3

The QCD story

We shall limit ourselves here to a qualitative survey of Quantum ChromoDynamics (QCD), with the aim to present in a short and simple way the main idea behind the theory. Many more complete and detailed reviews and books on QCD [2,3,42–52] and on Quantum Field Theory [53] exist in the literature, which the interested readers may consult, while recent results and developments in QCD both from theory and experiments may, for example, be found in many conferences, for instance, in the proceedings of the QCD–Montpellier Series of Conferences published regularly in *Nucl. Phys. B* (Proc. Suppl.) by Elsevier Publ. Co.

3.1 QCD and the notion of quarks

- QCD is by now expected (and widely accepted) to be the field theory describing the strong interactions of quarks q [8,9] (elementary constituents of the matter) having *three colours (blue, red, yellow)* which are glued together inside the nucleus by eight coloured *(chromo)* gluons which provide a vehicle for the Yukawa strong nuclear forces. However, the quark scheme is not only a pure mathematical concept for classifying the hadronic world. There is indirect evidence of the existence of quarks through the observation of two-jet events, such as the one from:
$$Z^0 \to \text{hadrons}\,, \tag{3.1}$$
as shown in Fig. 3.1.
- QCD originated from the natural development of the quark model of the early 1960s, where, as we have discussed in the previous chapter, hadrons were classified under the representations of an $SU(3)_F$ (now called a *flavour group*), the so-called *eightfoldway* of Gell-Mann and Ne'eman [7], where *ordinary* mesons and baryons of this $SU(3)_F$ classification are respectively bound states $\bar{q}q$ and qqq of the *light* quarks *up (u), down (d)* and *strange (s)*. The masses of these quarks,[1] which are given in the next section, are much lower than the value of the QCD scale $\Lambda \approx 300$ MeV, and have the values at the scale 2 GeV[2] [54]:
$$m_u \simeq 3.5\ \text{MeV}\,, \qquad m_d \simeq 6.3\ \text{MeV}\,, \qquad m_s \simeq 119\ \text{MeV}\,, \tag{3.2}$$
where one can notice that $m_s/m_d \simeq 20$ is a huge number obtained originally [21,55–57] from *current algebra* approaches [13]. These values have to be contrasted with the so-called *constituent* quark

[1] As quarks are not directly observed, the definitions of their masses are only theoretical. For light quarks, I will quote the values of running (or current) masses evaluated at a certain scale.
[2] The original choice of scale is 1 GeV. We take 2 GeV in order to follow current practice.

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Table 3.1. *Quantum numbers of the new quarks*

Quark	c	b	t
Charge Q	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
C-charm	1	0	0
B-bottomness or beauty	0	-1	0
T-topness or topless	0	0	1

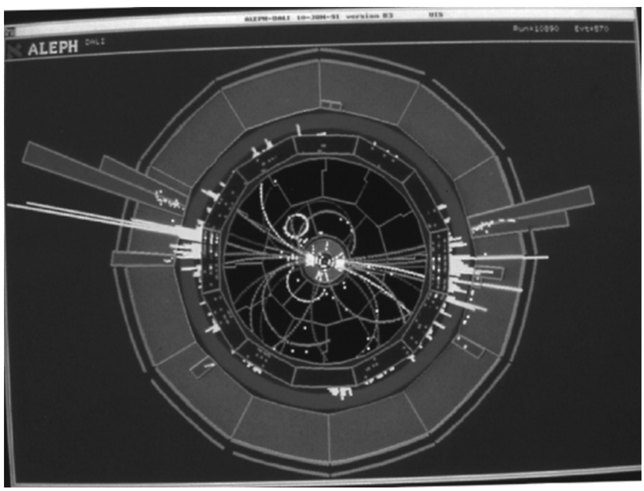

Fig. 3.1. Two-jet events from hadronic Z^0 decay.

values $M_q \approx 300$ MeV, used in the previous chapter for the case of the quark and potential model approaches [12], for explaining the mass-splittings of hadrons using the Gell-Mann–Okubo-like mass formulae [11].

- Including the previous three light quarks, at present *six quark flavours* have been found and classified according to their charge Q in units of the electron. They are:

$$Q = 2/3 \ : \ (u, c, t)$$
$$Q = -1/3 \ : \ (d, s, b) \,, \tag{3.3}$$

where, for instance, in these triplet representations, the neutral currents of electroweak interactions are flavour conserving. The new quarks c, b, t carry new quantum numbers as shown in Table 3.1.

