

MINIMAL PRESENTATIONS FOR GROUPS OF ORDER 2^n , $n \leq 6$

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1. Introduction

Let G be a finite 2-group having a minimal generating set $\{x_1, \dots, x_r\}$ so that $r = d(G)$ is an invariant of G . Suppose further that G has a presentation

$$G = \{x_1, \dots, x_n \mid R_1, \dots, R_m\}$$

then

$$n \geq d(G) = \dim_{\mathbb{Z}_2} H^2(G, \mathbb{Z}) \quad \text{and}$$

$$m \geq r(G) = \dim_{\mathbb{Z}_2} H^2(G, \mathbb{Z}) + \dim_{\mathbb{Z}_2} H^3(G, \mathbb{Z}).$$

The presentation is said to be minimal if $n = d(G)$ and $m = r(G)$.

A list of minimal presentations for groups of order 2^n , $n \leq 6$ is given. An individual group is given a designation such as $64, 30, (12^2, 12^2)$. Here the 64 gives the order of the group, 30 means it is number 30 of the groups of order 64 in [1], 12^2 is the terminology used in [1] to designate the abelian group $C_2 \times C_4 \times C_4$ where C_n is the cyclic group of order n . The first 12^2 means that $G/G' = H^2(G, \mathbb{Z}) = C_2 \times C_4 \times C_4$ and the second 12^2 means that the multiplier of $G = H^3(G, \mathbb{Z}) = C_2 \times C_4 \times C_4$. Then follows the minimal presentation given with $d(G)$ generators and $r(G)$ relations. For example $8, 5, (1^2, 0) = \{1^2 2^{-2}, [1, 2] 1^2\}$, means the fifth group of order 8 in [1] has minimal presentation

$$\{a_1, a_2 \mid a_1^2 a_2^{-2}, [a_1, a_2] a_1^2\}, \quad \text{or}$$

$$\{a_1, a_2 \mid a_1^2 = a_2^2, [a_1, a_2] = a_1^{-2}\}$$

where relations are used rather than relators.

2 The Table

2, 1, (1, 0)	=	$\{1^2\}$
4, 1, (1 ² , 1)	=	$\{1^2, 2^2, [1, 2]\}$
4, 2, (2, 0)	=	$\{1^4\}$
8, 1, (1 ³ , 1 ³)	=	$\{1^2, 2^2, 3^2, [1, 2], [1, 3], [2, 3]\}$
8, 2, (1 ² , 1)	=	$\{1^2, 2^4, [1, 2]\}$
8, 3, (3, 0)	=	$\{1^8\}$
8, 4, (1 ² , 1)	=	$\{1^4, 2^2, [1, 2]1^2\}$
8, 5, (1 ² , 0)	=	$\{1^{2^2-2}, [1, 2]1^2\}$
16, 1, (1 ⁴ , 1 ⁶)	=	$\{1^2, 2^2, 3^2, 4^2, [1, 2], [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]\}$
16, 2, (1 ² ² , 1 ³)	=	$\{1^2, 2^2, 3^4, [1, 2], [1, 3], [2, 3]\}$
16, 3, (2 ² , 2)	=	$\{1^4, 2^4, [1, 2]\}$
16, 4, (1 ³ , 1)	=	$\{1^2, 2^8, [1, 2]\}$
16, 5, (4, 0)	=	$\{1^{16}\}$
16, 6, (1 ³ , 1 ³)	=	$\{1^4, 2^2, 3^2, [1, 2]1^2, [1, 3], [2, 3]\}$
16, 7, (1 ³ , 1 ²)	=	$\{1^{2^2-2}, 3^2, [1, 2]1^2, [1, 3], [2, 3]\}$
16, 8, (1 ³ , 1 ²)	=	$\{2^2, 3^2, [2, 3]1^2, [1, 2], [1, 3]\}$
16, 9, (1 ² , 1 ²)	=	$\{1^2, 2^4, [1, 2, 1], [1, 2, 2]\}$
16, 10, (1 ² , 1)	=	$\{1^4, 2^4, [1, 2]1^2\}$
16, 11, (1 ² , 0)	=	$\{1^{2^2-8}, [1, 2]2^4\}$
16, 12, (1 ² , 1)	=	$\{1^8, 2^2, [1, 2]1^2\}$
16, 13, (1 ² , 0)	=	$\{2^2, [2, 1]1^2\}$
16, 14, (1 ² , 0)	=	$\{1^{4^2-2}, [1, 2]1^2\}$
32, 1, (1 ⁵ , 1 ¹⁰)	=	$\{1^2, 2^2, 3^2, 4^2, 5^2, [1, 2], [1, 3], [1, 4], [1, 5], [2, 3], [2, 4], [2, 5], [3, 4], [3, 5], [4, 5]\}$
32, 2, (1 ³ ² , 1 ⁶)	=	$\{1^2, 2^2, 3^2, 4^4, [1, 2], [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]\}$
32, 3, (1 ² ² , 1 ² ²)	=	$\{1^2, 2^4, 3^4, [1, 2], [1, 3], [2, 3]\}$
32, 4, (1 ² ³ , 1 ³)	=	$\{1^2, 2^2, 3^8, [1, 2], [1, 3], [2, 3]\}$
32, 5, (2 ³ , 2)	=	$\{1^4, 2^8, [1, 2]\}$
32, 6, (1 ⁴ , 1)	=	$\{1^2, 2^{16}, [1, 2]\}$
32, 7, (5, 0)	=	$\{1^{32}\}$
32, 8, (1 ⁴ , 1 ⁶)	=	$\{1^4, 2^2, 3^2, 4^2, [1, 2]1^2, [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]\}$
32, 9, (1 ⁴ , 1 ⁵)	=	$\{1^{2^2-2}, 3^2, 4^2, [1, 2]1^2, [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]\}$
32, 10, (1 ⁴ , 1 ⁵)	=	$\{2^2, 3^2, 4^2, [2, 3]1^2, [1, 2], [1, 3], [1, 4], [2, 4], [3, 4]\}$
32, 11, (1 ² ² , 1 ⁴)	=	$\{1^2, 2^4, 3^2, [1, 2, 1], [1, 2, 2], [1, 3], [2, 3]\}$
32, 12, (1 ² ² , 1 ³)	=	$\{1^4, 2^4, 3^2, [1, 2]1^2, [1, 3], [2, 3]\}$
32, 13, (1 ² ² , 1 ²)	=	$\{1^{2^2-8}, 3^2, [1, 2]2^4, [1, 3], [2, 3]\}$
32, 14, (1 ² ² , 1 ³)	=	$\{1^4, 2^2, 3^4, [1, 2]1^2, [1, 3], [2, 3]\}$
32, 15, (1 ² ² , 1 ²)	=	$\{1^{2^2-2}, 3^4, [1, 2]1^2, [1, 3], [2, 3]\}$
32, 16, (1 ² ² , 2 ¹)	=	$\{2^2, 3^4, [2, 3]1^2, [1, 2], [1, 3]\}$
32, 17, (1 ² ² , 1 ²)	=	$\{2^2, 3^2, [2, 3]1^4, [1, 2], [1, 3]\}$
32, 18, (2 ² , 1 ³)	=	$\{1^4, 2^4, [1, 2]^2, [1, 2, 1], [1, 2, 2]\}$
32, 19, (2 ² , 1)	=	$\{1^4, 2^8, [1, 2, 2]^4\}$
32, 20, (1 ³ , 1 ²)	=	$\{1^2, 2^8, [1, 2, 1], [1, 2, 2]\}$
32, 21, (1 ³ , 1)	=	$\{1^4, 2^8, [1, 2]1^2\}$
32, 22, (1 ³ , 0)	=	$\{1^{2^2-16}, [1, 2]2^8\}$
32, 23, (1 ³ , 1 ³)	=	$\{1^8, 2^2, 3^2, [1, 2]1^2, [1, 3], [2, 3]\}$

