PAPER 11

A CHARACTERISTIC LINE CURRENT IN A FULLY IONIZED GAS

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ABSTRACT

Standard formulae for the electrical resistance and for the radiating properties of a fully ionized gas have been combined with pinch effect relations to obtain the stationary state radial distribution functions and current—voltage characteristics of a filamentary current. The calculations suggest that the radiation cooling permits a pinched discharge to exist, with a maximum current of about one or two million amperes.

Alfvén [1] has drawn attention to the prevalence of line currents in cosmic physics and has pointed out the role which might be played by the pinch effect in producing them. Tonks [2] and Alfvén [3] have both discussed a possible disrupture of a high current discharge due to the pinch effect, when the current exceeds a certain value, such as is observed for instance in mercury arc rectifiers. Both these treatments appear to ignore radiation cooling, which is likely to be an important source of energy dissipation in cosmic physics. The present calculations of the characteristics of a filamentary current in a fully ionized gas assume that all the power input is dissipated in 'Bremsstrahlung' radiation.

We suppose the current filament extends over a cylinder of radius b containing N atoms/unit length, and is actuated by a uniform applied axial electric field of strength E. We suppose the gas is completely ionized hydrogen, and radiates power according to Cillié's [4] 'Bremsstrahlung' formula. The energy balance is then expressed by

$$EI = \int_0^b 2\pi r B n^2 T^{1/2}\, dr, \tag{1}$$

where I is the total current, n is the number of electrons (and of protons) per unit volume, T is the electron temperature, and B is a constant equal to $1{\cdot}4 \times 10^{-40}$ M.K.S. units; r is the distance from the axis of the current filament. In the axial direction, the momentum imparted to the electrons by the field is equal to that lost in collisions with protons; and from the work of Gvosdover [5] we obtain

$$EeN = \int_0^b \pi r n^2 e^2 w T^{-3/2} G \log X dr, \tag{2}$$

where e is the electronic charge, w is the electron drift velocity, G is a constant equal to 65 M.K.S. units, and $X = (kT/e^2 n^{1/3} . 137\beta)^2$ where β is the ratio of the electron speed to that of light; discussions of the factor 137β are given by Williams [6] and Spitzer [7]. Finally, in the radial direction, if the filament is neither expanding nor contracting there is a balance between the hydrostatic pressure and the pinch forces; and so:

$$\mu_0 i \frac{di}{dr} + 2\pi r^2 k \frac{d}{dr}(nT) = 0, \tag{3}$$

where μ_0 is the permeability of free space, i is the current enclosed within a radius r, and k is Boltzmann's constant. i is related to w by the formula

$$\frac{di}{dr} = 2\pi r n e w. \tag{4}$$

In (3) we have assumed that the ion temperature is equal to the electron temperature; if the temperature is zero, the factor of 2 is omitted from the second term of (3).

These equations cannot be solved fully without information about transport processes in the gas. We may solve however for certain simple cases, notably: zero viscosity and thermal conductivity; infinite viscosity and thermal conductivity; and zero viscosity and infinite thermal conductivity. When this is done, it is found that the solutions in each of these cases are very similar, and that the last case is the best of these simple approximations. The radial distribution functions for the case are:

$$n = \frac{2N}{b^2}(1 - r^2/b^2) \tag{5a}$$

and

$$\omega = \overline{\omega}/2(1 - r^2/b^2). \tag{5b}$$

T is, of course, a constant. So also is the current density (new). This configuration is the same as that discussed by Schlüter [8]. The radius of the filament is given by

$$b \doteq (2B/3\pi E)^{1/2}(\mu_0 N^3/k)^{1/4}. \tag{6}$$

The current is found to be a constant, independent of b, N and E provided the filament is supported entirely by the self-magnetic field, and is equal to

$$I = \frac{k}{\mu_0}(12G(\log X)/B)^{1/2}. \tag{7}$$

The effect of increasing the electric field is thus solely to increase the constriction of the current channel. The extra power fed in is dissipated solely by the consequent increase of the factor n^2 in equation (1). The term $\log X$ contains n, and the increased constriction lowers this term slightly: however, as has often been pointed out, $\log X$ is large and very insensitive to large changes of X, and so the current is virtually independent of E. If the electric field becomes very low, it must be expected that in practice some of the pressure is supported by means other than the self-magnetic field. It is easily shown that in such circumstances the current drops below the above value. With $\log X$ equal to 30, the current is about 2×10^6 amperes. In the other two cases mentioned, the numerical factors of Eqs. (6) and (7) are slightly different. When the constants G and B are expressed in terms of fundamental atomic constants, it is found that

$$I \sim me^{-2}\hbar^{1/2}c^{7/2} \quad \text{(e.s.u.)}.$$

This particular combination of constants can be written $(mc^3e^{-1})\,(e^2/\hbar c)^{-1/2}$. With an ionized gas containing positive ions of charge Z, the current is found to be altered by the factor $2Z/(Z+1)$.

These results depend on a number of assumptions implicit in Eqs. (1)–(3). Two particularly important ones are as follows. First, it is assumed that the gas has a Maxwellian velocity distribution superimposed on a slow drift velocity. Thus the results might well be inapplicable when a beam of fast electrons with predominantly ordered motion passes through a cloud of ions. Secondly, Eq. (2) implies that the electric current heats the gas entirely by collisions with the electrons and protons. In cases where there is a large electrodynamic voltage, the gas might be heated by collisions arising from the bulk motion of the gas. Such motions are known to arise from inherent instability of a current carrying plasma (Kruskal and Schwarzschild [9]).

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Discussion

Spitzer: Have you taken into account the reduction of thermal conductivity by the magnetic field? How does the temperature come out from that?