- The *charm quark* was proposed in [58], in which the name *charm* was adopted by Bjorken and Glashow [58]. The discovery of the charm quark through the finding of the $\bar{c}c$ bound state J/ψ meson [59] at 3.1 GeV, indicates that its mass is about 1/2 of the one of the meson.[3] Its discovery has been crucial for avoiding the flavour changing neutral current responsible for the excess of Z^0

[3] For heavy quarks ($m_q \gg \Lambda$), the mass is defined as the on-shell mass (pole mass) analogous to the one of the electron (see next section).

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exchange contributions in the K^0-$\bar{K}^0$ oscillations, and for the huge $K_L \to \mu^+\mu^-$ and $K^\pm \to \pi^\pm\nu\bar{\nu}$ experimentally unacceptable rates. The need for charm in this mechanism was indeed advocated a long time ago by Glashow–Iliopoulos–Maiani *(GIM suppression mechanism)* [60]. Then, after the charm discovery, the two generations of quarks (u, d) and (c, s) for the electroweak $SU(2)_L \times U(1)$ standard model (SM) of Glashow–Weinberg–Salam [61] were completed, and could be compared with the two lepton doublets (e, ν_e) and (μ, ν_μ). These two quark doublets mix through the Cabibbo mixing angle θ_c introduced a long time ago [15], and has the experimental value $\sin\theta_c = 0.220 \pm 0.003$ from, for example, the $K \to \pi^0 e^+\nu_e$ process [16].

- The discovery in 1974 of the third τ charged lepton [62], having a mass 1.8 GeV, was the first sign of the third generation, which was confirmed later on by the discovery of the Υ, which is a $\bar{b}b$ bound state [63] in 1977, with a b-mass $M_b \approx 4.6$ GeV, expected to be about 1/2 of the one of the Υ. More recently, the third family has been completed by the discovery of the t quark in 1995 [64] from the analysis of the lepton + jet and dilepton channels originated from $\bar{t}t \to W^- b W^+ b$ processes at the collider experiments. This gives a top mass $M_t \simeq (174.3 \pm 3.2 \pm 4.0)$ GeV [16]. The b and t quarks have been predicted by Kobayashi and Maskawa [65], and the names *bottom* and *top* were first used by Harari [66]. At present, we have found three families of leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \tag{3.4}$$

and analogous three families of quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} . \tag{3.5}$$

Quark families mix under a 3×3 unitary matrix, which is a generalization of the previous 2×2 Cabibbo unitary matrix, and which is called the CKM (Cabbibo–Kobayashi–Maskawa) mixing matrix [67]. This matrix has three real parameters (mixing angles) and one CP violating phase (see Appendix A3), which cannot be absorbed by a redefinition of the quark fields. LEP studies [68] of the Z^0 width also indicate that it is unlikely to have more than three (almost) massless neutrinos, such that, most probably, we only have these three generations in nature.

3.2 The notion of colours

- Historically [69], the introduction of colours has been motivated by the failure of the quark model to expain the peculiar nature of the pion-nucleon Δ^{++} baryon, which has a total zero angular momentum $J = 3/2$. In order to fulfill this property, one has to put its three u-quark constituents with spins aligned up. This requirement is not allowed by Dirac statistics as the quarks are supposed to be a spin 1/2 particle. This *wrong statistic* problem is solved when one gives three colours to the quarks,[4] such that the Δ^{++} can be represented as:

$$|\Delta^{++},\ J = 3/2\rangle = \frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}|u_\alpha \uparrow,\ u_\beta \uparrow,\ u_\gamma \uparrow,\ \rangle\ , \tag{3.6}$$

with an antisymmetric wave function ($\alpha,\ \beta,\ \gamma$ are colour indices).

[4] A possible solution, where quarks obey parastatistics of rank three, has been proposed by Greenberg [70], which can be satisfied by the attribution, by Gell-Mann *et al.* [69], of the new internal colour quantum number to the quarks.

- It is also known that quantum anomaly spoils the renormalizability of the $SU(2) \times U(1)$ Standard Model of Electroweak interactions. Its disappearance can only be achieved if the quark number of colour is 3.

