

ON THE METHOD OF TESTING THE SOLVENCY OF AN  
ASSURANCE COMPANY.

*To the Editor of the Assurance Magazine.*

SIR,—I have attentively read the letter of your correspondent H. A. S., a copy of which you were kind enough to send me, and I gladly avail myself of the opportunity thus afforded of replying to his strictures on my recent paper.

He begins by supposing that the paper referred to could only have been inserted "with a view to provoke discussion." I am quite at a loss to conceive upon what such a supposition is based. Had the paper contained anything essentially new, or anything at variance with universally acknowledged truths, there might have been room for dissent; but to my mind it contains little more than a collection of facts. The formula\* for  $f_n$  at p. 186, may seem at first sight to be original; but the very same idea has frequently been stated in a different form, and no one ever thought of disputing its accuracy. Neglecting for a moment the effect of lapsed and surrendered policies on the reserve of a Life Office, and supposing the rate of interest assumed to be that actually obtained, has it not been frequently urged that the marginal additions to the net premiums are alone available for working expenses and bonuses? Have not various conclusions been drawn from this assumption? Was not an elaborate paper, by a masterly hand, read before the Institute of Actuaries and published in this *Journal* not long since, showing the bonus-giving power of an Assurance Office, based upon this supposition? and wherein, I would ask your correspondent, does my formula differ, which simply states that in the long run all the net premiums will be required for policy claims; and that if, at the end of the  $n$ th year, the net premiums duly accumulated at interest exceed the claims which have been paid by  $f_n$ , that sum should still be in the Company's possession?

In speaking of the formula, he asks—"if the mortality should have been excessive, how can the amount thus found be said to be that necessary to provide for the sums originally assured under existing policies?" Your correspondent does not seem to understand the formula, or he could scarcely put such a question. Perhaps the following explanation will suffice. Every premium received during the first year is separated into two sums, one being the net premium and the other the marginal addition: out of the latter all expenses of management are paid; but the former is left untouched till the end of the year, and then all the policy claims which have accrued during the year are paid out of it; what remains of this fund is immediately invested at the rate of interest assumed in the calculation of the premiums. The same thing is done every subsequent year; and thus the net premium fund accumulates until the termination of the  $n$ th year,

\* By mistake,  $\frac{k-1}{k}$  was written for  $\frac{k}{k+1}$  throughout pp. 186 and 187; the formula spoken of should have been

$$f_n = \left( \frac{ka_1}{k+1} - \delta_1 \right) \mu^{n-1} + \left( \frac{ka_2}{k+1} - \delta_2 \right) \mu^{n-2} \dots + \frac{ka_n}{k+1} - \delta_n.$$

I noticed the oversight soon after the paper was published, and, as you are aware, requested an erratum to be inserted in the following Number, many weeks before I received the copy of your correspondent's letter.

when it reaches the amount expressed by the formula. Now if the net premiums be sufficient for the risks, does it not necessarily follow that  $f_n$  correctly represents the reserve which the Company should have at the time spoken of, to meet its policy engagements—and that, too, whatever the mortality may have been? The formula is framed in close accordance with the assumptions made in calculating premiums; it differs only in one particular, namely, the supposition virtually made that the net premiums are invested at the *end* of the year in which they are received: whereas, in forming tables, each net premium is supposed to yield interest from the day on which it is paid to the Company. The formula is therefore somewhat below the truth, supposing no lapses or surrenders; but when the reserve necessary to be made on account of these is unknown, the formula already given will probably be nearer the truth than the more rigorous expression

$$f_n = \left( \frac{ka_1}{k+1} \mu^{\delta} - \delta_1 \right) \mu^{n-1} + \left( \frac{ka_2}{k+1} \mu^{\delta} - \delta_2 \right) \mu^{n-2} \dots + \frac{ka_n}{k+1} \mu^{\delta} - \delta_n,$$

in which the net premiums are assumed to accumulate at interest from the time at which they are paid, and that time is supposed to be identical with the period when they become due.

With regard to the subject of a transfer of business at the termination of the  $n$ th year, your correspondent could hardly imagine that I intended the sum  $f_n$  to represent the amount which should be handed over to an Office taking the policies: it is not very likely that a Company, able to do this, would desire to part with its business at all. I merely suggested that the formula (as a ready method) might be found of use, in the way pointed out, in guiding an actuary to the proper terms of a transfer.

