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Numerical simulations of semiconvection

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Abstract. Using a semiconvective model based on thermohaline convection, we investigate the case of an expanding core of a main-sequence massive star. The numerical simulations at high Prandtl number show a flow consistent with the assumption that a dynamically neutral layer sits between the core and the radiative envelope. More simulations at low Prandtl number are needed to infer scaling laws applicable to astrophysical regimes.

Keywords. Convection, diffusion, hydrodynamics, stellar dynamics, instabilities

1. Introduction

Some fifty years ago, Schwarzschild & Härm (1958) discovered an inconsistency in the classical treatment of convection. They found that in main-sequence massive stars, for example, as the fully mixed convective core expands, the compositional jump across its boundary causes the helium-rich core to be less opaque than the surrounding envelope. This can render it impossible to determine the position of the boundary on the basis of a simple reversal of a local dynamical stability criterion. Under somewhat different circumstances, a similar difficulty arises when the core retreats. In order to solve the inconsistency, they postulated a region of partial mixing outside the core. This region, called semiconvective zone, would have a compositional gradient of precisely the magnitude that would render the fluid marginally stable to convection. Subsequently, there has been disagreement as to what the condition for marginal stability should be. Typically astronomers choose between two extreme assumptions for the outcome of the semiconvective instability in order to build evolution models for giant stars. The two outcomes are as follows: the interface slowly mixes the helium to establish a gradient such that: (i) the layer is adiabatically stratified or (ii) the stratification is dynamically neutrally stable. Under some circumstances, these two assumptions lead to quite different evolutionary consequences for the star. Therefore this study is of great astrophysical importance.

2. Semiconvective model

Since opacity laws and double diffusion play the only key roles in semiconvection, we developed a simplified model based on thermohaline convection (salt and water) where opacity laws are embedded in a relation between the thermal conductivity K_T and the salt content S:

$$K_T = K_{T_0} f(z)(1 + \mu S) \tag{2.1}$$

2D plane-parallel geometry with constant gravity is assumed, but a geometry factor $f(z)$ is used to provoke a radiative zone on top of a convective zone. The salinity S plays the role of the helium content. Since opacity decreases, equivalently conductivity increases, when the helium content increases, the coefficient μ has to be positive.

Navier-Stokes equations for thermohaline convection are employed in the Boussinesq approximation for gases:

$$\frac{1}{\sigma}\left(\frac{\partial \underline{u}}{\partial t}+(\underline{u}.\nabla)\underline{u}\right)=-\nabla p+\left(R_T T-R_S S\right)\underline{z}+\nabla^2\underline{u} \tag{2.2}$$

$$\nabla.\underline{u}=0 \tag{2.3}$$

$$\frac{\partial T}{\partial t}+(\underline{u}.\nabla)T+W=\nabla.\left(f(z)(1+\mu S)\nabla T\right) \tag{2.4}$$

$$\frac{\partial S}{\partial t}+(\underline{u}.\nabla)S=\tau\nabla.\left(f(z)\nabla S\right) \tag{2.5}$$

Where $\underline{u}$ is the velocity, p the pressure, W the vertical velocity, z the vertical coordinate, t the time, T the temperature and S the salinity.

Length is scaled out with the height of the layer H, time with the thermal diffusion time across the layer H^2/κ_T , temperature with the adiabatic temperature jump across the layer $\Delta T_{ad}=gH/c_p$, and salinity with the salinity jump that would create the same effect on the fluid's buoyancy $\Delta S_{eq}=\Delta T_{ad}\alpha/\alpha_S$. The thermal expansion coefficient α and the saline expansion coefficient α_S are defined by:

$$\alpha=-\frac{1}{\rho}\left(\frac{\rho}{\partial T}\right)_{S,P} \qquad \alpha_S=\frac{1}{\rho}\left(\frac{\rho}{\partial S}\right)_{T,P}$$

Four dimensionless numbers appear in the equations: the Prandtl number, the Lewis number, the thermal and the saline Rayleigh numbers. They are respectively:

$$\sigma=\frac{\nu}{\kappa_T} \qquad \tau=\frac{\kappa_S}{\kappa_T} \qquad R_T=\frac{g\alpha\Delta T H^3}{\kappa_T\nu} \qquad R_S=\frac{g\alpha_S\Delta S H^3}{\kappa_T\nu}$$

These equations were solved numerically with a code employing 6th-order compact finite differences in the vertical on a stretched mesh, Fourier expansion in the horizontal and 3rd-order Runge-Kutta time step. The non-linear terms are treated explicitly, while the linear terms are treated implicitly. With the geometry factor f(z) set to one and the heat conductivity K_T to a constant, the code reproduced both kinds of double-diffusive instabilities: salt fingers and overstable motions leading to diffusive interfaces.

3. Expanding core

A small random velocity field is added to a layer of static fluid with neutral stratification, adiabatic temperature gradient and absence of salt. Salt and heat fluxes are imposed at the bottom boundary in order to simulate nuclear burning in stars. The expansion of the core is achieved by increasing the heat flux imposed at the bottom with time. Periodic boundary conditions are used in the horizontal, and free-free boundary conditions are used in the vertical.

The salinity fields of two different runs are depicted on the next page. In the first simulation, figure (1), the parameters are $\sigma=7$, $\tau=0.01$, $R_T=10^7$, and $R_S=2\times10^6$. In the second simulation, figure (2), the parameters are $\sigma=0.03$, $\tau=0.01$, $R_T=10^8$, and $R_S=10^8$. In the first simulation, the Prandtl number and the Lewis number have the same values as in the heat-salt system. In the second simulation, their ratio has the actual computed value for massive stars, and the Prandtl number is closer to its stellar value. In the pictures, light regions are salty while dark regions are fresh.

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Figure 1. salt content at an early stage and a late stage of the first simulation.

Figure 2. salt content at an early stage and a late stage of the second simulation.

4. Conclusions

In the simulation at high Prandtl number, a thermal instability on the lower boundary generates hot saline rising plumes, thus producing an advancing convective mixed layer, representing the stellar core. In the stable layer above the core, a vacillating flow induces a descending cold fresh plume on top of the downwelling flow of an eddy, while entraining a small amount of salt from the other into a layer above the boundary of the true convective zone. That layer is in hydrostatic equilibrium, not unlike the extreme recipe (ii) mentioned above. However, the computations extend to too short a time for the layer to have achieved a diffusive equilibrium. Therefore we cannot say whether the final state is like (i), (ii), or something else. The low Prandtl number run shows similar features to those seen in the high Prandtl number run. However, the driving of the counter flow cannot be achieved by viscosity at low Prandtl numbers. Therefore, thermal driving has to be instrumental in generating the counter flow.

More simulations are needed to infer scaling laws for the fluxes when the Prandtl number and the Lewis number have their astrophysical values. Also the influence of large aspect ratios needs to be probed. The results of this model can be extended to other scenarios of semiconvection, including the case of the expanding cores of horizontal-branch stars.

References

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