37

Gluonia correlators

37.1 Pseudoscalar gluonia

We shall be concerned with the correlator:

$$\chi(k^2) = \int dx\ e^{ik.x} i\langle 0|\mathcal{T} Q_R(x)\ Q_R(0)|0\rangle\ , \tag{37.1}$$

where $Q_R(x)$ is the renormalized gluon topological density which mixes under renormalization with the divergence of the flavour singlet axial current $J^0_{\mu 5R}$ mix as follows [129] (see Section 10.3.3 in Part III):

$$\begin{aligned} J^0_{\mu 5R} &= Z J^0_{\mu 5B} \\ Q_R &= Q_B - \frac{1}{2n_f}(1-Z)\partial^\mu J^0_{\mu 5B}\ , \end{aligned} \tag{37.2}$$

where:

$$\begin{aligned} J^0_{\mu 5B} &= \sum \bar{q}\gamma_\mu\gamma_5 q \\ Q_B &= \frac{\alpha_s}{8\pi}\mathbf{Tr}\ G^{\mu\nu}\tilde{G}_{\mu\nu} \end{aligned} \tag{37.3}$$

and we have quoted the formulae for n_f flavours. The correlation function $\chi(k^2)$ obeys the inhomogeneous RGE [260]:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(\alpha_s)\alpha_s\frac{\partial}{\partial\alpha_s} - 2\gamma\right)\chi(k^2) = -\frac{1}{(2n_f)^2}2\beta^{(L)}k^4\ . \tag{37.4}$$

The anomalous dimension is:

$$\gamma \equiv \mu\frac{d}{d\mu}\log Z = -\left(\frac{\alpha_s}{\pi}\right)^2\ . \tag{37.5}$$

The extra RG function $\beta^{(L)}$ (so called because it appears in the longitudinal part of the Green function of two axial currents) is given by

$$\frac{1}{(2n_f)^2}\beta^{(L)} = -\frac{1}{32\pi^2}\left(\frac{\alpha_s}{\pi}\right)^2\left[1+\frac{29}{4}\left(\frac{\alpha_s}{\pi}\right)\right]. \tag{37.6}$$

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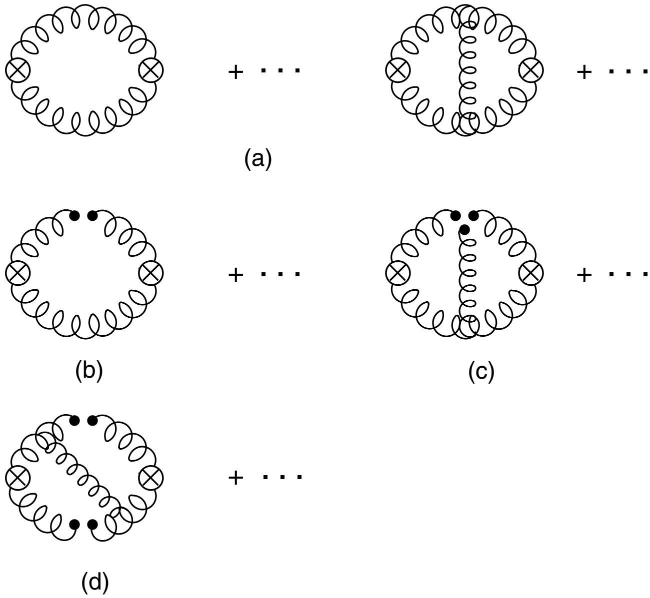

Fig. 37.1. Feynman diagrams corresponding to the OPE of the gluonium correlator: (a) perturbative; (b) two-gluon condensate; (c) three-gluon condensate; (d) four-gluon condensate.