- 32, 24, $(1^3, 1^2)$ = $\{2^2, 3^2, [2, 1]1^2, [1, 3], [2, 3]\}$
- 32, 25, $(1^3, 1^2)$ = $\{142^{-2}, 3^2, [1, 2]1^2, [1, 3], [2, 3]\}$
- 32, 26, $(1^3, 1^2)$ = $\{122^{-4}, 3^2, [2, 3]2^2, [1, 2], [1, 3]\}$
- 32, 27, $(1^2, 1^2)$ = $\{2^2, [2, 1]2^{14}, [1, 2, 2]1^{-4}, [1, 2, 1]\}$
- 32, 28, $(1^2, 1)$ = $\{142^{-2}, [2, 1]2^{14}, [1, 2, 1]\}$
- 32, 29, $(1^2, 1)$ = $\{1^8, 2^4, [1, 2]1^2\}$
- 32, 30, $(1^2, 1)$ = $\{1^8, 2^4, [2, 1]1^2\}$
- 32, 31, $(1^2, 1)$ = $\{1^4, 2^2, [1, 2, 1]\}$
- 32, 32, $(1^2, 0)$ = $\{142^{-4}, [1, 2]1^2\}$
- 32, 33, $(1^3, 1^4)$ = $\{1^2, 2^2, 3^2, [1, 2], [1, 3, 1], [2, 3, 1], [2, 3, 2]\}$
- 32, 34, $(1^3, 1^{22})$ = $\{1^4, 2^4, 3^2, [1, 2], [1, 3]1^2, [2, 3]2^2\}$
- 32, 35, $(1^3, 1^2)$ = $\{2^4, 1^{23-2}, [1, 2], [3, 1]1^2, [2, 3]2^2\}$
- 32, 36, $(1^3, 1^3)$ = $\{1^4, 2^2, 3^2, [1, 3]1^2, [1, 2], [2, 3, 3]\}$
- 32, 37, $(1^3, 1^2)$ = $\{1^{23-2}, 2^2, [3, 1]1^2, [1, 2], [2, 3, 2]\}$
- 32, 38, $(1^3, 1^2)$ = $\{1^2, 3^2, [3, 1]2^2, [1, 2], [2, 3, 3]\}$
- 32, 39, $(1^3, 1^2)$ = $\{2^4, 3^2, [3, 1]1^2, [2, 3]2^{21-2}, [1, 2]\}$
- 32, 40, $(1^3, 1)$ = $\{1^{23}2^{2-2}, [3, 1]1^2, [2, 3]3^2, [1, 2]\}$
- 32, 41, $(1^3, 1)$ = $\{3^2, [3, 2]1^2, [3, 1]1^{22}, [1, 2]\}$
- 32, 42, $(1^4, 1^5)$ = $\{1^{22-2}, 1^{23-2}, 1^{24-2}, [3, 2]1^2, [4, 1]1^2, [1, 2], [1, 3], [2, 4], [3, 4]\}$
- 32, 43, $(1^4, 1^5)$ = $\{4^2, 1^{22-2}, 1^{23-2}, [3, 2]1^2, [4, 1]1^2, [1, 2], [1, 3], [2, 4], [3, 4]\}$
- 32, 44, $(1^3, 1^2)$ = $\{1^2, 3^2, [2, 1]2^4, [2, 3]2^2, [1, 3]\}$
- 32, 45, $(1^3, 1^2)$ = $\{1^2, 2^{43-2}, [2, 1]2^4, [2, 3]2^2, [1, 3]\}$
- 32, 46, $(1^2, 1^2)$ = $\{1^2, 2^4, [1, 2, 1], [1, 2, 2, 2]\}$
- 32, 47, $(1^2, 1)$ = $\{1^2, [1, 2, 1], [1, 2, 2]2^{-4}\}$
- 32, 48, $(1^2, 1)$ = $\{1^{22-4}, [1, 2, 1], [1, 2, 2]2^{-4}\}$
- 32, 49, $(1^2, 1)$ = $\{1^{16}, 2^2, [1, 2]1^2\}$
- 32, 50, $(1^2, 0)$ = $\{1^{16}2^{-2}, [2, 1^3]1^2\}$
- 32, 51, $(1^2, 0)$ = $\{1^{82-2}, [2, 1^7]1^2\}$
- 64, 1, $(1^6, 1^{15})$ = $\{1^2, 2^2, 3^2, 4^2, 5^2, 6^2, [1, 2], [1, 3], [1, 4], [1, 5], [1, 6], [2, 3], [2, 4], [2, 5], [2, 6], [3, 4], [3, 5], [3, 6], [4, 5], [4, 6], [5, 6]\}$
- 64, 2, $(142, 1^{10})$ = $\{1^2, 2^2, 3^2, 4^2, 5^4, [1, 2], [1, 3], [1, 4], [1, 5], [2, 3], [2, 4], [2, 5], [3, 4], [3, 5], [4, 5]\}$
- 64, 3, $(1^{222}, 1^{52})$ = $\{1^2, 2^2, 3^4, 4^4, [1, 2], [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]\}$
- 64, 4, $(1^{33}, 1^6)$ = $\{1^2, 2^2, 3^2, 4^8, [1, 2], [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]\}$
- 64, 5, $(2^3, 2^3)$ = $\{1^4, 2^4, 3^4, [1, 2], [1, 3], [2, 3]\}$
- 64, 6, $(123, 1^{22})$ = $\{1^2, 2^4, 3^8, [1, 2], [1, 3], [2, 3]\}$
- 64, 7, $(1^{24}, 1^3)$ = $\{1^2, 2^2, 3^{16}, [1, 2], [1, 3], [2, 3]\}$
- 64, 8, $(3^2, 3)$ = $\{1^8, 2^8, [1, 2]\}$
- 64, 9, $(24, 2)$ = $\{1^4, 2^{16}, [1, 2]\}$
- 64, 10, $(15, 1)$ = $\{1^2, 2^{32}, [1, 2]\}$
- 64, 11, $(6, 0)$ = $\{1^{64}\}$
- 64, 12, $(1^5, 1^{10})$ = $\{1^4, 2^2, 3^2, 4^2, 5^2, [1, 2]1^2, [1, 3], [1, 4], [1, 5], [2, 3], [2, 4], [2, 5], [3, 4], [3, 5], [4, 5]\}$
- 64, 13, $(1^5, 1^9)$ = $\{1^{22-2}, 3^2, 4^2, 5^2, [1, 2]1^2, [1, 3], [1, 4], [1, 5], [2, 3], [2, 4], [2, 5], [3, 4], [3, 5], [4, 5]\}$
- 64, 14, $(1^5, 1^9)$ = $\{2^2, 3^2, 4^2, 5^2, [2, 3]1^2, [1, 2], [1, 3], [1, 4], [1, 5], [2, 4], [2, 5], [3, 4], [3, 5], [4, 5]\}$
- 64, 15, $(1^{32}, 1^7)$ = $\{1^2, 2^4, 3^2, 4^2, [1, 2, 1], [1, 2, 2], [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]\}$
- 64, 16, $(1^{32}, 1^6)$ = $\{1^4, 2^4, 3^2, 4^2, [1, 2]1^2, [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]\}$

