Can. J. Math., Vol. XXVII, No. 2, 1975, pp. 311–318

EXTENSIVE SUBCATEGORIES OF THE CATEGORY OF T_0-SPACES

SUNG SA HONG

Introduction. It is well known that epimorphisms in the category Top (Top_1, respectively) of topological spaces (T_1-spaces, respectively) and continuous maps are precisely onto continuous maps. Since every mono-reflective subcategory of a category is also epi-reflective and every embedding in Top (Top_1, respectively) is a monomorphism, there is no nontrivial reflective subcategory of Top (Top_1 respectively) such that every reflection is an embedding. However, in the category Top_0 of T_0-spaces and continuous maps as well as in the category Haus of Hausdorff spaces and continuous maps, there are epimorphisms which are not onto. Moreover, every reflection of a reflective subcategory of Top_0, which contains a non T_1-space, is an embedding [**16**]. For an epi-reflective subcategory $\mathscr{B}$ of Haus, there is a hereditary subcategory $R\mathscr{B}$ of Haus such that $\mathscr{B}$ is extensive in $R\mathscr{B}$, i.e. every reflection is an extension. Using extensive operators, we have been able to characterize every extensive subcategory of a hereditary subcategory of Haus [**11**; **13**]. In this paper, we deal with all extensive subcategories of Top_0. We introduce idempotent semi-limit-operators. With these, we can also characterize all extensive subcategories of Top_0. In this approach, one of the main advantages is that by using the trace filters, in this case union filters, one can easily characterize extensive subcategories of Top_0 and every reflection is precisely given. Moreover, one can associate an extensive subcategory of Top_0 with a coreflective subcategory of Top. We obtain also some interesting results about the front closure operator which determines the coreflective subcategory of Top generated by indiscrete spaces.

All topological and categorical concepts will be used in the sense of N. Bourbaki [**4**] and H. Herrlich [**7**], respectively. In particular, we assume throughout this paper that a subcategory of a category is full and isomorphism-closed. The closure of a subset A of a topological space X will be written $\text{cl}_X A$ (cl A when no confusion is possible) and for $x \in X$, cl x means cl $\{x\}$, which will be called point closure.

The author takes this opportunity to thank Professor L. D. Nel for many helpful discussions.

1. Front closure. The following definition is due to S. Baron [**3**] and L. Skula [**16**].

Received August 28, 1973 and in revised form, November 18, 1974.

1.1 *Definition.* Let X be a topological space. For $A \subseteq X$, let

$$\mathrm{fcl}_X A = \{x \in X \mid \text{for any neighborhood } V \text{ of } x,\ V \cap \mathrm{cl}\, x \cap A \neq \emptyset\}.$$

Then the operator fcl is called the *front closure operator.*

It is known that the front closure operator fcl satisfies the Kuratowski axioms. We shall give the usual names preceded by front to the topological concepts with respect to the topology defined by the front closure operator. H. Herrlich [**8**] has defined limit-operators with which he has been able to construct every coreflective subcategory of Top. The second Proposition on p. 205 and the first statement of the Theorem on p. 206 [**8**], were incorrect (see the following proposition).

1.2 PROPOSITION. *The front closure operator* fcl *is an idempotent limit-operator and the subcategory* $\mathscr{C}$(fcl) *of* Top *determined by*

$$\{X \in \mathrm{Top} \mid \text{each subset } A \text{ of } X \text{ with } \mathrm{fcl}_X A = A \text{ is closed in } X\}$$

is the category of all coproducts of indiscrete spaces.

Proof. The first part is immediate from the fact that $g(\mathrm{fcl}_X A) \subseteq \mathrm{fcl}_Y g(A)$ for any continuous map $g: X \to Y$ and $A \subseteq X$.