The RGE is solved in the standard way, giving

$$\chi(k^2, \alpha_s; \mu) = e^{-2\int_0^t dt' \gamma(\alpha_s(t'))} \Bigg[\chi(k^2, \alpha_s(t); \mu e^t) \\ -2 \int_0^t dt'' \beta^{(L)}(\alpha_s(t'')) e^{2\int_0^{t''} dt' \gamma(\alpha_s(t'))} \Bigg], \tag{37.7}$$

where $\alpha_s(t)$ is the running coupling. The different QCD diagrams contributing to the correlator are shown in Fig. 37.1.

The perturbative expression for the two-point correlation function in the $\overline{MS}$ scheme is [455]:

$$\chi(k^2)_{\text{P.T.}} \simeq -\left(\frac{\alpha_s}{8\pi}\right)^2 \frac{2}{\pi^2} k^4 \log \frac{-k^2}{\mu^2} \left[1 + \left(\frac{\alpha_s}{\pi}\right) \left(\frac{1}{2}\beta_1 \log \frac{-k^2}{\mu^2} + \frac{29}{4}\right) + \cdots\right] \tag{37.8}$$

The non-perturbative contribution from the gluon condensates (coming from the next lowest dimension operators in the OPE) is [382]:

$$\chi(k^2)_{\text{N.P.}} \simeq -\frac{\alpha_s}{16\pi^2} \left[\left(1 + \frac{1}{2}\beta_1 \left(\frac{\alpha_s}{\pi}\right) \log \frac{-k^2}{\mu^2}\right) \langle \alpha_s G^2 \rangle - 2\frac{\alpha_s}{k^2} \langle g G^3 \rangle\right]. \tag{37.9}$$

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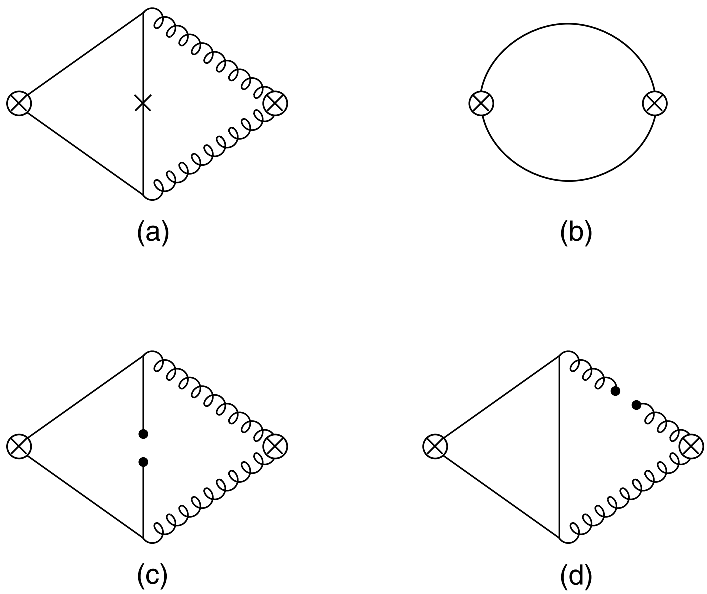

Fig. 37.2. Feynman diagrams corresponding to the OPE of the meson-gluonium correlator: (a) perturbative; (b) diagram which mixes with (a) under renormalization; (c) quark condensate; (d) gluon condensate.

37.2 Pseudoscalar meson-gluonium mixing

Let us consider the off-diagonal two-point correlator:

$$\Psi^-_{gq}(q^2) = \int dx\, e^{iq.x} i\langle 0|\mathcal{T}\, Q_R(x)\, \partial_\mu J_5^\mu(0)|0\rangle \tag{37.10}$$

shown in Fig. 37.2.