In order to be explicit upon another point alluded to by H. A. S., I will call the selling Office, S; and the purchasing Office, P. Now he states that the common method of valuing policies is not open to either of the objections I bring against it; “since, if P gets paid over to it the difference between the present value of the sums assured and that of the premiums, what loss can result to P if half the policies are forfeited by non-renewal on the following day?” Here he is assuming that S pays P for taking its business, but a less hasty perusal of the paper would have shown him that I was referring to the case in which P pays S; in fact, nowhere, from first to last, do I discuss any other case—except, indeed, that in which no money passes: therefore the objections I raised remain untouched by his criticism.

With respect to premiums paid upon lapsed policies, I had no intention of departing from the *opinion*, which I believe is almost universal, and in which I fully concur—that only the balance of such premiums is profit, that remains after deducting the value of the risk actually borne. It is rather surprising that, while your correspondent maintains this doctrine, he should advocate a method of valuation which repudiates it altogether. In raising the question as to whether a Life Office could spend the whole of the premiums on lapsed policies without injuring its stability, I was speaking of the practical possibility—theoretically, I know it could not be done; but when we see Companies thriving after an existence of half a century or more, notwithstanding they have always, by their system of valuation, regarded these premiums as entire profit, it is clear that, however impossible the success of an Office acting on this principle may be in theory, it is by no means impossible in practice. This seeming contradiction does

not, however, throw any doubt upon the theory of the subject. The only conclusion I have been able to arrive at, which appears to explain away the difficulty, is this—the rate of interest realized is usually in excess of that assumed in the valuations; and this surplus rate, operating on very large investments, creates an unanticipated fund, which continues to increase during the interval between any two successive divisions of profits—thus supplying the deficiency which existed immediately after the last distribution of surplus, from the omission of the premiums on lapsed policies. A Company with large investments might therefore spend all such premiums, and yet flourish: but this in no way upsets the theoretical view, it merely shows that, while the Society spends certain premiums that are *not* profit, it compensates that error by using money that really *is* profit in the payment of policy claims. If an Office realized precisely the rate of interest assumed in the calculation of the premiums, and the marginal additions to the net premiums were entirely absorbed in expenses of management and bonuses—and if that Office, adopting the present customary mode of valuation at each division of profits, were to reserve merely the excess of the value of the risks over that of the future premiums, giving to the future expenditure and bonus funds whatever it possessed over and above the reserve so calculated—such an Office, it is certain, would be ruined in the long run, and the cause of its downfall would be the habitual neglect to include in the various estimated reserves the proper proportion of premiums on lapsed and surrendered policies.

I believe I have now fully answered all your correspondent's objections; but before closing this letter, I would respectfully remind him that the cause of truth—if that prompted him to write—is not advanced by the use of strong and unseemly language; and I regret that his communication should have been couched in such an uncharitable tone as to savour more of a desire to find fault than of a wish to place the several matters in their true light.

I am, Sir, your obedient Servant,

*Engineers' Life Office,*  
1st March, 1858.

SAMUEL YOUNGER.

ON THE COMMUTATION TABLES RECENTLY PUBLISHED BY  
MR DAVID CHISHOLM.

*To the Editor of the Assurance Magazine.*

SIR,—Permit me to bring under the notice of your readers one of the many facilities which will be now afforded to the actuary by the publication of Mr. Chisholm's Tables of Survivorship Assurances. Besides introducing many new kinds of transactions, they greatly abbreviate the labour of the calculation of those transactions with which we are already acquainted. In "post obits," however, they afford a still greater advantage; they enable the actuary to ascertain *more correctly* the amount which should be charged the heir of entail in repayment of the sums or annuity advanced to him. The obit charged contains, besides the sum advanced and the redemption of the interest during the joint lives, the redemption of the premiums of insurance to assure the obit on the death of the heir, should he predecease the "life in possession." But while the amount of the outlay is *progressive*, the sum usually assured is maintained at its maximum from the beginning; and thus considerable injustice is done to the heir, by making