For the second part, it is enough to show that a topological space X belongs to $\mathscr{C}$(fcl) if and only if cl x is open for every $x \in X$ (see [**8**, II, (2)]). Suppose X belongs to $\mathscr{C}$(fcl). For any $x \in X$ and any $y \in \mathrm{cl}\, x$, $\mathrm{cl}\, y \cap \mathbf{C}\, \mathrm{cl}\, x = \emptyset$, where $\mathbf{C}A$ means the complement of A. Hence $y \notin \mathrm{fcl}_X \mathbf{C}\, \mathrm{cl}\, x$, i.e., $\mathbf{C}\, \mathrm{cl}\, x$ is closed; cl x is open. Noting that fcl_X in a space X is precisely the original closure operator provided that every point closure in X is open, the converse is also true.

It is well known [**5**] that any morphism $f: X \to Z$ in the category Haus, whose restriction to a dense subset Y of X is a homeomorphism, carries $X - Y$ into $Z - f(Y)$. Obviously it is not so in the category Top_0.

1.3 THEOREM. *Let Y be a front-dense subspace of a T_0-space X and let $f: X \to Z$ be a continuous map. If the restriction of f to Y is a homeomorphism, then* $f(X - Y) \subseteq Z - f(Y)$.

Proof. Suppose, on the contrary, that $f(x) = f(y)$ for some $y \in Y$ and $y \neq x \in X$. Suppose $y \notin \mathrm{cl}\, x$. Then there is an open neighborhood W of $f(y)$ with $f(Y \cap \mathbf{C}\, \mathrm{cl}\, x) = W \cap f(Y)$. Since Y is front-dense in X, every neighborhood of x contains points of $Y \cap \mathrm{cl}\, x$. Then the hemeomorphism $f|Y$ takes all such points into $Z - W$; f is not continuous at x. Hence $y \in \mathrm{cl}\, x$. Since X is T_0-space, $x \notin \mathrm{cl}\, y$. Since $\mathbf{C}\, \mathrm{cl}\, y$ is a neighborhood of x, $Y \cap \mathrm{cl}\, x \cap \mathbf{C}\, \mathrm{cl}\, y \neq \emptyset$. Let p be an element of the set. Then $f(p)$ does not belong to $\mathrm{cl}_{f(Y)} f(y)$. On the other hand, $f(p)$ belongs to $\mathrm{cl}_Z f(x) \cap f(Y) = \mathrm{cl}_{f(Y)} f(x) = cl_{f(Y)} f(y)$, which is a contradiction. This completes the proof.

By the same argument as in [**5**], one has:

https://doi.org/10.4153/CJM-1975-037-7 Published online by Cambridge University Press

1.4 COROLLARY. *Every retract of a T_0-space X is front-closed in X. In particular, the graph of a morphism $f: X \to Y$ in* Top_0 *is front-closed in $X \times Y$, and if a T_0-space Y contains a product $X = \times X_\iota$ and each projection $pr_\iota: X \to X_\iota$ has a continuous extension to Y, then X is front-closed in Y.*

2. Quasi-sobre spaces and sobre spaces.

2.1 *Definition.* A closed subset of a topological space is said to be *irreducible* if it cannot be expressed as the union of two proper closed subsets. A topological space is said to be *quasi-sobre* if every non-empty irreducible closed subset of the space is a point closure. A T_0 quasi-sobre space is called *sobre.*

We note that sobre spaces have been also called *pc*-spaces in [**14**], or spectral spaces in [**9**].

2.2 PROPOSITION. *A topological space is quasi-sobre if and only if its* Top_0-*reflection space is sobre.*

Proof. For any topological space X, let $\tau_X: X \to \tau X$ be a Top_0-reflection of X. It is known that τX is a quotient space $X/\sim$ of X, where $x \sim y$ if and only if $\mathrm{cl}\, x = \mathrm{cl}\, y$. Suppose X is quasi-sobre. Let F be a non-empty irreducible closed subset of τX. Since τ_X is closed and onto, and every closed subset of X is saturated with respect to the equivalence relation $\sim$, $\tau_X{}^{-1}(F)$ is also a non-empty irreducible closed subset; $\tau_X{}^{-1}(F) = \mathrm{cl}_X x$ for some $x \in X$. Hence $F = \tau_X(\tau_X{}^{-1}(F)) = \tau_X(\mathrm{cl}_X x) = \mathrm{cl}_{\tau X}\tau_X(x)$.