- 64,17,(132,15) = {122⁻⁸, 32, 4², [1, 2]2⁴, [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]}
- 64,18,(132,16) = {14, 2², 3², 4⁴, [1, 2]1², [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]}
- 64,19,(132,15) = {122⁻², 3², 4⁴, [1, 2]1², [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]}
- 64,20,(132,142) = {2², 3⁴, 4², [2, 3]1², [1, 2], [1, 3], [1, 4], [2, 4], [3, 4]}
- 64,21,(132,15) = {2², 3², 4², [2, 3]1⁴, [1, 2], [1, 3], [1, 4], [2, 4], [3, 4]}
- 64,22,(122,15) = {14, 2⁴, 3², [1, 2]2², [1, 2, 1], [1, 2, 2], [1, 3], [2, 3]}
- 64,23,(122,13) = {14, 2⁸, 3², [1, 2]2⁴, [1, 3], [2, 3]}
- 64,24,(123,14) = {12, 2⁸, 3², [1, 2, 1], [1, 2, 2], [1, 3], [2, 3]}
- 64,25,(123,13) = {14, 2⁸, 3², [1, 2]1², [1, 3], [2, 3]}
- 64,26,(123,12) = {122⁻¹⁶, 3², [1, 2]2⁸, [1, 3], [2, 3]}
- 64,27,(132,15) = {2², 3², 4⁴, [2, 3]1², [1, 2], [1, 3], [1, 4], [2, 4], [3, 4]}
- 64,28,(122,132) = {12, 2⁴, 3⁴, [1, 2, 1], [1, 2, 2], [1, 3], [2, 3]}
- 64,29,(122,122) = {14, 2⁴, 3⁴, [1, 2]1², [1, 3], [2, 3]}
- 64,30,(122,122) = {14, 2⁴, 3⁴, [2, 3]1², [1, 2], [1, 3]}
- 64,31,(122,12) = {122⁻⁸, 3⁴, [1, 2]2⁴, [1, 3], [2, 3]}
- 64,32,(122,12) = {2², 3⁴, [3, 2]1⁴, [1, 2], [1, 3]}
- 64,33,(123,12) = {2², 3⁸, [3, 2]1², [1, 2], [1, 3]}
- 64,34,(123,13) = {14, 2², 3⁸, [1, 2]1², [1, 3], [2, 3]}
- 64,35,(123,12) = {122⁻², 3⁸, [1, 2]1², [1, 3], [2, 3]}
- 64,36,(123,12) = {2², 3², [3, 2]1⁸, [1, 2], [1, 3]}
- 64,37,(23,13) = {14, 2⁸, [1, 2]2², [1, 2, 1], [1, 2, 2]}
- 64,38,(23,2) = {1⁸, 2⁸, [1, 2]1⁴}
- 64,39,(23,1) = {14, 2¹⁶, [1, 2]2⁸}
- 64,40,(14,12) = {12, 2¹⁶, [1, 2, 1], [1, 2, 2]}
- 64,41,(14,1) = {14, 2¹⁶, [1, 2]1²}
- 64,42,(14,0) = {122⁻³², [2, 1]2¹⁶}
- 64,43,(14,16) = {18, 2², 3², 4², [1, 2]1², [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]}
- 64,44,(14,15) = {2², 3², 4², [2, 1]1², [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]}
- 64,45,(14,15) = {142⁻², 3², 4², [1, 2]1², [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]}
- 64,46,(14,15) = {122⁻⁴, 3², 4², [2, 3]2², [1, 2], [1, 3], [1, 4], [2, 4], [3, 4]}
- 64,47,(122,14) = {2², 3², [2, 1]2¹⁴, [1, 2, 2]1⁻⁴, [1, 2, 1], [1, 3], [2, 3]}
- 64,48,(122,13) = {142⁻², 3², [2, 1]2¹⁴, [1, 2, 1], [1, 3], [2, 3]}
- 64,49,(122,13) = {1⁸, 2⁴, 3², [1, 2]1², [1, 3], [2, 3]}
- 64,50,(122,13) = {1⁸, 2⁴, 3², [2, 1]1², [1, 3], [2, 3]}
- 64,51,(122,13) = {14, 2², 3², [1, 2, 1], [1, 3], [2, 3]}
- 64,52,(122,12) = {142⁻⁴, 3², [1, 2]1², [1, 3], [2, 3]}
- 64,53,(122,13) = {3², 122⁻⁴, [2, 3]2¹⁻², [1, 2], [1, 3], [2, 3, 2]}
- 64,54,(122,12) = {3⁴, 122⁻⁴, [2, 3]2², [1, 2], [1, 3]}
- 64,55,(122,13) = {1⁸, 2², 3⁴, [1, 2]1², [1, 3], [2, 3]}
- 64,56,(122,12) = {2², 3⁴, [2, 1]1², [1, 3], [2, 3]}
- 64,57,(122,12) = {142⁻², 3⁴, [1, 2]1², [1, 3], [2, 3]}
- 64,58,(122,12) = {3², 142⁻⁴, [2, 3]2², [1, 2], [1, 3]}
- 64,59,(22,13) = {1⁸, 2⁴, [1, 2]2¹⁴, [1, 2, 2]1⁴, [1, 2, 1]}
- 64,60,(22,12) = {14, 2⁴, [1, 2, 2], [2, 1]2², [1, 2, 1]}
- 64,61,(22,12) = {142⁻⁴, [2, 1]2¹⁴, [1, 2, 2]2⁻⁴, [1, 2, 1]}
- 64,62,(13,12) = {1⁸, 2², [1, 2, 1], [1, 2, 2]2¹2²}
- 64,63,(13,1) = {1⁸, [2, 1]2²², [1, 2, 1]}
- 64,64,(13,1) = {1⁸, 2⁸, [1, 2]1²}
- 64,65,(13,1) = {1⁸, 2⁸, [2, 1]1²}