Its QCD expression reads [458]:

$$\begin{aligned}(8\pi)^2\Psi^-_{gq}(q^2) = {}& \alpha_s\left(\frac{\alpha_s}{\pi}\right)\frac{3}{\pi^2}m_s^2q^2\log-\frac{q^2}{\nu^2}\left[\log-\frac{q^2}{\nu^2}-\frac{2}{3}\left(\frac{11}{4}-3\gamma_E\right)\right]\\ & -8\alpha_s\left(\frac{\alpha_s}{\pi}\right)m_s\langle\bar{s}s\rangle\log-\frac{q^2}{\nu^2}\\ & +2\left(\frac{\alpha_s}{\pi}\right)\langle\alpha_s G^2\rangle\left(\frac{m_s^2}{q^2}\right)\log-\frac{q^2}{m_s^2}\,,\end{aligned} \tag{37.11}$$

where one can notice that the mixing from the OPE vanishes in the chiral limit. However, one should notice that this mixing acts on the gluonium propagator, that is, it affects the mass splitting but not its decay width which is governed by a three-point function. Unfortunately, several authors mix these two features in the literature. This feature may justify why the lattice prediction in the world without quark can give a prediction that is almost compatible with the experimentally observed gluonium candidate.

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37.3 Scalar gluonia

We shall be concerned with the correlator:

$$\Psi_s(q^2) \equiv 16i \int d^4x\, e^{iqx}\, \langle 0|\mathcal{T}\theta_\mu^\mu(x)\theta_\mu^\mu(0)^\dagger|0\rangle\ , \tag{37.12}$$

where $\theta_{\mu\nu}$ is the improved QCD energy-momentum tensor (neglecting heavy quarks) whose anomalous trace reads, in standard notations:

$$\theta_\mu^\mu(x) = \frac{1}{4}\beta(\alpha_s)G^2 + (1+\gamma_m(\alpha_s))\sum_{u,d,s} m_i\bar{\psi}_i\psi_i\ . \tag{37.13}$$

Its leading-order perturbative and non-perturbative expressions in α_s have been obtained by the authors of [382]. To two-loop accuracy in the $\overline{MS}$ scheme, its perturbative expression has been obtained by [455], while the radiative correction to the gluon condensate has been derived in [456]. Using a simplified version of the OPE:

$$\Psi_s(q^2) = \sum_{D=0,4,\cdots} C_D\langle O_D\rangle\ , \tag{37.14}$$

one obtains for three flavours and by normalizing the result with $(\beta(\alpha_s)/\alpha_s)^2$:

$$\begin{aligned}
C_0 &= -2\left(\frac{\alpha_s}{\pi}\right)^2(-q^2)^2\log -\frac{q^2}{\nu^2}\left\{1+\frac{59}{4}\left(\frac{\alpha_s}{\pi}\right)+\frac{\beta_1}{2}\left(\frac{\alpha_s}{\pi}\right)\log -\frac{q^2}{\nu^2}\right\}\\
C_4\langle O_4\rangle &= 4\alpha_s\left\{1+\frac{49}{12}\left(\frac{\alpha_s}{\pi}\right)+\frac{\beta_1}{2}\left(\frac{\alpha_s}{\pi}\right)\log -\frac{q^2}{\nu^2}\right\}\langle\alpha_s G^2\rangle\\
C_6\langle O_6\rangle &= 2\alpha_s\left\{1-\frac{29}{4}\alpha_s\log -\frac{q^2}{\nu^2}\right\}g^3 f_{abc}\langle G^aG^bG^c\rangle\\
C_8\langle O_8\rangle &= 14\left\langle\left(\alpha_s f_{abc}G^a_{\mu\rho}G^{b\rho}_\nu\right)^2\right\rangle - \left\langle\left(\alpha_s f_{abc}G^a_{\mu\nu}G^b_{\rho\lambda}\right)^2\right\rangle .
\end{aligned} \tag{37.15}$$