Conversely, let G be a non-empty irreducible closed subset of X. Since τ_X is closed, $\tau_X(G)$ is also a non-empty irreducible closed subset of τX. Since τX is sobre, there is a point x of G with $\tau_X(G) = \mathrm{cl}_{\tau X}\tau_X(x)$. Using the fact that τ_X is open, it is easy to show that $G = \mathrm{cl}_X x$. We omit the detail of the proof.

It is well known [**6**; **9**; **14**] that the subcategory Sob of Top_0 determined by sobre spaces is epi-reflective in Top_0. Using the limit-operator fcl, we give here another proof.

For any set X, let $\mathscr{P}_0(X)$ denote the set of all non-empty subsets of X. Let X be a non-empty topological space. We define a space $\tilde{X}$ as follows: its underlying set is $\mathscr{P}_0(X)$ and its topology has $\{\mathscr{P}_0(F)|F$: closed subset of $X\}$ as a subbase for the closed subsets. It is then obvious that the map $x \mapsto \{x\}$ is an embedding of X into $\tilde{X}$ and for any continuous map $f: X \to Y$, the map $\tilde{f}: \tilde{X} \to \tilde{Y}$ defined by $M \mapsto f(M)$ is also continuous (see [**4**, Ex. 7, §2]). We now show that $\tilde{X}$ is a quasi-sobre space. Let Λ be a non-empty irreducible closed subset of $\tilde{X}$;

$$\Lambda = \bigcap_{\iota \in I} \left\{ \bigcup_{j=1}^{n_\iota} \mathscr{P}_0(F_{\iota,j}) | n_\iota\text{: natural number and } F_{\iota,j}\text{: non-empty closed subset of } X \right\}.$$

https://doi.org/10.4153/CJM-1975-037-7 Published online by Cambridge University Press

Since Λ is irreducible, we may assume that

$$\Lambda = \bigcap_{\iota\in I} \mathscr{P}_0(F_\iota) = \mathscr{P}_0\left(\bigcap_{\iota\in I} F_\iota\right).$$

Since $\mathrm{cl}_{\tilde{X}}M = \mathscr{P}_0(\mathrm{cl}_{\tilde{X}}M)$ for every $M \in \tilde{X}$, $\Lambda = \mathrm{cl}_{\tilde{X}}\{\bigcap_{\iota\in I}F_\iota\}$. We note that $\tilde{X}$ is T_0 if and only if X is discrete.

Let $\mathscr{E}(X)$ be the Top_0-reflection space of $\tilde{X}$. By Lemma 1 in [**2**], the map $\mathscr{E}_X: X \to \mathscr{E}(X) = X \to \tilde{X} \to \mathscr{E}(X)$ is also an embedding if X is T_0. Since we may assume that $\mathscr{E}_X$ is the natural embedding, one has by Theorem 3.2 in [**14**] and Proposition 2.2 the following.

2.3 Proposition. *A* T_0-*space X is sobre if and only if X is front-closed in* $\mathscr{E}(X)$.

2.4 Theorem. *The subcategory* Sob *of all sobre spaces is epi-reflective in the category* Top_0.

Proof. For any $X \in \mathrm{Top}_0$, let πX be the subspace of $\mathscr{E}(X)$ whose underlying set is $\mathrm{fcl}_{\mathscr{E}(X)}X$ and $\pi_X: X \to \pi X$ be the natural embedding of X into πX. Since πX is a front-closed subspace of the sobre space $\mathscr{E}(X)$, πX is sobre. Moreover, the map $\pi_X: X \to \pi X$ is an epimorphism (see [**3**]). For any $Y \in$ Sob and any $g: X \to Y$ in Top_0, we have the following commutative diagram:

$$\begin{array}{ccccc} X & \xrightarrow{\pi_X} & \pi X & \xhookrightarrow{j} & \mathscr{E}(X) \\ {\scriptstyle g}\downarrow & & & & \downarrow{\scriptstyle \tilde{g}^{\sim}} \\ Y & = & \mathrm{fcl}_{\mathscr{E}(Y)}Y & \hookrightarrow & \mathscr{E}(Y), \end{array}$$

where $\tilde{g}^{\sim}$ is determined by $\tilde{g}$ and the reflection property. Since fcl is a limit-operator, $\tilde{g}^{\sim}(\pi X) = \tilde{g}^{\sim}(\mathrm{fcl}_{\mathscr{E}(X)}X) \subseteq \mathrm{fcl}_{\mathscr{E}(Y)}\tilde{g}^{\sim}(X) \subseteq \mathrm{fcl}_{\mathscr{E}(Y)}Y = Y$. Hence there is a unique morphism $g^\pi: \pi X \to Y$ such that $\tilde{g}^{\sim}j$ is g^π followed by the natural embedding $Y \to \mathscr{C}(Y)$. It is obvious that $g^\pi\pi_X = g$.

2.4 *Remark.* (1) The space $\mathscr{C}(X)$ is actually the subspace of $\tilde{X}$ with the set of all non-empty closed subsets as its underlying set and the map $\mathscr{E}_X: X \to \mathscr{C}(X)$ is defined by $x \mapsto \mathrm{cl}\, x$.

(2) A non-empty closed subset A belongs to the front-closure πX of X in $\mathscr{E}(X)$ if and only if for every finite family $\{U_1, \ldots, U_n\}$ of open subsets with $U_\iota \cap A \neq \emptyset$ $(i = 1, \ldots, n)$, $\bigcap_\iota U_\iota \cap A \neq \emptyset$ if and only if $\bigcup_{x\in A}O(x)$ is a (proper) open filter on X, where $O(x)$ denotes the open neighborhood filter of x if and only if A is irreducible. Hence πX is the subspace of $\mathscr{E}(X)$ with the set of all non-empty irreducible closed subsets as its underlying set (see [**6**]). For any closed subset F of X, let Σ_F denote $\mathscr{P}_0(F) \cap \pi X$. Then it is obvious that $\{\Sigma_F | F$: closed subset of $X\}$ is precisely the family of closed subsets of πX. Moreover, the correspondence $F \mapsto \Sigma_F$ is a lattice isomorphism. Hence X is

quasi-compact (Lindelöf, of second countability, connected respectively) if and only if πX is (see [**17**]).

2.5 *Definition.* An open filter on a T_0-space X is called a *union filter* if it is a union of open neighborhood filters on X.

2.6 *Remark.* For any $A \in \pi X$, its trace filter $T(A)$ on X is precisely the union filter $\cup_{x\in A} O(x)$ and πX is the strict extension of X with all union filters as the filter trace (see [**1**]). Hence a T_0-space X is sobre if and only if every union filter on X is already the open neighborhood filter of some point of X.

Likewise union filters, if $\vee_{x\in A} O(x)$ (see [**2**]) is a (proper) open filter, then so is $\vee_{x\in \mathrm{cl} A} O(x)$ and

$$\bigvee_{x\in \mathrm{cl} A} O(x) = \bigvee_{x\in A} O(x).$$

We note that a non-empty closed set A belongs to the closure of X in $\mathscr{C}(X)$ if and only if $\vee_{x\in A} O(x)$ is a proper open filter, and that the trace filter of $A \in \mathrm{cl}_{\mathscr{C}(X)} X$ on X is precisely the join filter $\vee_{x\in A} O(x)$. Contrary to the case of πX, the extension $\mathrm{cl}_{\mathscr{C}(X)} X$ of X is not relatively T_0 (see [**1**]).