3.3 The confinement hypothesis

- However, the theory is amusing as one has to avoid the existence of coloured states, i.e., they should have infinite energy, such that all asymptotic states should be colourless. This leads to the *confinement hypothesis* implying the non-observability of free quarks. There is indeed an indication of such a property from a lattice measurement of heavy quark-antiquark bound state potential, where it is found to be Coulomic at short distances and increases linearly at long distances (see also Section 3.8):

$$V_{\bar{Q}Q} \sim C_{\mathrm{F}} \frac{\alpha_s(r)}{r} + \sigma r \tag{3.7}$$

  with $C_{\mathrm{F}} = 4/3$ and σ is the QCD string tension. The linear rising term renders the separation of the $\bar{Q}Q$ pair energetically impossible.
- The confinement assumption also implies that QCD should be a *local field theory* that leads to local observables described by local operators or currents built with gluons and/or quark fields. This locality property is one of the basis of the current algebras that we have outlined in the previous chapter.
- Confinement is also essential for explaining the short-range nature of the nuclear forces, while massless gluons exchange is a long-range process. This is because nucleons are colour singlet states which cannot exchange colour octet gluons but only coloureds states.
- Some qualitative ideas on the nature of confinement lead to the picture that quarks are bound by strings or chromelectric flux tubes. Indeed, if a $\bar{Q}Q$ pair is created at one space-time point in a given process, and the quark and antiquark start to move away from each other in the centre of mass of the system, then it soon becomes energetically possible to create additional pairs smoothly distributed in rapidity between the two leading charges, which neutralize colour and allow the final state to be reorganized into two jets of coloured hadrons, which communicate in the central region by a number of *wee hadrons*. This phenomena is very similar to the case of broken magnet, where an attempt to isolate a magnetic monopole by stretching a dipole, leads to the breaking of the magnet into two new monopoles at the breaking point. with small energy. Alas, nobody has succeeded yet in proving this scenario, which remains a great challenge due to the peculiar IR properties (*infrared slavery*) of the theory. At present, the *confinement hypothesis* can still be considered a *postulate*.

3.4 Indirect evidence of quarks

Prior QCD, constituent quark models have been used for predicting some processes. The calculations assume that one can simply produce free quarks, which, a priori, is in contradiction with the *confinement postulate*. (Indirect) evidence[5] of quarks have been observed at LEP from two hadronic jet events in the decay of the weak boson Z^0 through the intermediate process $Z^0 \to \bar{q}q$, where the quarks hadronize later on. However, one should remember

[5] Some direct searches based on the expectation to observe spin $1/2$ quark were not successful.

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that, in these hard processes, experimentalists only detect hadrons (pions, kaons, ...), but neither quarks nor gluons. It is impressive that these hard processes can be nicely explained by perturbative QCD [52].

3.5 Evidence for colours

- In an analogous way, the existence of gluons has been seen in three hadronic jet decays of the Z^0 through the process $Z^0 \to \bar{q}qg$.
- The number of colours has also been tested from different experiments. Classical examples are:

♣ The $e^+e^- \to$ hadrons total cross-section $R_{e^+e^-}$ normalized to the $e^+e^- \to \mu^+\mu^-$ cross-section is expected to be equal to the number of colours N_c times the sum of the square of the quark charge, if one assumes the production of free $\bar{q}q$ pairs (*parton model*) before hadronization (see Fig. 3.2):

$$R_{e^+e^-} \equiv \frac{\sigma(e^+e^- \to \gamma,\ Z^0 \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx N_c \sum_{u,d,s...} Q_i^2 \ . \tag{3.8}$$

This fact has been observed in e^+e^- experiments for sufficiently large energy beyond the resonances structure as shown in Fig. 3.3.

♢ Similarly, the decay rate of the weak Z^0 boson shown in Fig. 3.4 is also controlled by N_c. Its hadronic branching ratio reads:

$$R_Z \equiv \frac{\Gamma(Z^0 \to \text{hadrons})}{\Gamma(Z^0 \to e^+e^-)} \approx \frac{N_c}{\left(v_e^2 + a_e^2\right)} \sum_{u,d,s...} \left(v_i^2 + a_i^2\right) , \tag{3.9}$$

where v_i and a_i are the electroweak vector and axial-vector couplings of the $\bar{q}q$ or e^+e^- pairs to the Z^0.