37.4 Scalar meson-gluonium mixing

Let's consider the off-diagonal two-point correlator:

$$\Psi^+_{gq}(q^2) = \int dx\, e^{iq.x} i\langle 0|\mathcal{T} J_{2g}(x)\, J_q^\dagger(0)|0\rangle\ , \tag{37.16}$$

where:

$$J_{2g} = \alpha_s G^2\ , \qquad J_q = 2m\bar{q}q\ . \tag{37.17}$$

Its perturbative QCD expression reads [458]:

$$\Psi^+_{gq}(q^2) = \alpha_s\left(\frac{\alpha_s}{\pi}\right)\frac{3}{\pi^2}m_s^2q^2\log -\frac{q^2}{\nu^2}\left[\log -\frac{q^2}{\nu^2}-\frac{2}{3}(4-3\gamma_E)\right]. \tag{37.18}$$

The evaluation of the quark and gluon condensates is very similar to the case of the pseudoscalar channel, which the reader can easily evaluate as an exercise. The result

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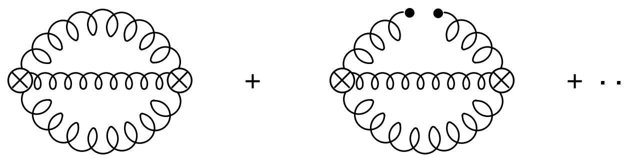

Fig. 37.3. Feynman diagrams corresponding to the OPE of the tri-gluonium correlator: (a) perturbative; (b) gluon condensate.

indicates that the mixing also vanishes in the chiral limit like in the case of the pseudoscalar channel.

37.5 Scalar tri-gluonium correlator

Here, one studies the correlator in Fig. 37.3 associated to the interpolating trigluon current:

$$J_{3g} = g^3 f_{abc} \langle G^a G^b G^c \rangle \tag{37.19}$$

Its QCD expression reads [457]:

$$\begin{aligned}\psi_3(q^2) = -\alpha_s^2 \Bigg\{ & \frac{3\alpha_s}{10\pi} q^8 \log -\frac{q^2}{\nu^2} + 18\pi q^4 \langle \alpha_s G^2 \rangle \\ & - \frac{27}{2}\left(q^2 \log -\frac{q^2}{\nu^2}\right) g^3 f_{abc} \langle G^a G^b G^c \rangle + \alpha_s \pi^3 36 \times 64 \, (\phi_7 - \phi_5) \Bigg\} ,\end{aligned} \tag{37.20}$$

with:

$$\begin{aligned}\phi_5 &= \frac{1}{16} \mathbf{Tr} \langle G_{\nu\mu} G^{\mu\rho} G_{\rho\tau} G^{\tau\nu} \rangle \\ \phi_7 &= \frac{1}{16} \mathbf{Tr} \langle G_{\nu\mu} G^{\nu\rho} G^{\mu\rho} G_{\mu\tau} \rangle .\end{aligned} \tag{37.21}$$

37.6 Scalar di- and tri-gluonium mixing

We shall be concerned with the off-diagonal correlator:

$$\psi_{23}(q^2) \equiv i \int d^4x \, e^{iqx} \, \langle 0 | \mathcal{T} J_{2g}(x) J_{3g}(0)^\dagger | 0 \rangle . \tag{37.22}$$

Its QCD expression reads [457]:

$$\psi_{23}(q^2) = \alpha_s^2 \left\{ \log -\frac{q^2}{\nu^2} \left[\frac{9}{4\pi^3} g^2 q^6 - \frac{9}{4\pi} g^2 \langle G^2 \rangle q^2 \right] - 24\pi g \langle f_{abc} G^a G^b G^c \rangle \right\} . \tag{37.23}$$

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37.7 Tensor gluonium

We shall be concerned with the two-point correlator:

$$\begin{aligned}\Psi^T_{\mu\nu\rho\sigma} &\equiv i\int d^4x\ e^{iqx}\langle 0|\mathcal{T}\theta^g_{\mu\nu}(x)\theta^g_{\rho\sigma}(0)^\dagger|0\rangle\\ &= \frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma}\right)\psi_T(q^2)\,, \end{aligned}\tag{37.24}$$

where:

$$\theta^g_{\mu\nu} = -G^\alpha_\mu G_{\nu\alpha} + \frac{1}{4}g_{\mu\nu}G_{\alpha_\beta}G^{\alpha\beta}\,. \tag{37.25}$$

and:

$$\eta_{\mu\nu} \equiv g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\,. \tag{37.26}$$