64,66,(13,1)	=	$\{1^{16}2^{-2}, [2,1]^2 1^8, [1,2,1]\}$
64,67,(13,0)	=	$\{142^{-8}, [1,2]1^{12}\}$
64,68,(14,17)	=	$\{12, 2^2, 3^2, 4^2, [1,2], [1,3,1], [2,3,1], [2,3,2], [1,4], [2,4], [3,4]\}$
64,69,(14,15 ²)	=	$\{14, 2^4, 3^2, 4^2, [1,2], [1,3]1^2, [2,3]2^2, [1,4], [2,4], [3,4]\}$
64,70,(14,1 ⁵)	=	$\{2^4, 123^{-2}, 4^2, [1,2], [3,1]1^2, [2,3]2^2, [1,4], [2,4], [3,4]\}$
64,71,(14,1 ⁶)	=	$\{14, 2^2, 3^2, 4^2, [1,3]1^2, [1,2], [2,3,3], [1,4], [2,4], [3,4]\}$
64,72,(14,1 ⁵)	=	$\{123^{-2}, 2^2, 4^2, [3,1]1^2, [1,2], [2,3,2], [1,4], [2,4], [3,4]\}$
64,73,(14,1 ⁵)	=	$\{12, 3^2, 4^2, [3,1]2^2, [1,2], [2,3,3], [1,4], [2,4], [3,4]\}$
64,74,(14,14 ²)	=	$\{2^4, 3^2, 4^2, [3,1]1^2, [2,3]2^2 1^{-2}, [1,2], [1,4], [2,4], [3,4]\}$
64,75,(14,14)	=	$\{1232^2 2^{-2}, 4^2, [3,1]1^2, [2,3]3^2, [1,2], [1,4], [2,4], [3,4]\}$
64,76,(14,14)	=	$\{3^2, 4^2, [3,2]1^2, [3,1]1^2 2^2, [1,2], [1,4], [2,4], [3,4]\}$
64,77,(14,1 ⁵)	=	$\{3^2, 12^2 2^{-2}, 4^2, [4,2]1^2, [1,2], [1,3], [1,4], [2,3], [3,4,3]\}$
64,78,(14,1 ⁵)	=	$\{2^2, 3^4, 4^2, [4,2]1^2, [3,4]3^2, [1,2], [1,3], [1,4], [2,3]\}$
64,79,(14,14)	=	$\{2^2, 3^2, 4^{-2}, [4,2]1^2, [4,3]3^2, [1,2], [1,3], [1,4], [2,3]\}$
64,80,(14,14)	=	$\{3^2, 4^2, [4,2]1^2, [4,3]2^2, [1,2], [1,3], [1,4], [2,3]\}$
64,81,(122,1 ⁶)	=	$\{12, 2^2, 3^4, [1,3,1], [1,3,2], [1,3,3], [2,3,2], [2,3,3], [1,2]\}$
64,82,(122,12 ²)	=	$\{14, 2^4, 3^4, [1,3]1^2, [2,3]2^2, [1,2]\}$
64,83,(122,14)	=	$\{14, 2^2, 3^4, [1,3]1^2, [1,2], [2,3,2], [2,3,3]\}$
64,84,(122,1 ⁵)	=	$\{12, 2^4, 3^2, [1,3,1], [1,3,2], [2,3,2], [2,3,3], [1,2]\}$
64,85,(122,1 ³ 2)	=	$\{14, 2^4, 3^2, [1,3]1^2, [2,3,2], [2,3,3], [1,2]\}$
64,86,(122,14)	=	$\{12, 2^4, 3^4, [2,3]3^2, [1,3,1], [1,3,3], [1,2]\}$
64,87,(122,12 ²)	=	$\{123^{-2}, 2^4, [3,1]3^2, [2,3,2], [2,3,3], [1,2]\}$
64,88,(122,1 ³)	=	$\{14, 2^4, 3^4, [1,3], 1^2 [2,3]3^2, [1,2]\}$
64,89,(122,14)	=	$\{12, 3^4, [2,3]2, [3,1]2^2, [2,3,2], [2,3,3], [1,2]\}$
64,90,(122,12 ²)	=	$\{14, 2^4, 3^4, [1,3]1^2, [2,3]1^2 2^2, [1,2]\}$
64,91,(122,12 ²)	=	$\{2^4, 3^2, [3,2]1^2, [1,3,1], [1,3,3], [1,2]\}$
64,92,(122,12)	=	$\{2^4, 3^4, [1,3]3^2, [3,2]1^2, [1,2]\}$
64,93,(122,1 ³)	=	$\{14, 2^4, 3^4, [1,3]1^2 2^2, [2,3]1^2, [1,2]\}$
64,94,(122,1 ³)	=	$\{2^2, 12^3 3^{-4}, [3,1]1^2, [2,3,2], [2,3,3], [1,2]\}$
64,95,(122,12)	=	$\{12, 2^4, [3,1]3^4, [2,3]2^2, [1,2]\}$
64,96,(122,1 ³)	=	$\{12, 3^2, [3,1]2^4, [2,3,2], [2,3,3], [1,2]\}$
64,97,(122,12)	=	$\{12, 3^4, [3,1]2^4, [2,3]3^2, [1,2]\}$
64,98,(122,12)	=	$\{2^2, 3^8, [1,3]3^4, [3,2]1^2, [1,2]\}$
64,99,(122,12)	=	$\{2^2, 3^2, [1,3]1^4, [2,3,2], [1,2]\}$
64,100,(122,12)	=	$\{2^4, 3^2, [1,3]1^4, [2,3]2^2, [1,2]\}$
64,101,(122,1)	=	$\{22^3 2^{-2}, [1,3]1^4, [3,2]2^2, [1,2]\}$
64,102,(122,2)	=	$\{3^2, [3,1]2^4, [3,2]1^2, [1,2]\}$
64,103,(1 ⁵ ,1 ⁹)	=	$\{12^2 2^{-2}, 12^3 2^{-2}, 12^4 2^{-2}, 5^2, [3,2]1^2, [4,1]1^2, [1,2], [1,3], [1,5], [2,4], [2,5], [3,4], [3,5], [4,5]\}$
64,104,(1 ⁵ ,1 ⁹)	=	$\{4^2, 12^2 2^{-2}, 12^3 2^{-2}, 5^2, [3,2]1^2, [4,1]1^2, [1,2], [1,3], [1,5], [2,4], [2,5], [3,4], [3,5], [4,5]\}$
64,105,(1 ⁵ ,1 ⁹)	=	$\{2^2, 3^2, 4^2, 5^2, [4,3]1^2, [5,2]1^2, [1,2], [1,3], [1,4], [1,5], [2,3], [2,4], [3,5], [4,5]\}$
64,106,(1 ³ 2,1 ⁵)	=	$\{12, 3^2, 4^4, [3,2]2^2, [4,1]2^2, [1,2], [1,3], [2,4], [3,4]\}$
64,107,(1 ³ 2,1 ⁵)	=	$\{12^2 2^{-2}, 12^3 2^{-2}, 4^4, [3,2]1^2, [4,1]1^2, [1,2], [1,3], [2,4], [3,4]\}$
64,108,(1 ³ 2,1 ⁵)	=	$\{12, 22^3 2^{-2}, 4^4, [3,2]2^2, [4,1]2^2, [1,2], [1,3], [2,4], [3,4]\}$
64,109,(1 ³ 2,1 ⁵)	=	$\{12, 2^2, 3^2, [3,2]4^4, [4,1]4^4, [1,2], [1,3], [2,4], [3,4]\}$
64,110,(14,1 ⁵)	=	$\{12, 3^2, 4^2, [2,1]2^4, [2,3]2^2, [1,3], [1,4], [2,4], [3,4]\}$
64,111,(14,1 ⁵)	=	$\{12, 2^4 3^{-2}, 4^2, [2,1]2^4, [2,3]2^2, [1,3], [1,4], [2,4], [3,4]\}$
64,112,(14,1 ⁵)	=	$\{12^3 2^{-4}, 2^2, 4^2, [3,2]3^4, [3,4]3^2, [1,2], [1,3], [1,4], [2,4]\}$