Experimentally, one has [16]:

$$R_Z = 20.77 \pm 0.08 \ . \tag{3.10}$$

♡ The inclusive heavy lepton τ semi-hadronic rate R_τ normalized to its semi-leptonic one, shown in Fig. 3.5, is expected to be equal to the colour number 3 from the parton model:

$$R_\tau \equiv \frac{\Gamma(\tau \to \nu_\tau + \text{hadrons})}{\Gamma(\tau \to \nu_\tau + l + \bar{\nu}_l)} \approx N_c \ . \tag{3.11}$$

Experimentally, one has:

$$R_\tau = \frac{1 - \sum_{e,\mu} B_r(\tau \to \nu_\tau + l\bar{\nu}_l)}{B_r(\tau \to \nu_\tau + l\bar{\nu}_l)} = 3.647 \pm 0.05 \tag{3.12}$$

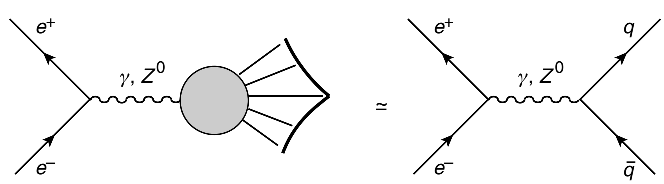

Fig. 3.2. $e + e- \to$ hadrons process.

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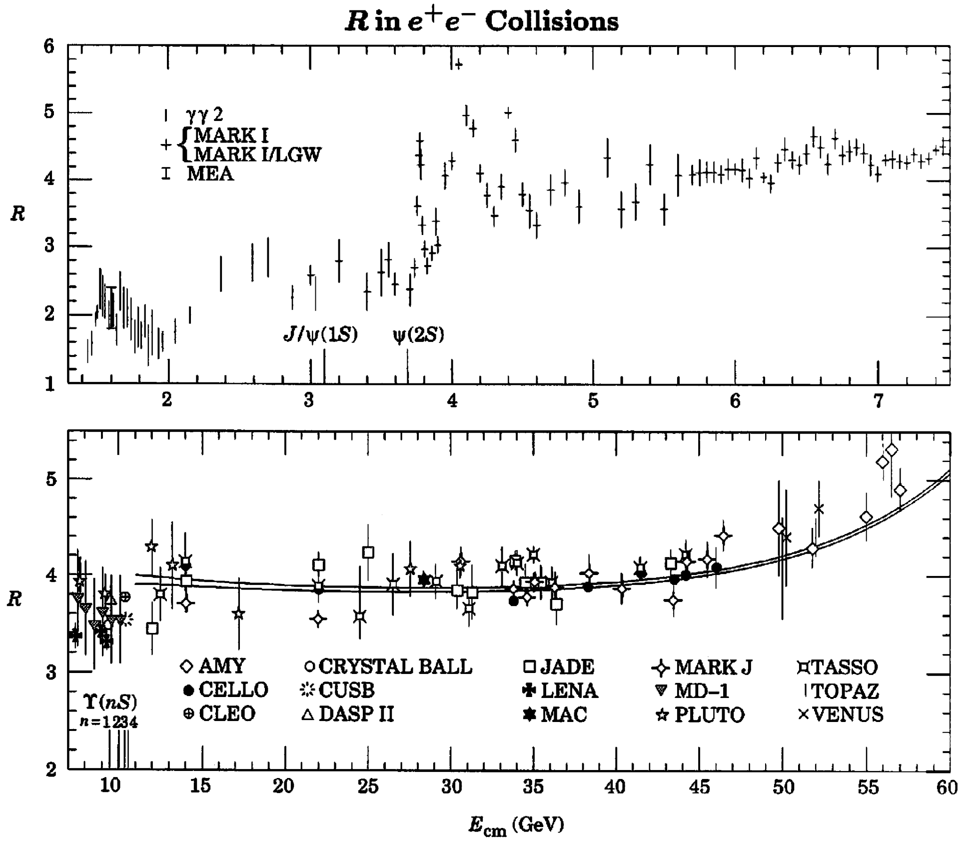

Fig. 3.3. $e + e- \to$ hadrons data. The continuous lines are QCD fit.

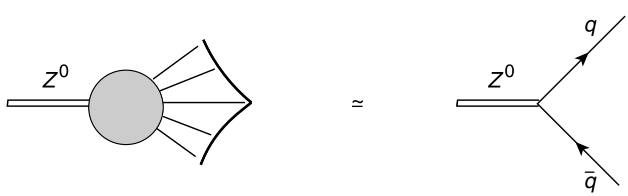

Fig. 3.4. $Z^0 \to$ hadrons decay.

