. (Letters),* **277**, L61.

Fontaine, G., and Wesemael, F. 1987, in *I.A.U. Coll. 95, The Second Conference on Faint Blue Stars*, ed. A.G.D. Philip, D.S. Hayes, and J. Liebert (Schenectady: Davis), p. 319.

Forestini, M. 1991, unpublished M.Sc. Thesis, Université de Montréal.

Koester, D. 1976, *Astr. Ap.,* **52**, 415.

Koester, D., Weidemann, V., and Zeidler-K.T., E.M. 1982, *Astr. Ap.,* **116**, 147.

Pelletier, C. 1986, unpublished Ph.D. Thesis, Université de Montréal.

Pelletier, C., Fontaine, F., and Wesemael, F. 1989, in *I.A.U. Coll. 114, White Dwarfs*, ed. G. Wegner (New York: Springer Verlag), p. 249.

Pelletier, C., Fontaine, G., Wesemael, F., Michaud, G., and Wegner, G. 1986, *Ap.J.,* **307**, 242.

Schatzman, E. 1958, *White Dwarfs* (Amsterdam: North Holland).

Tassoul, M., Fontaine, G., and Winget, D.E. 1990, *Ap.J. Suppl.,* **72**, 335.

Vauclair, G., and Reisse, C. 1977, *Astr. Ap.,* **61**, 415.

Vennes, S., Chayer, P., Fontaine, G., and Wesemael, F. 1989, *Ap.J. (Letters),* **336**, L25.

Vennes, S., Fontaine, G., and Wesemael, F. 1989, in *I.A.U. Coll. 114, White Dwarfs*, ed. G. Wegner (New York: Springer Verlag), p. 368.

Vennes, S., Pelletier, C., Fontaine, G., and Wesemael, F. 1988, *Ap.J.,* **331**, 876.