- 64,113,(122,122) = $\{12, 3^2, [2, 1]2^4, [3, 2]^2 2^4, [1, 3], [2, 3, 2]\}$
- 64,114,(122,13) = $\{12^2 - 4, 12^3 - 2, [2, 1]2^4, [3, 2]^2 1^2, [1, 3], [2, 3, 2]\}$
- 64,115,(122,13) = $\{1^2, 2^4 3^{-2}, [2, 1]3^2, [3, 2]^2 2^4, [1, 3], [2, 3, 2]\}$
- 64,116,(122,12) = $\{1^2, 3^4, [2, 1]2^4, [2, 3]2^2, [1, 3]\}$
- 64,117,(122,13) = $\{1^4, 2^8, 3^2, [1, 2]2^4, [2, 3]2^2, [1, 3]\}$
- 64,118,(122,12) = $\{1^4, 3^2, [2, 1]2^4, [3, 2]2^2, [1, 3]\}$
- 64,119,(122,12) = $\{1^4, 2^4 3^{-2}, [2, 1]2^4, [2, 3]2^2, [1, 3]\}$
- 64,120,(122,13) = $\{1^2, 2^4, 3^2, [2, 1][2, 3]^2, [1, 3], [2, 3, 2]\}$
- 64,121,(122,12) = $\{1^2, 2^4 3^{-4}, [2, 1]2^4, [2, 3]2^2, [1, 3]\}$
- 64,122,(122,12) = $\{1^4 2^{-4}, 3^2, [2, 1]2^4, [2, 3]2^2, [1, 3]\}$
- 64,123,(122,14) = $\{1^2, 2^4, 3^2, [1, 2, 1], [1, 2, 2, 2], [1, 3], [2, 3]\}$
- 64,124,(122,13) = $\{1^2, 3^2, [1, 2, 1], [1, 2, 2]2^{-4}, [1, 3], [2, 3]\}$
- 64,125,(122,13) = $\{1^2 2^{-4}, 3^2, [1, 2, 1], [1, 2, 2]2^{-4}, [1, 3], [2, 3]\}$
- 64,126,(122,122) = $\{2^2, 3^4, [2, 3, 3]1^{-2}, [1, 2], [1, 3], [2, 3, 2]\}$
- 64,127,(122,13) = $\{1^2 3^{-4}, 2^2, [2, 3, 3]1^{-2}, [1, 2], [1, 3], [2, 3, 2]\}$
- 64,128,(22,13) = $\{1^4, 2^4, [1, 2]2^2, [1, 2, 2]^2, [1, 2, 2]2^{-1}\}$
- 64,129,(22,12) = $\{1^4, [1, 2]2^2, [1, 2, 2]2^{-4}, [1, 2, 1]\}$
- 64,130,(22,12) = $\{2^4, [1, 2]2^2, [1, 2, 2]1^{-4}, [1, 2, 1]\}$
- 64,131,(13,12) = $\{1^2, 2^8, [1, 2, 2]^2, [1, 2, 2]2^{-1}\}$
- 64,132,(13,12) = $\{2^8, [1, 2]2^2, [1, 2, 2]1^{-2}, [1, 2, 1]\}$
- 64,133,(13,1) = $\{1^2, [1, 2, 2]2^{-8}, [1, 2, 1]\}$
- 64,134,(13,13) = $\{1^{16}, 2^2, 3^2, [1, 2]1^2, [1, 3], [2, 3]\}$
- 64,135,(13,12) = $\{1^{16} 2^{-2}, 3^2, [2, 1^3]1^2, [1, 3], [2, 3]\}$
- 64,136,(13,12) = $\{1^{18} 2^{-2}, 3^2, [2, 1^7]1^2, [1, 3], [2, 3]\}$
- 64,137,(13,12) = $\{1^{22} 2^{-8}, 3^2, [2, 3]2^2, [1, 2], [1, 3]\}$
- 64,138,(12,12) = $\{1^{16}, 2^2, [2, 1]2^4, [1, 2, 1]\}$
- 64,139,(12,1) = $\{1^{18} 2^{-2}, [2, 1]2^4, [1, 2, 1]\}$
- 64,140,(12,1) = $\{1^{16}, 2^4, [1, 2]1^2\}$
- 64,141,(12,1) = $\{1^{16}, 2^4, [1, 2]1^{10}\}$
- 64,142,(12,1) = $\{2^2, [2, 1]2^4, [1, 2, 1]\}$
- 64,143,(12,0) = $\{1^{18} 2^{-4}, [1, 2]1^2\}$
- 64,144,(13,15) = $\{1^2, 2^2, 3^2, [1, 2]^2, [1, 3]^2, [2, 3]^2, [1, 2, 3], [1, 3, 2]\}$
- 64,145,(13,13) = $\{1^{22} 2^{-2}, 3^4, [2, 1]1^2, [3, 1][2, 3]3^2, [2, 3, 2], [2, 3, 3]\}$
- 64,146,(13,14) = $\{1^2, 2^2, 3^4, [3, 2][1, 3]3^2, [1, 2]^2, [1, 3]^4, [1, 3, 3]\}$
- 64,147,(13,14) = $\{1^{22} 2^{-2}, 3^2, [2, 1]1^2, [1, 3, 1], [1, 3, 3], [2, 3, 3]\}$
- 64,148,(13,13) = $\{2^4, 3^2, [1, 2, 3]1^2, [1, 3, 2]1^2, [1, 3][3, 2]2^2 1^2, [1, 2, 3]\}$
- 64,149,(13,12) = $\{1^4, 2^4, [1, 2]2^2 3^2, [1, 3]1^2, [3, 2]3^2\}$
- 64,150,(13,13) = $\{1^4, 2^4, 3^4, [1, 2]1^2, [1, 3]3^2, [2, 3]2^2\}$
- 64,151,(13,13) = $\{1^2, 2^4, [2, 1]3^2, [1, 3]2^2, [2, 3]^2, [2, 3, 3]\}$
- 64,152,(13,12) = $\{2^4, 3^4, [1, 2]3^2, [1, 3]1^2, [3, 2]2^2 1^2\}$
- 64,153,(13,12) = $\{1^{22} 3^{-2}, [2, 1][2, 3]1^{-2}, [3, 1][2, 3]2^2, [2, 3, 1], [2, 3, 2]\}$
- 64,154,(14,16) = $\{1^4, 2^2, 3^4, 4^2, [1, 2]1^2, [3, 4]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
- 64,155,(14,15) = $\{1^4, 2^2, 3^2 4^{-2}, [1, 2]1^2, [3, 4]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
- 64,156,(14,14) = $\{1^{22} 2^{-2}, 3^2 4^{-2}, [1, 2]1^2, [3, 4]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
- 64,157,(14,15) = $\{2^2, 3^2, 1^2 4^{-2}, [2, 1]1^2, [3, 4]^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
- 64,158,(14,15) = $\{3^4, 1^{22} 2^{-2}, 1^2 4^{-2}, [2, 1]1^2, [3, 4]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
- 64,159,(14,15) = $\{3^2, 1^{22} 2^{-2}, 1^2 4^{-2}, [2, 1]1^2, [3, 4]^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
- 64,160,(14,15) = $\{2^2, 3^4, 1^2 4^{-2}, [2, 1]1^2, [3, 4]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
- 64,161,(14,14) = $\{2^2, 1^2 3^2 4^{-2}, [2, 1]1^2, [4, 3]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$