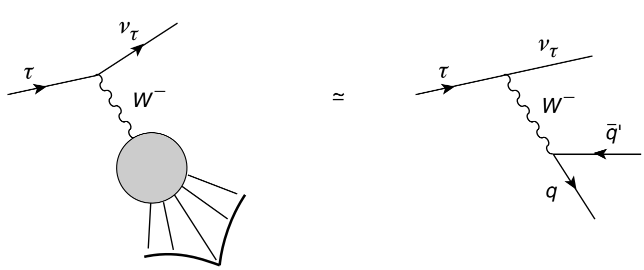

Fig. 3.5. $\tau \to \nu_\tau +$ hadrons decay.

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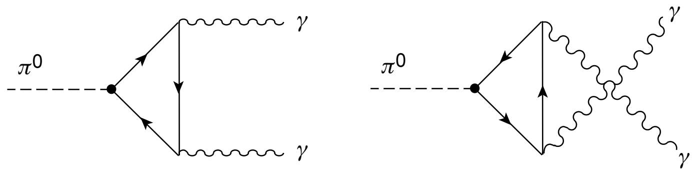

Fig. 3.6. $\pi^0 \to \gamma\gamma$ decay from the quark triangle.

where B_r is the leptonic branching ratio. We shall see later on that the QCD radiative corrections explain the 20% discrepancy between the parton model prediction and the data.

♠ The decay rate of the neutral pion into two photons which occurs through the quark triangle loop (Abelian anomaly) shown in Fig. 3.6 is controlled by the square of the colour [71]:

$$\Gamma(\pi^0 \to \gamma\gamma) = \left[N_c\left(Q_u^2 - Q_d^2\right)\right]^2 \left(\frac{\alpha^2}{64\pi^3}\right)\frac{m_\pi^3}{f_\pi^2} = 7.7 \text{ eV}\,, \tag{3.13}$$

where $f_\pi = 92.4$ MeV is the pion decay constant controlling the decay $\pi^- \to \mu\nu$. It was shown a long time before QCD that this prediction is not affected by quantum corrections [72]. This prediction is in remarkable agreement with the data of (7.7 ± 0.6) eV [16].

3.6 The $SU(3)_c$ colour group

The previous properties:

- Quarks with three colours
- Quarks and anti-quarks are different objects
- Exact colour symmetry (hadrons have no colour multiplicity)

are sufficient to select the $SU(3)_c$ symmetric colour group for desribing the theory of strong interactions, instead of the other Lie group candidates $SO(3)$ and its isomorphic $SU(2) \simeq Sp(1)$, which have real representations, and then cannot distinguish the particle from its anti-particle. In this $SU(3)_c$ unitary group, quarks (anti-quarks) then belong to the fundamental presentation $\underline{3}$ (resp $\underline{3}^*$), whereas gluons are in the adjoint $\underline{8}$. The previous *Gell-Mann eightfoldway* [7] quark model classification, can now be viewed in a modern way, where *hadrons should be colour-singlet* states. The $SU(3)_c$ decomposition into products of $\underline{3}$ and $\underline{3}^*$ representations gives for mesons:

$$\bar{q}q \;:\; \underline{3}^* \otimes \underline{3} = \underline{1} \oplus \underline{8} \tag{3.14}$$

and for baryons:

$$qqq \;:\; \underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10}\,, \tag{3.15}$$

which guarantee the colour-singlet configurations of hadrons required by the *confinement postulate*. and which are satisfied by the experimentally observed hadrons. On the contrary,

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some exotic combinations like diquarks:

$$qq\ :\ \underline{3}\otimes\underline{3}=\underline{3}^*\oplus\underline{6}\,, \tag{3.16}$$

and four-quark states:

$$qqqq\ :\ \underline{3}\otimes\underline{3}\otimes\underline{3}\otimes\underline{3}=3\oplus\underline{3}\oplus\underline{3}\oplus 6^*\oplus\underline{6^*}\oplus\underline{15}\oplus 15\oplus\underline{15}\oplus\underline{15'} \tag{3.17}$$

do not satisfy the colour-singlet confinement constraints, and can induce coloured states in the spectrum [73].