To leading order in α_s and including the non-perturbative condensates, the QCD expression of the correlator reads [382]:

$$\Psi_T(q^2\equiv -Q^2) = -\frac{1}{20\pi^2}(Q^4)\log\frac{Q^2}{\nu^2}+\frac{5}{12}\frac{g^2}{Q^4}\langle 2O_1 - O_2\rangle\,, \tag{37.27}$$

where:

$$O_1 = (f_{abc}G_{\mu\alpha}G_{\nu\alpha})^2 \quad\text{and}\quad O_2 = (f_{abc}G_{\mu\nu}G_{\alpha\beta})^2\,. \tag{37.28}$$

Using the vacuum saturation hypothesis, one can write:

$$\langle 2O_1 - O_2\rangle \simeq -\frac{3}{16}\langle G^2\rangle^2\,. \tag{37.29}$$

37.8 Tensor meson-gluonium mixing

We shall be concerned with the off-diagonal two-point correlator:

$$\begin{aligned}\Psi^T_{gq,\mu\nu\rho\sigma} &\equiv i\int d^4x\ e^{iqx}\langle 0|\mathcal{T}\theta^g_{\mu\nu}(x)\theta^q_{\rho\sigma}(0)^\dagger|0\rangle\\ &= \frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma}\right)\Psi^T_{gq}(q^2)\,, \end{aligned}\tag{37.30}$$

where

$$\theta^q_{\mu\nu}(x) = i\bar{q}(x)(\gamma_\mu\bar{D}_\nu + \gamma_\nu\bar{D}_\mu)q(x)\,. \tag{37.31}$$

Here, $\bar{D}_\mu \equiv \vec{D}_\mu - \overleftarrow{D}_\mu$ is the covariant derivative, and the other quantities have already been defined earlier. Taking into account the mixing of the currents, one obtains [452]:

$$\Psi^T_{gq}(q^2\equiv -Q^2) \simeq \frac{q^4}{15\pi^2}\left(\frac{\alpha_s}{\pi}\right)\left(\log^2\frac{Q^2}{\nu^2}-\frac{91}{15}\log\frac{Q^2}{\nu^2}\right)-\frac{7}{36}\log\frac{Q^2}{\nu^2}\left(\frac{\alpha_s}{\pi}\right)\langle G^2\rangle\,. \tag{37.32}$$

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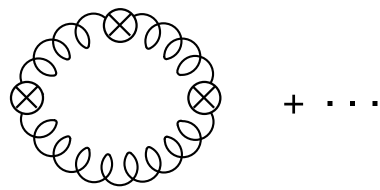

Fig. 37.4. Lowest order tachyonic gluon contribution to the gluonic correlator. The cross in the internal gluon propagator corresponds to the tachyonic gluon mass insertion λ^2.

37.9 Contributions beyond the OPE: tachyonic gluon mass

As we have seen previously, there are also contributions beyond the SVZ-expansion. We shall first be concerned with the two-point correlator:

$$\Psi_s(Q^2) \equiv i \int d^4x\, e^{iqx}\, \langle 0|\mathcal{T} J_{2g}(x)(J_{2g}(0))^\dagger|0\rangle \tag{37.33}$$

associated to the scalar gluonium current:

$$J_{2g} = \alpha_s \left(G^a_{\mu\nu}\right)^2 . \tag{37.34}$$

Its evaluation leads to:

$$\frac{1}{\pi}\mathrm{Im}\Psi_s(s) \approx (\text{parton model})\left(1 - \frac{6\lambda^2}{s} + \cdots\right). \tag{37.35}$$

The tachyonic gluon contribution comes from the diagram in Fig. 37.4.

