THE VOID PROBABILITY FUNCTION AS A STATISTICAL INDICATOR

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Recent results concerning the galaxy distribution at scales < 100 h^{-1} Mpc (H_o = 100 h kms^{-1} Mpc^{-1}) show a number of characteristics which cannot be described by conventional statistical models. Correlation functions, for instance, can in no way give account of the presence of voids of the cellular (or spongy) appearance of the local galaxy distribution (M. Geller, this conference). There is clearly a need for new kinds of statistical models and statistical indicators.

Among those we wish to emphasize the advantages of the void probability function (VPF), and its particular convenience for studying the galaxy distribution.

For a given sample of galaxies, the VPF is defined as the probability that a randomly selected volume, of specified size V and shape contains no galaxy. This quantity has many advantages. Firstly it can be calculated from dynamical or statistical models. It can be predicted, if necessary, from the set of all correlation functions, or from the set of count probabilities, for which it acts as a generator function (see White 1979)[12]. Moreover it can be used as a link between various approaches like BBGKY hierachy and correlation functions, presence of voids, and percolation.

The VPF is not difficult to calculate from any sample and is not very sensitive to the particular features of the galaxy distribution. It also presents the advantage that results in 1, 2 and 3 dimensions are easy to link. For these reasons, we developed a method for calculating the VPF (and associated indicators like low-order number counts) from any sample. Applications of this method to the 2-dimensional center for Astrophysics (CfA) catalog have already been published[1]. We present below some first results for the 3-dimensional CfA catalog.

SCALING

An important remark should be made, that the VPF is basically a function of 2 variables, the density of the sample and the size V of the volume. This means for instance that, for 2 different samples similarly clustered but with different densities, the VPF take different

A. Hewitt et al. (eds.), Observational Cosmology, 359–362.
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https://doi.org/10.1017/S0074180900159327 Published online by Cambridge University Press

values for each volume. This contrasts with the 2 point correlation functions ξ which does not vary with the density of the sample.

For this reason, 2 approaches exist for the VPF. Either to consider it as a function of n at a fixed V, or as a function of V at fixed n.

The first approach, essentially used by Saslaw et al.[6], involves different samples (or subsamples of a same catalog), assumed to be similarly clustered. The VPF is then evaluated for each of them. A function of n (or of x = nV, but where V remains fixed) is obtained, which can be compared to the predictions of some models.

The second method is more natural and significant since it does not imply the analysis of a serie of different samples. Its results are well suited for comparison with models which, in general, concentrate on the V-dependence of the structure, for a given density. There is however one difficulty: being a function of the 2 variables n and V (and not only of V like $\xi(V)$), the VPF does not allow, a priori, to compare various samples with different densities ; or to analyse a catalog where the density varies from place to place, like any non volume-limited catalog, the CfA in particular.

There is however one way of surmounting such difficulties, if what we will call a scaling hypothesis is verified[10]. This latter can be expressed by the fact that the VPF (or more exactly its logarithm, normalized to the poissonian value - nV, i.e. $\chi \equiv \frac{-\log(\mathrm{VPF})}{nV}$) can be described as a function of some scaling variable q only, constructed from V and n. In other words, the V-dependence and the n-dependence of the VPF can be deduced each one from each other. This scaling hypothesis plays a very important role for the following reasons. Firstly it allows us to reconcile the two approaches, since the V and n dependence describe the same information. Secondly, it will allow us to compare the clustering properties of different samples having different densities and to check, for instance, if they are or not similarly clustered. Similarly it gives the possibility to explore different parts of a non homogeneous catalog (like for instance the CfA), and to combine them for a study of the whole catalog. The only condition is to plot χ as a function, not of n or V, but of the scaling variable q. Moreover the scaling hypothesis is predicted by a whole class of statistical models, called hierarchical models. These have been recently reviewed by Fry[5] as well as their predictions for the VPF. This makes very usefull to test if the scaling hypothesis is verified for the real galaxy distribution[10,11].

Before knowing the validity of the scaling hypothesis, and in order to check it, we must define complete volume-limited samples. We extracted three samples from the CfA catalog that we call:

faint	$-17 < M < -19$,	$D < 20$ Mpc,	366 galaxies
medium	$-17.5 < M$	, $D < 27$ Mpc,	488 galaxies
and bright	$-18.5 < M$	, $D < 40$ Mpc,	396 galaxies.

For each of them we derived the VPF and tested the scaling hypothesis. For this task, we constructed catalogs with the same clustering properties but different densities, just throwing away randomly a given proportion of the galaxies present.

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The result is that for each sample the scaling property is very well verified with the scaling variable $q = nV\bar{\xi}(v)$, as proposed in the Schaeffer models, $\bar{\xi}(v)$ being defined as $\frac{1}{V^2}\int_V d^3V_1 d^3V_2 \xi(r_{12})$,[10] where the evaluation of ξ by Davis and Peebles[8] have been used. Independently of any model, this implies that the clustering properties of the matter in universe, at a given scale, are related to its properties at an other scale. A result that any proposed model has to take into account.

This scaling property allows to compare the different samples previously defined. We have been able to show that, for any value of the scaling variable q, χ decreases with the faintness of the samples.

In other words, bright galaxies appear much more clustered than faint ones. Note that we used the same value of ξ to calculate the scaling variable q for the 3 samples. This may be not correct if faint and bright galaxies have not the same correlation functions. However in such a case the conclusion that they are differently clustered would be unchanged.

Finally we have compared our measures with the available theoretical predictions. The result is a good agreement with the $\nu = 1$ model proposed by Schaeffer[10]. Additionally a similarity in shape appears with our calculations for biased galaxy formation (Maurogordato + Lachièze-Rey[7]), although the level is not the same. It should be remarked however that the comparison between theory and observations presently involves a normalization by the correlation functions which forbids any definitive conclusion.

In conclusion, the galaxy distribution appears to be scale invariant, in the sense previously defined. A result favouring the class of hierarchical models (including biased galaxy formation) and allowing more specific use of the VPF.

It appears also that bright galaxies are more clustered than faint galaxies in the CfA catalog.

Comparison with theoretical models is only tentative but predictions from the Schaeffer model ($\nu = 1$) and the biased galaxy formation both appear attractive.

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