SCIENCE EXTENDS ITS HORIZONS

Mathematics is on the March

Academician I. Petrovskiǐ[1]

Recently, doubts have arisen among some scientists as to whether mathematical congresses are necessary. It seems to me that such congresses are necessary because they provide an opportunity for broad, personal contacts among mathematicians of various countries. Mathematics has grown so enormously that no one can have an equally good command of all its branches. Consequently, meetings of mathematicians engaged in the different branches of their science are very useful, for the permeation of the methods of some branches of mathematics into others is extremely productive.

Since the conclusion of such a congress of mathematicians, just held in Moscow, sufficient time has passed to permit a calm assessment of its results. And, in reflecting on this, observe that it demonstrated particularly clearly the necessity of regular meetings of mathematicians working in the different regions of their science. As an example, one can mention at least the lecture of Professor Atiyah, who, applying methods of geometry (topology), obtained valuable results in the theory of differential equations and, conversely, using such equations, obtained important conclusions for geometry.

The second general question, which applies not only to mathematics, but also to almost all other sciences as well, is the question of the relationship of so-called abstract science and applied science. In this connection, there are very many different opinions. I think it not necessary that all discoveries of science should have immediate application; it is only important that they should be the results of an objective study of existing reality. And every such result, even a very abstract one, is essential.

In the course of its development, every science poses problems which are important for its further progress. And now, following the

[1] This is a complete but unofficial translation of an article in Pravda (10 September, 1966, p.2) by Academician I.G. Petrovskiǐ, Rector of Moscow University, who was President of the International Congress of Mathematicians (16-26 August, 1966). It is an editorial combination of translations made independently by Dr. M.D. Greendlinger (Ivanova, USSR) and Dr. Fred Ustina (University of Alberta, Edmonton), each at the request of Professor Lee Lorch (Edmonton) who learned of Academician Petrovskiǐ's article after his own report on the Congress had been submitted for publication.

congress, it is possible to state with even greater conviction that in recent times a series of outstanding discoveries have been made in the field of mathematics. In one of the leading places I would put the work of the American mathematician Cohen, who proved the independence of the continuum hypothesis, i.e., the consistency of the conjecture that there exist sets whose cardinality is intermediate between the cardinality of the set of natural numbers and the cardinality of the totality of all real numbers.

The Swedish mathematician Carleson proved that a trigonometric series, constructed for an arbitrary function whose square is integrable, converges to it almost everywhere. The proof of this definitive result has long been sought unsuccessfully by many mathematicians all over the world.

I should like to mention some remarkable work of young Soviet mathematicians. V.I. Arnol'd has conducted qualitative investigations of differential equations which occur in mechanics. One example consists of the equations appearing in the description of the motion of the planets in the solar system. For many years, the following question remained unanswered: do there exist stable motions in such systems? Although the astronomers were always convinced of this, there was no mathematical proof. Developing the methods of Andrei Nikolayevich Kolmogorov, Arnol'd was able to prove that there do exist stable motions in such systems, and that they are sufficiently numerous.

The young Soviet mathematician S.P. Novikov took an important step forward in the development of that branch of geometry which is called differential topology. Differential topology is the science of smooth surfaces.

Mathematical methods are now widely applied in other sciences. Thus, for a long time mathematical logic was considered as one of the abstract sciences. At present, it has become a science with direct applications. It is of great significance in the construction of computing machines and in working with them.

The development of modern, high speed electronic machines has made it possible to carry out an enormous number of mathematical operations very rapidly. This has opened up the possibility of solving problems which, although yielding previously to a theoretical solution, in practice required so much work that it was impossible to effectuate it. To a great extent, we are indebted to these machines for the successful solution of intricate problems of production control, planning of transportation, etc.

To indicate the power of modern computing machines, I shall deal with only two examples.

In his day, the mathematician Shanks spent a lifetime computing a number [π - trans.] to 707 decimal places. Nowadays a calculator computes this number to an accuracy of 2000 decimal places in a rather short time. Moreover, it turns out that of the 707 decimal places computed by Shanks, only the first 200 were correct.

It is only the modern calculating machines which made it possible to compute the trajectories of sputniks of the earth and the moon, and to send sputniks into the desired orbits.

It is often said that there is classical mathematics and modern mathematics. The well-known French professor Henri Cartan, elected at the recently held congress as president of the International Mathematical Union, spoke very well in this connection. The sense of his statement is as follows: there is no old and no new mathematics; there is a developing mathematics.

Far from all of the older discoveries in mathematics die out after the appearance of new ideas, new discoveries. Euclid's geometry, created over two thousand years ago, is used by all engineers to the present day. In the past century, N.I. Lobachevski demonstrated the existence of geometries other than euclidean. After the discoveries of Einstein, it became clear that in the vast cosmic space, non-euclidean geometry has a proper place.

In comparison with other natural sciences, mathematics has one fortunate advantage: for the development of a majority of its branches, laboratories, auxiliary scientific personnel, expeditions are not needed. Only time for research and information about what is being done in the science are required. That is why scientific work in mathematics is conducted with the same success in our institutions of higher learning as in the special mathematical scientific-research institutes. In my view, mathematical scientific-research institutes, where problems of pure mathematical science are investigated, and not only its applications, should be few in number, with small staffs. Only a few mathematicians have the right to be free from teaching, and from being engaged directly with its applications.

In order to create the most favourable working conditions for professors and instructors engaged in serious mathematical research in institutions of higher learning, they must not be overloaded with pedagogical duties and it is necessary to give them the opportunity to receive reprints of all works of interest to them, brief accounts of which appear in the mathematical reference journal which we publish.

At the congress there was a section devoted to the question of teaching mathematics in high school. Ordinarily, the high school programs in mathematics provide only information that was already known in the science some 200 to 300 years ago. Programs in physics, chemistry and biology in high school include the achievements of the most recent decades. But, from another point of view, every student who passes through high school mathematics must master and know how to use this material. In any institution of higher learning, high school mathematics is not repeated, while the study of physics, chemistry, biology always begins all over again.

The task of our high schools consists of imparting to the students the ability and understanding to master the mathematical learning they study, and at the same time to bring them closer to modern science.