3. Extensive subcategories of Top_0.

3.1 *Definition.* A subcategory $\mathscr{B}$ of Top_0 is said to be *extensive* in Top_0 if $\mathscr{B}$ is a reflective subcategory of Top_0 such that every $\mathscr{B}$-reflection map $r_X: X \to rX$ is an embedding for each $X \in \mathrm{Top}_0$.

3.2 *Remark.* (1) Since every extensive subcategory $\mathscr{B}$ is epi-reflective and epimorphisms in Top_0 are exactly front-dense continuous maps (see [**3**]), every $\mathscr{B}$-reflection map $r_X: X \to rX$ of $X \in \mathrm{Top}_0$ is an extension.

(2) Likewise H-closed spaces, every sobre space is front-closed in its T_0 superspace (see [**14**]). Thus every extensive subcategory of Top_0 contains all sobre spaces and obviously the category Sob is the smallest extensive subcategory of Top_0. Moreover every reflective subcategory of Top_0 containing Sob is also extensive in Top_0.

(3) Using Theorem 1.3 and Corollary 1.4, one can directly show that every extensive subcategory of Top_0 is front-closed-hereditary and productive.

3.3 *Definition.* An operator l which associates every pair (X, A), where X is a sobre space and A is a subset of X, a subset $l_X A$ of X is said to be an *idempotent semi-limit-operator* if l satisfies the following conditions:

(1) if A is a subset of a sobre space X, then $A \subseteq l_X A \subseteq \mathrm{cl}_X A$;

(2) if $f: X \to Y$ is a morphism in the category Sob and A is a subset of X then $f(l_X A) \subseteq l_Y f(A)$;

(3) if A is a subset of a sobre space X, then $l_X(l_X A) = l_X A$.

It is obvious that the restriction of an idempotent limit-operator to the category Sob is an idempotent semi-limit-operator.

https://doi.org/10.4153/CJM-1975-037-7 Published online by Cambridge University Press

For an idempotent semi-limit-operator l, a subset A of a sobre space X with $l_X A = A$ will be called l-closed.

Hereafter, by an extension space of a space X is meant a space of which X is a dense subspace.

Let l be an idempotent semi-limit-operator and let Sob_l be the subcategory of Top_0 determined by T_0-spaces which are l-closed in their Sob-reflection spaces.

3.4 THEOREM. *A subcategory $\mathscr{B}$ of* Top_0 *is extensive if and only if $\mathscr{B}$ is of the form* Sob_l *for some idempotent semi-limit-operator l.*

Proof. For any $X \in \mathrm{Top}_0$, let $\pi_X: X \to \pi X$ be the Sob-reflection of X such that X is a subspace of πX and π_X is the natural embedding.

$\Leftarrow$ Let $\pi^l X$ be the subspace of πX with $l_{\pi X} X$ as its underlying set. Obviously, the natural embedding $j: \pi^l X \to \pi X$ is a Sob-reflection of $\pi^l X$; hence $\pi^l X$ belongs to Sob_l. Now we wish to show that the natural embedding $\pi_X{}^l: X \to \pi^l X$ is the Sob_l-reflection of X. For any $Y \in \mathrm{Sob}_l$ and any morphism $f: X \to Y$ in Top_0, there is a unique $\bar{f}: \pi X \to \pi Y$ with $\bar{f}\pi_X = \pi_Y f$. Since $\bar{f}$ is a morphism in Sob and $l_{\pi Y} Y = Y$,

$$\bar{f}(\pi^l X) = \bar{f}(l_{\pi X} X) \subseteq l_{\pi Y}\bar{f}(X) \subseteq l_{\pi Y} Y = Y.$$

Let f^l be the restriction and corestriction of $\bar{f}$ to $\pi^l X$ and Y respectively. Then it is obvious that $f^l \pi_X{}^l = f$ and it is unique.