Thus, one can expect that the λ^2 correction in this channel is relatively much larger since it is not proportional to an extra power of α_s. Let us consider now the case of the tensor gluonium with the correlator:

$$\begin{aligned}
\psi^T_{\mu\nu\rho\sigma}(q) &\equiv i \int d^4x\, e^{iqx} \langle 0|\mathcal{T}\theta^g_{\mu\nu}(x)\theta^g_{\rho\sigma}(0)^\dagger|0\rangle \\
&= \psi^T_4\left(q_\mu q_\nu q_\rho q_\sigma - \frac{q^2}{4}(q_\mu q_\nu g_{\rho\sigma} + q_\rho q_\sigma g_{\mu\nu}) + \frac{q^4}{16}(g_{\mu\nu}g_{\rho\sigma})\right) \\
&\quad + \psi^T_2\left(\frac{q^2}{4}g_{\mu\nu}g_{\rho\sigma} - q_\mu q_\nu g_{\rho\sigma} - q_\rho q_\sigma g_{\mu\nu} + q_\mu q_\sigma g_{\nu\rho} + q_\nu q_\sigma g_{\mu\rho}\right. \\
&\quad \left. + q_\mu q_\rho g_{\nu\sigma} + q_\nu q_\rho g_{\mu\sigma}\right) \\
&\quad + \psi^T_0\left(g_{\mu\sigma}g_{\nu\rho} + g_{\mu\rho}g_{\nu\sigma} - \frac{1}{2}g_{\mu\nu}g_{\rho\sigma}\right),
\end{aligned} \tag{37.36}$$

where $\theta^g_{\mu\nu}$ has been defined in Eq. (37.25). A direct calculation gives the following results

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for the structure functions ψ_i^T and their respective Borel/Laplace transforms:

$$\pi^2\psi_4^T = \frac{l_{\mu Q}}{15} + \frac{17}{450} - \lambda^2\frac{1}{3Q^2} \Longrightarrow \frac{1}{15}\left(1 - 5\frac{\lambda^2}{M^2}\right), \tag{37.37}$$

$$\pi^2\psi_2^T = \frac{Q^2 l_{\mu Q}}{20} + \frac{9Q^2}{200} + \lambda^2\left(\frac{l_{\mu Q}}{6} - \frac{2}{9}\right) \Longrightarrow -\frac{M^2}{20}\left(1 - \frac{10}{3}\frac{\lambda^2}{M^2}\right), \tag{37.38}$$

$$\pi^2\psi_0^T = \frac{Q^4 l_{\mu Q}}{20} + \frac{9Q^4}{200} + \lambda^2 Q^2\left(\frac{l_{\mu Q}}{4} - \frac{1}{12}\right) \Longrightarrow \frac{M^4}{20}\left(1 - \frac{5}{2}\frac{\lambda^2}{M^2}\right). \tag{37.39}$$

If, instead of considering $\theta^g_{\mu\nu}$, we would introduce the total energy-momentum tensor of interacting quarks and gluons $\theta_{\mu\nu}$, then various functions components of $\psi_{\mu\nu\rho\sigma}$ are related to each other because of the energy-momentum conservation. Indeed, requiring that

$$\psi^T_{\mu\nu\rho\sigma} q_\mu \equiv 0$$

we immediately obtain:

$$\psi_2^T = \frac{3}{4}Q^2\psi_4^T \quad \text{and} \quad \psi_0^T = \frac{3}{4}Q^4\psi_4^T\,,$$

and, as a consequence, the following representation of the function in Eq. 37.36:

$$\psi^T_{\mu\nu\rho\sigma}(q) = \left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma}\right)\psi^T(Q^2)\,, \tag{37.40}$$

where:

$$\psi^T(Q^2) \equiv Q^4\frac{3}{4}\psi_4^T(Q^2), \qquad \eta_{\mu\nu} \equiv g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\,. \tag{37.41}$$