64,162,(14,14)	=	$\{122^{-2}, 123^2 4^{-2}, [2, 1]1^2, [4, 3]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
64,163,(14,14)	=	$\{1^2, 3^2, [2, 1]4^2, [4, 3]2^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
64,164,(14,14)	=	$\{3^2, 124^{-2}, [2, 1]1^2, [4, 3]2^2 4^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
64,165,(14,14)	=	$\{123^2 2^{-2}, 224^{-2}, [2, 1]1^2, [4, 3]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
64,166,(14,132)	=	$\{124^{-2}, 223^{-2}, [2, 1]1^2, [4, 3]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
64,167,(14,14)	=	$\{124^{-2}, 3^2, [2, 1]1^2, [4, 3]2^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
64,168,(14,14)	=	$\{124^{-2}, 123^2 2^{-2}, [2, 1]1^2, [4, 3]3^2, [1, 3], [1, 4], [2, 3], [2, 4]\}$
64,169,(14,15)	=	$\{1^2, 2^2, 3^4, 4^2, [3, 4]3^2, [4, 1]2^2, [2, 3], [1, 2], [1, 3], [2, 4]\}$
64,170,(14,142)	=	$\{1^2, 3^2, 4^2, [3, 4]2^2, [4, 1]2^2, [3, 2]2^2, [1, 2], [1, 3], [2, 4]\}$
64,171,(14,15)	=	$\{1^2, 3^4, 4^2, [3, 4]3^2, [4, 1]2^2, [3, 2]2^2, [1, 2], [1, 3], [2, 4]\}$
64,172,(14,15)	=	$\{1^2, 4^2, 2^2, 3^{-2}, [3, 4]2^2, [4, 1]2^2, [3, 2]2^2, [1, 2], [1, 3], [2, 4]\}$
64,173,(14,14)	=	$\{1^2, 2^2, 3^2 4^{-2}, [4, 3]3^2, [1, 4]3^2, [1, 2], [1, 3], [2, 4]\}$
64,174,(14,14)	=	$\{1^2, 3^2 4^{-2}, [4, 1]2^2, [3, 2]2^2, [4, 3]3^2, [1, 2], [1, 3], [2, 4]\}$
64,175,(14,14)	=	$\{1^2, 3^2, 4^2, [4, 3]2^2, [4, 1]2^2, [2, 3], [1, 2], [1, 3], [2, 4]\}$
64,176,(14,14)	=	$\{3^2, 2^2 4^{-2}, [4, 1]1^2, [3, 2]1^2, [4, 3]4^2, [1, 2], [1, 3], [2, 4]\}$
64,177,(14,14)	=	$\{1^2, 4^2, [4, 1]3^2, [3, 2]3^2, [4, 3]2^2, [1, 2], [1, 3], [2, 4]\}$
64,178,(14,14)	=	$\{3^2, 4^2, [4, 1]1^2, [3, 2]1^2, [4, 3]2^2, [1, 2], [1, 3], [2, 4]\}$
64,179,(14,14)	=	$\{3^2, 1^2 2^2 4^{-2}, [4, 1]1^2, [3, 2]1^2, [4, 3]2^2, [1, 2], [1, 3], [2, 4]\}$
64,180,(22,12)	=	$\{2^4, [2, 1]2^2 1^4, [1, 2, 1], [1, 2, 2]\}$
64,181,(22,12)	=	$\{14^2^{-4}, [2, 1]2^2 1^4, [1, 2, 1], [1, 2, 2]\}$
64,182,(22,0)	=	$\{14^2^{-16}, [2, 1]2^4\}$
64,183,(14,1222)	=	$\{3^2, 4^2, [3, 1]1^2, [1, 4]2^{-2} 1^2, [2, 3]2^{21-2}, [4, 2]2^2, [1, 2], [3, 4]\}$
64,184,(14,14)	=	$\{3^2, 4^2, [3, 1]1^2, [4, 1]2^2, [3, 2]2^2, [2, 4]2^{21-2}, [1, 2], [3, 4]\}$
64,185,(14,14)	=	$\{4^2, 1^2 2^2 3^{-2}, [3, 1]1^2, [1, 4]2^{-2} 1^2, [2, 3]2^{21-2}, [4, 2]2^2, [1, 2], [3, 4]\}$
64,186,(14,14)	=	$\{3^2, 1^2 4^{-2}, [3, 1]1^2, [1, 4]2^{-2} 1^2, [2, 3]2^{21-2}, [4, 2]2^2, [1, 2], [3, 4]\}$
64,187,(14,14)	=	$\{1^2 2^2 4^{-2}, 2^2 3^{-2}, [3, 1]1^2, [1, 4]2^{-2} 1^2, [2, 3]2^{21-2}, [4, 2]2^2, [1, 2], [3, 4]\}$
64,188,(13,122)	=	$\{14, 2^8, 3^2, [1, 2], [1, 3]1^2, [2, 3]2^2\}$
64,189,(13,12)	=	$\{14, 3^2, [1, 2], [1, 3]1^2, [3, 2]2^2\}$
64,190,(13,12)	=	$\{123^{-2}, 2^8, [1, 2], [3, 1]1^2, [2, 3]2^2\}$
64,191,(13,2)	=	$\{123^{-2}, [1, 2], [3, 1]1^2, [3, 2]2^2\}$
64,192,(13,12)	=	$\{14, 2^4 3^{-2}, [1, 2], [1, 3]1^2, [2, 3]2^2\}$
64,193,(13,12)	=	$\{2^8, 3^2, [1, 2], [3, 1]1^2, [2, 3]2^{21-2}\}$
64,194,(13,2)	=	$\{2^4 3^{-2}, [1, 2], [3, 1]1^2, [2, 3]1^{-2} 2^2\}$
64,195,(13,13)	=	$\{1^2, 2^8, 3^2, [1, 2], [2, 3]2^2, [1, 3, 3]\}$
64,196,(13,12)	=	$\{1^2, 3^2, [1, 2], [3, 2]2^2, [1, 3, 3]\}$
64,197,(13,12)	=	$\{1^2, 2^4 3^{-2}, [1, 2], [2, 3]2^2, [1, 3, 3]\}$
64,198,(13,12)	=	$\{14, 3^2, [1, 2], [2, 3]2^2, [1, 3]2^{-4} 1^2\}$
64,199,(13,1)	=	$\{1^2 3^{-2} 2^4, [1, 2], [1, 3]3^2, [2, 3]2^2\}$
64,200,(13,2)	=	$\{3^2, [1, 2], [3, 1]1^2 2^4, [3, 2]1^2 2^2\}$
64,201,(13,14)	=	$\{1^2, 2^8, 3^2, [1, 2]2^2, [2, 3]2^2, [1, 3], [1, 2, 2]\}$
64,202,(13,13)	=	$\{1^2, 3^2, [1, 2]2^2, [3, 2]2^2, [1, 3], [1, 2, 2]\}$
64,203,(13,13)	=	$\{1^2, 2^4 3^{-2}, [1, 2]2^2, [3, 2]2^2, [1, 3], [1, 2, 2]\}$
64,204,(13,13)	=	$\{2^4 3^{-2}, 1^2, [1, 2]2^2, [2, 3]2^2, [1, 3], [1, 2, 2]\}$
64,205,(13,13)	=	$\{14, 2^8, 3^2, [1, 2]1^2, [2, 3]2^2, [1, 3]\}$
64,206,(13,12)	=	$\{14, 2^4 3^{-2}, [1, 2]1^2, [3, 2]2^2, [1, 3]\}$
64,207,(13,12)	=	$\{14, 3^2, [1, 2]1^2, [3, 2]2^2 1^2, [1, 3]\}$
64,208,(13,12)	=	$\{14, 3^2, [1, 2]1^2, [3, 2]2^2, [1, 3]\}$
64,209,(13,12)	=	$\{14, 3^2, [1, 2]1^2, [3, 2]1^2 2^2, [1, 3]\}$
64,210,(13,1)	=	$\{2^4 3^{-2}, [1, 2]1^2, [3, 2]2^2 1^2, [1, 3]\}$