$\Rightarrow$ For any subset A of a sobre space X, define

$$l_X A = \cap\{B | A \subseteq B \text{ and } B \text{ is an object of } \mathscr{B} \text{ as a subspace of } X\}.$$

Since every extensive subcategory of Top_0 is front-closed-hereditary, $\mathscr{B}$ is, in particular, closed-hereditary. Hence $A \subseteq l_X A \subseteq \mathrm{cl}_X A$. For any $f: X \to Y$ in Sob and $B \subseteq Y$ with $B \in \mathscr{B}$ as a subspace of Y, $f^{-1}(B)$ is also an object of $\mathscr{B}$ as a subspace of X. Take $x \in l_X A$ and $B \supseteq f(A)$ with $B \in \mathscr{B}$. Since $f^{-1}(B) \supseteq A$ and $f^{-1}(B) \in \mathscr{B}$, $x \in f^{-1}(B)$, i.e., $f(x) \in B$. Hence $f(l_X A) \subseteq l_Y f(A)$. Since $\mathscr{B}$ is closed under the intersections, $l_X A = A$ if and only if $A \in \bar{B}$. Hence $l_X(l_X A) = l_X A$ and $\mathscr{B} = \mathrm{Sob}_l$.

3.5 *Remark.* For any extensive subcategory $\mathscr{B}$ of Top_0 and for any $X \in \mathrm{Top}_0$, its $\mathscr{B}$-reflection space is given by the intersection of all subspaces of πX which belong to $\mathscr{B}$ and contain X.

3.6 COROLLARY. *For any coreflective subcategory $\mathscr{C}$ of* Top, *let* $\mathrm{Sob}_{\mathscr{C}}$ *be the subcategory of* Top_0 *determined by T_0-spaces which are closed in the $\mathscr{C}$-coreflection spaces of their* Sob-*reflection spaces. Then* $\mathrm{Sob}_{\mathscr{C}}$ *is also an extensive subcategory of* Top_0.

3.7 *Examples.* (1) Let k be an infinite cardinal number. For $A \subseteq X \in \mathrm{Top}$, we define $l^k{}_X A = \{x \in X |$ for any family $(U_\iota)_{\iota \in I}$ of open neighborhoods of x with $|I| < k$, $\cap_\iota U_\iota \cap A \neq \emptyset\}$. Then it is known [**10**] that l^k is an idempotent limit-operator. Moreover, for any extension Y of a space X, $l^k{}_Y X = X$ if and only if any point y of Y whose trace filter on X has the k-intersection property is already a point of X (see [**10**]). Hence a T_0-space X belongs to Sob_{l^k} if and only if every union filter with the k-intersection property is itself the open neighborhood filter if and only if every non-empty irreducible closed subset A such that every open filter $\mathscr{U}$ on X with $U \cap A \neq \emptyset$ for every $U \in \mathscr{U}$ has the k-intersection property, is a point closure. We note that $\text{Sob}_{l\aleph_0} = \text{Sob}$, $\text{Sob}_{l\aleph_1} =$ the category of fc-spaces (see [**14**]), and more generally Sob_{l^k} is the category of $\varphi(k)$-spaces which is simply generated by $E(k)$: $E(\aleph_0) = \mathbf{2}$, $E(k^+) = \mathbf{2}^k - \{1^k\}$, and

$$E(k) = \times_{m<k} E(m^+)$$

for a limit cardinal number k, where $\mathbf{2}$ is the space $\{0, 1\}$ with the topology $\{\phi, \{1\}, \{0, 1\}\}$ (see [**15**]).

(2) For $A \subseteq X \in \text{Top}$, let

$$l_X A = \{x \in X | \text{ there is a point } b \text{ of } A \text{ with } x \in \text{cl } b\}.$$

Then it is known [**8**] that l is an idempotent limit-operator. For any extension Y of a space X, $l_Y X = X$ if and only if any point y of Y whose trace filter on X is contained in the open neighborhood filter of some point of X is already a point of X. Hence a T_0-space X belongs to Sob_l if and only if every union filter which is contained in an open neighborhood filter on X is itself an open neighborhood filter if and only if every non-empty irreducible closed subset which is contained in a point closure is already a point closure.