3.7 Asymptotic freedom

Gell-Mann postulated that, at short distances, the commutation relations of the *local* hadronic currents imply that the quark fields entering them are free particles (*asymptotic freedom*). These assumptions led to the success of the different current algebra superconvergent sum rules and to the Bjorken scaling. However, such assumptions a priori contradict the previous confinement postulate. As we shall see, QCD can satisfy simultaneously the two conditions thanks to the property of the QCD gauge coupling g, which is the only parameter that controls the QCD Lagrangian in the massless quarks limit (as we shall see in the next chapter). 't Hooft observed [74] that the slope of the first coefficient (N_c and n_f are respectively the colour and flavour numbers):

$$\beta_1=-\frac{1}{2}\left(11\frac{N_c}{3}-\frac{2}{3}n_f\right) \tag{3.18}$$

of the β-function [75,76] is negative at the origin of the coupling constant for a $SU(3)_c$ *Yang–Mills gauge theory*, while, independently, Gross, Wilczek and Politzer [77] discovered that for non-Abelian gauge theories, the origin of the coupling constant is an UV stable fixed point in the deep Euclidian region. This *asymptotic freedom*[6] property thus states, after solving the renormalization group equation (RGE) (resummation of all leading logs corrections), as we shall see in Section 11.7, that at large momenta Q, the *running* QCD coupling falls off as:

$$\alpha_s\equiv\frac{g^2}{4\pi}\simeq\frac{\pi}{-\beta_1\ln Q/\Lambda}\,,$$

where Λ is the characteristic QCD scale, which indicates that below its value, the perturbative approximation breaks down. The situation in QCD is the opposite of the familiar QED described by the $U(1)$ Abelian theory, in which the *effective charge* α increases slowly for increasing Q^2 because the corresponding β function is positive ($\beta_1=2/3$).[7] At the electron mass, α has the value $1/137$, while it is $1/129$ at an Z^0 mass of a distance of $1/500$ fm (It becomes infinite (so-called *Landau pole* [79]) at an energy much higher than

6 For historical reviews on the discovery of asymptotic freedom, see the talks given by David Gross and Gerard 't Hooft at the QCD 98 Montpellier Euroconference [78].
7 More discussions on QED will be given later.

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the mass of the universe). An intuitive understanding of this decrease of the QED effective coupling at long distance is provided by the dielectric screening due to the cloud of virtual e^+e^- pairs created in the vacuum, through quantum effects, surrounding the electric charge. For QCD, and more generally, for non-Abelian theories, one then expects an anti-screening effect generated by the gauge self-interactions of gluons, which spread out the QCD colour charge, and makes the Yang–Mills vacuum like a paramagnetic substance implying an anti-screening charge through relativistic invariance. This anti-screening or the asymptotic freedom property are only true for non-Abelian theories [80]. This remarkable asymptotic freedom property of QCD then permits a simple treatment of the different *QCD hard processes*, which can be approximated by perturbative series in the strong coupling α_s at large momenta. This feature also confirms the success of the parton model in describing (to lowest order of the α_s-series expansion in the perturbative QCD language), the examples of QCD processes $R_{e^+e^-}$, R_Z, R_τ and DIS mentioned previously, but also implies that for $Q^2 \to \infty$, quarks become *free particles*.

3.8 Quantum mechanics and non-relativistic aspects of QCD

We have learned from previous sections that quarks are free at very short distances but tightly bounded at long distances. For an heavy $\bar{Q}Q$ bound state, the QCD potential is Coulomic at short distances and increases linearly at long distances. This behaviour is typical for quantum mechanical systems bound together by a potential which is not singular at short distance and increases infinitely with distance at large distances. This is, for instance, the case of the harmonic oscillator where its potential reads:

$$V(r) = \frac{1}{2}m\omega^2 r^2 \tag{3.19}$$

The corresponding Green's function of the system is:

$$G(\vec{x}, \vec{x}', t) = \left(\frac{m\omega}{2\pi\hbar \sin \omega t}\right)^{3/2} \exp\left\{\frac{im\omega}{2\hbar \sin \omega t}((\vec{x}^2 + \vec{x}'^2)\cos \omega t - 2\vec{x}\vec{x}')\right\}, \tag{3.20}$$

which, for small t ($\omega t \ll 1$), is well approximated by the function for the free particle:

$$G_0(\vec{x}, \vec{x}', t) = \left(\frac{m}{2\pi\hbar}\right)^{3/2} \exp\left\{\frac{im\omega}{2\hbar t}\left(\vec{x} - \vec{x}'\right)^2\right\}. \tag{3.21}$$

Therefore, it is not so surprising that non-relativistic potential models of quarks [12, 81–94] were able to describe some characteristic features of the systems, and successfully explain the complex hadron spectra made with heavy quarks. However, a purely quantum mechanical description of the theory is not fully satisfactory, as it does not incorporate Lorentz invariance. We shall come back to this subject in a future section.