64,211,(1 ³ ,1 ²)	=	{1 ⁴ ,2 ⁴³ - ² ,[1,2]1 ² ,[2,3]2 ² ,[1,3]}
64,212,(1 ³ ,1)	=	{2 ⁴³ - ² ,[1,2]1 ² ,[3,2]2 ⁻² 1 ² ,[1,3]}
64,213,(1 ³ ,1 ²)	=	{1 ² ,2 ⁸ ,[2,1]3 ² ,[2,3]2 ² ,[1,3]}
64,214,(1 ³ ,1 ²)	=	{1 ² ,2 ⁸ ,[2,1]3 ² 2 ⁴ ,[3,2]2 ² ,[1,3]}
64,215,(1 ³ ,1)	=	{1 ² ,[2,1]3 ² ,[3,1]2 ⁸ ,[3,2]2 ² }
64,216,(1 ³ ,1)	=	{1 ² ,[2,1]2 ⁴³ 2,[1,3],[2,3]2 ² }
64,217,(1 ³ ,1 ³)	=	{1 ²² - ⁴ ,3 ² ,[2,3]2 ² ,[1,2,1],[1,2,2],[1,3]}
64,218,(1 ³ ,1 ³)	=	{1 ²² - ⁴ ,1 ²³ - ² ,[2,3]2 ² ,[1,2,1],[1,2,2],[1,3]}
64,219,(1 ³ ,1 ²)	=	{2 ⁸ ,3 ² ,[2,1]1 ² 2 ⁴ ,[2,3]2 ² ,[1,3]}
64,220,(1 ³ ,1 ²)	=	{1 ⁴ ,2 ⁴³ - ² ,[2,1]2 ⁴ 1 ² ,[2,3]2 ² ,[1,3]}
64,221,(1 ³ ,1)	=	{3 ² ,[1,2]2 ⁻⁴ 1 ² ,[1,3],[2,3]1 ⁻² 2 ⁶ }
64,222,(1 ³ ,1)	=	{2 ⁴³ - ² ,[1,2]2 ⁻⁴ 1 ² ,[2,3]1 ⁻² 2 ⁶ ,[1,3]}
64,223,(1 ³ ,2)	=	{1 ²² - ⁴ ,[2,1]2 ⁴³ 2,[2,3]2 ² ,[1,3]}
64,224,(1 ³ ,1)	=	{1 ²² - ⁴ ,[2,1]3 ² ,[2,3]2 ² ,[1,3]}
64,225,(1 ³ ,1 ²)	=	{1 ⁴ ,3 ² ,[2,1]2 ⁴ ,[1,3]1 ² ,[3,2]2 ⁶ }
64,226,(1 ³ ,1 ²)	=	{1 ⁴ ,2 ⁴³ - ² ,[2,1]2 ⁴ ,[1,3]1 ² ,[2,3]2 ² }
64,227,(1 ³ ,1)	=	{1 ²³ - ² ,[2,1]2 ⁴ ,[1,3]1 ² ,[2,3]2 ² }
64,228,(1 ³ ,1 ²)	=	{1 ² ,3 ² ,[1,3]2 ² ,[2,1]2 ⁴ ,[2,3]2 ² }
64,229,(1 ³ ,1 ²)	=	{1 ² ,3 ⁴ ,[2,1]2 ⁴ ,[1,3]3 ² ,[2,3]2 ² }
64,230,(1 ³ ,1 ²)	=	{1 ² ,2 ⁴³ - ² ,[1,3]2 ² ,[2,1]2 ⁴ ,[2,3]2 ² }
64,231,(1 ³ ,2)	=	{3 ² ,[2,1]2 ⁴ ,[1,3]1 ² ,[2,3]2 ¹ - ² 2}
64,232,(1 ³ ,2)	=	{1 ²³ - ² ,[2,1]2 ⁴ ,[1,3]1 ² ,[3,2]2 ⁻¹ 1 ² 2 ⁷ }
64,233,(1 ³ ,2)	=	{2 ⁴³ - ² ,[2,1]2 ⁴ ,[1,3]1 ² ,[2,3]2 ¹ - ² 2}
64,234,(1 ² ,1 ²)	=	{2 ⁸ ,[2,1]2 ¹ 1 ² ,[1,2,1]1 ⁻² ,[1,2,2]2 ⁴ 1 ⁻² }
64,235,(1 ² ,1)	=	{1 ²² - ⁴ ,[1,2,1][2,1]2 ² ,[1,2,2][2,1]2 ² - ⁴ }
64,236,(1 ² ,1)	=	{[2,1]2 ² 1 ² ,[1,2,1]1 ⁻² ,[1,2,2]2 ⁻⁴ 1 ⁻² }
64,237,(1 ² ,1 ²)	=	{1 ² ,2 ⁴ ,[1,2,2,1],[1,2,2,2]}
64,238,(1 ² ,1)	=	{2 ⁴ ,[2 ² ,1 ⁻¹ 2 ¹],[2 ¹ 1 ⁻¹ 2 ¹]2 ⁴ }
64,239,(1 ² ,1)	=	{1 ² ,[1,2 ⁴],[2,1 ⁻¹ 2 ¹]2 ⁴ }
64,240,(1 ² ,0)	=	{[1,2,1]1 ⁻⁴ ,[1,2]2 ² - ² 1 ⁴ }
64,241,(1 ⁴ ,1 ⁵)	=	{1 ⁴² - ² ,1 ⁴³ - ² ,1 ⁴⁴ - ² ,[1,4]1 ² ,[3,2]1 ⁴ ,[1,2],[1,3],[2,4],[3,4]}
64,242,(1 ⁴ ,1 ⁵)	=	{1 ⁴² - ² ,1 ⁴³ - ² ,4 ² ,[4,1]1 ² ,[3,2]1 ⁴ ,[1,2],[1,3],[2,4],[3,4]}
64,243,(1 ⁴ ,1 ⁵)	=	{1 ⁴² - ² ,1 ⁴³ - ² ,4 ² ,[1,4]1 ² ,[3,2]1 ⁴ ,[1,2],[1,3],[2,4],[3,4]}
64,244,(1 ³ ,1 ²)	=	{1 ² ,3 ² ,[2,1]2 ⁸ ,[3,2]2 ¹⁴ ,[1,3]}
64,245,(1 ³ ,1 ²)	=	{1 ² ,2 ⁸³ - ² ,[2,1]2 ⁸ ,[2,3]2 ² ,[1,3]}
64,246,(1 ² ,0)	=	{1 ¹⁶² - ⁴ ,[2,1]1 ² }
64,247,(1 ² ,1)	=	{2 ² ,[2,1]2 ¹⁴ ,[1,2,1]1 ⁻⁸ }
64,248,(1 ² ,1)	=	{2 ² ,[1,2]2 ¹⁴ ,[1,2,1]1 ⁻⁸ }
64,249,(1 ² ,1)	=	{2 ²¹ - ⁸ ,[2,1]2 ¹⁴ ,[1,2,1]1 ⁻⁸ }
64,250,(1 ² ,1 ²)	=	{1 ² ,2 ⁴ ,[1,2 ⁻¹ 1 ²],[1,2 ⁻² 1 ² 2]}
64,251,(1 ² ,1)	=	{2 ⁴ ,[1,2 ⁻¹ 1 ²],[1 ² - ¹ 1 ²]3 ¹ - ¹ 2 ⁻¹ }
64,252,(1 ² ,1 ²)	=	{1 ⁴ ,2 ⁴ ,[1,2,1],[1,2,2][2,1]1 ² }
64,253,(1 ² ,1 ²)	=	{1 ⁴ ,2 ⁴ ,[1,2,1],[1,2,2]1 ² }
64,254,(1 ² ,1)	=	{[2,1]2 ²⁴ ,[1,2,1],[1,2,2]1 ⁻² }
64,255,(1 ² ,1)	=	{[2,1]2 ²⁴ ,[1,2,1],[1,2,2]1 ⁻² 2 ⁻⁴ }
64,256,(1 ³ ,1 ²)	=	{1 ⁴² - ² ,1 ⁴³ - ⁴ ,[1,2],[1,3]1 ² ,[2,3]3 ² }
64,257,(1 ³ ,1 ²)	=	{1 ⁴² - ² ,1 ⁴³ - ⁴ ,[1,2],[3,1]1 ² ,[3,2]3 ² }
64,258,(1 ³ ,1 ²)	=	{1 ⁴² - ² ,1 ⁴³ - ⁴ ,[1,2],[1,3]1 ² ,[3,2]3 ² }
64,259,(1 ³ ,1 ⁴)	=	{1 ² ,2 ² ,3 ² ,[1,2]2 ² ,[1,3]2 ² ,[2,3],[1,2,3][1,3,2]1 ⁻¹ }

