Preface

The style of this book is a bit idiosyncratic. The results that interest us belong to number theory, but the emphasis in the proofs will be on the probabilistic aspects and on the interaction between number theory and probability theory. In fact, we attempt to write the proofs so that they use *as little arithmetic as possible*, in order to clearly isolate the crucial number-theoretic ingredients that are involved.

This book is quite short. We attempt to foster an interest in the topic by focusing on a few key results that are accessible and at the same time particularly appealing, in the author's opinion, without targeting an encyclopedic treatment of any. We also try to emphasize connections to other areas of mathematics – first, to a wide array of arithmetic topics, but also to some aspects of ergodic theory, expander graphs, and so on.

In some sense, the ideal reader of this book is a student who has attended at least one introductory advanced undergraduate or beginning graduate-level probability course, including especially the Central Limit Theorem, and maybe some aspects of Brownian motion, and who is interested in seeing how probability interacts with number theory. For this reason, there are almost no number-theoretic prerequisites, although it is helpful to have some knowledge of the distribution of primes.

Probabilistic number theory is currently evolving very rapidly, and uses more and more refined probabilistic tools and results. For many number theorists, we hope that the detailed and motivated discussion of basic probabilistic facts and tools in this book will be useful as a basic "toolbox".

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$$\sum_{n=1}^{+\infty} \frac{1}{n^2} = \frac{\pi^2}{6},$$

in Exercise 1.3.4. Thanks to V. Tassion for help with the proof of Proposition B.11.11 and to G. Ricotta and E. Royer for pointing out a small mistake in [79].

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