Pease: The mean temperature comes out from the pinch relation solely. You have $\mu_0 I^2 = 4Nk\overline{T}$ where N is the ion density per unit length of the column. For $N = 10^{20}\ \text{m}^{-1}$ the temperature comes out about 5×10^7 °K. Regarding the thermal conductivity question, I have taken the reduction of thermal conductivity by the magnetic field into account when estimating the effect of conductivity on the radial temperature gradients. The gradients obtained when the conductivity is assumed zero, in fact require heat flow from each plasma element which is comparable with $Bn^2T^{\frac{1}{2}}$, the radiation loss. This is calculated for the thermal conductivity of the electrons. But the reduction of the thermal conductivity of the positive ions is not so great. Hence I conclude that radial temperature gradients are largely eliminated by radial thermal conduction.

Artsimovich: Have you also investigated other forms when the radius varies with time? If it varies with time, and the contraction is strong enough, the temperature will increase considerably and 'Bremsstrahlung' can probably not be neglected.

Pease: I have not studied the non-stationary case, which I believe you have treated, i.e. where the radius collapses. There is no inertial term in my equations.

Artsimovich: If you neglect the 'Bremsstrahlung' effects, have you then investigated the time variation rates?

Pease: No, but your colleagues have done it, surely.

Artsimovich: I should think that these investigations are too specialized and have only historical interest. In reality the plasma column is not stable.

A great number of solutions can be obtained by means of relations which express a current-compression. If the temperature is low enough for 'Bremsstrahlung' not to be of importance, then the radius of the pinch can be determined and the current becomes proportional to $T^{1/7}$.

Pease: I have investigated cases where the energy dissipation is by other mechanisms, for instance when the 'Bremsstrahlung' radiation is not produced by proton-electron collisions but is produced by magnetic spiralling.

Artsimovich. If this investigation is carried on still further then cases can be found in which the limiting current is larger than the values you have found.

Pease: Is this the situation when one is working with transient conditions?

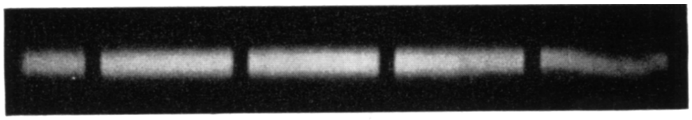
Artsimovich: Yes. Such investigations are based upon simple conditions expressing thermal equilibrium between ions and electrons.

Spitzer: What is the time dependence of the transient solutions which you have obtained?

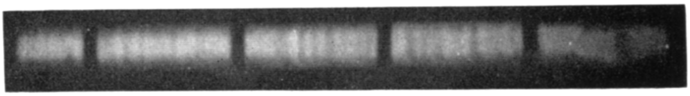
Artsimovich: We have got many different solutions the forms of which depend on the boundary conditions; I do not think that I can give a simple answer to Dr Spitzer's question.

Thonemann: Pease talked about the steady state limiting current. Instabilities do in fact develop rapidly as the following figures illustrate. Plate I *a* shows a straight discharge in a tube with only slight perturbations near the cathode end. As time proceeds and the current increases, striations appear (Plate I *b*). Finally at a still later time the line current breaks up or develops into a badly defined helix with a pitch of about 45° (Plate I *c*). Plate II shows another example of an unstable discharge in a toroidal glass tube, again of helical form.

https://doi.org/10.1017/S0074180900237674 Published online by Cambridge University Press



(a)



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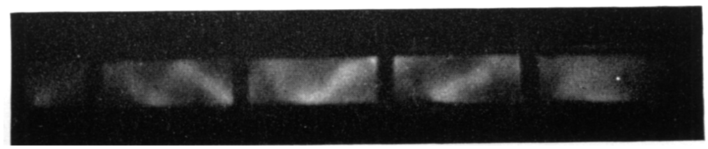

(c)

Plate I. Gaseous discharge in a cylindrical tube. (a) The current is relatively weak. A slight perturbation is seen near the cathode in the right-hand corner of the picture. (b) Striations occur when the current is raised. (c) If the current is raised further it breaks up and a helix is formed.

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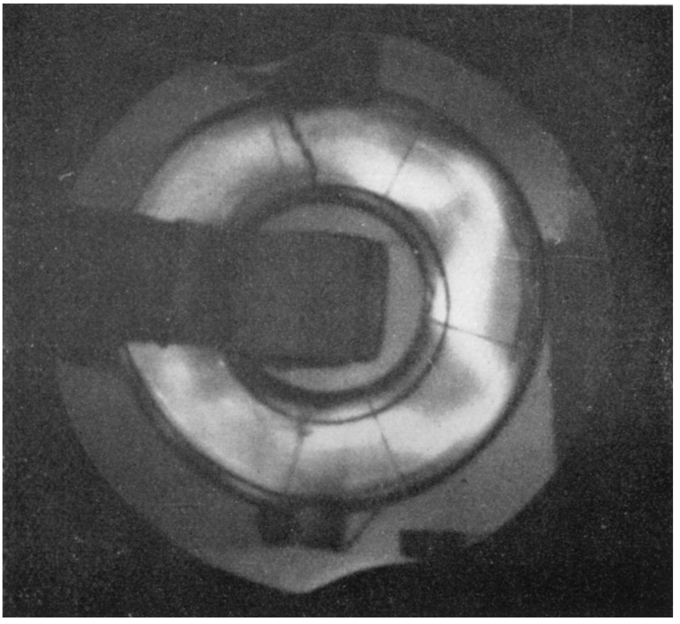

Plate II. Gaseous discharge in a toroidal tube without electrodes. The gas forms the secondary of a pulse transformer. For strong currents a helical configuration arises as seen from the picture.