$$\begin{aligned}
64,260, (1^3, 1^3) &= \{2^2, 3^2, [2, 3], [1, 2, 2], [1, 3, 3], [1, 3, 2]1^{-2}\} \\
64,261, (1^3, 1^3) &= \{1^4, 2^2, 3^2, [1, 2]1^2, [2, 3], [1, 3, 1]\} \\
64,262, (1^3, 1^3) &= \{1^4, 2^2, 3^2, [2, 1][1, 3]1^2, [1, 3, 1], [2, 3]\} \\
64,263, (1^3, 1^3) &= \{1^4, 3^2, [2, 1]1^2 2^2, [3, 1]2^2 2^2, [2, 3], [1, 3, 1]\} \\
64,264, (1^3, 1^3) &= \{1^4, 3^2, [3, 1]2^2 2^2, [1, 2]1^2, [2, 3], [1, 3, 1]\} \\
64,265, (1^2, 1) &= \{1^{3^2}, 2^2, [1, 2]1^2\} \\
64,266, (1^2, 0) &= \{1^{3^2} 2^{-2}, [2, 1]1^2\} \\
64,267, (1^2, 0) &= \{2^{2^1-16}, [1, 2]1^2\}
\end{aligned}$$

Reference

- [1] Marshall Hall, Jr. & James K. Senior, *The groups of order 2^n ($n \leq 6$)* (Macmillan).

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