Let X be an ordered set. Let X_+ denote the space with the right topology on X, i.e. the topology whose base is $\{[x, \rightarrow[\,|\, x \in X\}$. Then $\mathbf{R}_+ \in \text{Sob}_l - \text{Sob}$ and $\mathbf{R}_+ - \{0\} \in \text{Top}_0 - \text{Sob}_l$, where $\mathbf{R}$ is the real line with the usual order.

(3) For $A \subseteq X \in \text{Top}$, let

$$l_X A = \{x \in X | \text{ there is a sequence in } A \text{ which converges to } x\}.$$

For the limit-operator l and for any extension Y of a space X, $l_Y X = X$ if and only if any point y of Y whose trace filter on X is contained in a filter with a countable base is already a point of X (see [**12**]). Hence a T_0-space X belongs to $\text{Sob}_{\bar{l}}$, where $\bar{l}$ is the associated idempotent limit-operator with l (see [**8**]) if and only if every union filter which is contained in a filter with a countable base is already an open neighborhood filter. It is obvious that $\mathbf{N}_+ \in \text{Top}_0 - \text{Sob}_{\bar{l}}$, where $\mathbf{N}$ is the set of natural numbers with the usual order. Let $W(\omega_1)$ be the set of ordinals $< \omega_1$ with the usual order. Then $W(\omega_1)_+ \in \text{Sob}_{\bar{l}} - \text{Sob}$.

References

1. B. Banaschewski, *Extensions of topological spaces*, Can. Math. Bull. *7* (1964), 1–22.
2. ——— *Essential extensions of T_0-spaces* (to appear).
3. S. Baron, *Note on epi in $\mathscr{T}_0$*, Can. Math. Bull. *11* (1968), 503–504.
4. N. Bourbaki, *Topologie générale* (Hermann, Paris, 1960).

5. L. Gillman and M. Jerrison, *Rings of continuous functions* (D. Van Nostrand Co., Princeton, 1960).

6. A. Grothendieck and J. A. Dieudonné, *Eléments de géométrie algébrique* (Springer, New York, 1971).

7. H. Herrlich, *Topologische Reflexionen und Coreflexionen*, Lecture Notes in Math. *78* (Springer, New York, 1968).

8. ——— *Limit-operators and topological coreflections*, Trans. Amer. Math. Soc. *146* (1969), 203–210.

9. K. H. Hofmann and K. Keimel, *A general character theory for partially ordered sets and lattices*, Mem. Amer. Math. Soc. *122* (1972).

10. S. S. Hong, *On k-compactlike spaces and reflective subcategories*, General Topology and Appl. *3* (1973), 319–330.

11. ——— *Studies in categorical topology*, Doctoral dissertation, McMaster University, 1973.

12. ——— *Limit-operators and reflective subcategories*, Lecture Notes in Math. *378* (Springer, New York, 1974), 219–227.

13. ——— *Extensive subcategories and extensive operators* (unpublished manuscript).

14. L. D. Nel and R. G. Wilson, *Epireflections in the category of T_0-spaces*, Fund. Math. *75* (1972), 69–74.

15. L. D. Nel and R. G. Wilson, *A class of epireflective subcategories of* $\mathscr{T}_0$ (unpublished manuscript).

16. L. Skula, *On a reflective subcategory of the category of all topological spaces*, Trans. Amer. Math. Soc. *142* (1969), 37–41.

17. Yim-Ming Wong, *Lattice-invariant properties of topological spaces*, Proc. Amer. Math. Soc. *26* (1970), 206–208.

Carleton University,
Ottawa, Ontario;
Sogang University,
Seoul, Korea

https://doi.org/10.4153/CJM-1975-037-7 Published online by Cambridge University Press