PRELIMINARY DESIGN OF NON-LINEAR SYSTEMS BASED ON GLOBAL SENSITIVITY ANALYSIS AND MODELICA LANGUAGE

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ABSTRACT

In the last few years, the growing need of highly reliable and time-effective strategies to perform preliminary design of complex systems has led industries to adopt the Model Based System Engineering (MBSE) approach. In MBSE, systems are split into multiple sub-systems and the relevant physical phenomena are described via analytical or numerical models. When a significant number of design variables are to be considered, a smart approach to reduce the number of analyses to perform would be to make use of the Global Sensitivity Analysis (GSA) to higlight those variables that have a more significant influence on the system output. Moreover, an even more significant reduction of computational cost to perform the GSA can be achieved if the complex system modelled via the MBSE approach is exported under the Functional Mock-Up Interface (FMI) norm. In this context, this paper proposes an original approach to address the study of two constructive solutions of an acceleration measuring device typically used on airbags for which the use of a new solution characterized by a porous material is compared with a classical one.

Keywords: Systems Engineering (SE), Functional modelling, Open source design, Global Sensitivity Analysis, Functional Mock-up Interface

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1 INTRODUCTION

When dealing with the design of complex systems, one of the main issues is to determine the influence of each design variable on the outputs of the system. Global sensitivity analysis (GSA) methods can help the designer to achieve this task. In the last two decades, several methods for GSA have been developed (Saltelli et al., 2008; Razavi et al., 2021). GSA methods can be grouped in four classes as presented by Razavi et al. (2021): a) the *derivative-based* approaches, which are multi-local techniques firstly introduced by Morris (1991); b) the *distribution-based* approaches, which consist in studying conditional model output variances, based on the Hoeffding (1948) decomposition and introduced by Sobol (1993) (the so-called Sobol's indices); c) the *variogram-based* approaches, linking derivative-based and distribution-based approaches (Razavi and Gupta, 2016a,b); d) the *regression-based* approaches Kleijnen (1995).

In the literature, one can find several algorithms to perform GSA based on the notion of Sobol's indices (Tissot and Prieur, 2012; Tarantola et al., 2006). The most popular ones are those developed by Saltelli et al. (Saltelli, 2002; Saltelli et al., 2010). Today, they are available in classical Python libraries, together with other strategies, like the Fourier Amplitude Sensitivity Test (FAST) (Cukier et al., 1973). The main difference between Saltelli's algorithms and FAST method is related to the calculation of the Sobol's indices: the former relies on the standard analysis of variance (ANOVA), while the latter is based on a frequency analysis of the output. Saltelli's algorithms are more accurate than FAST method, but at the price of a higher computational effort. Consequently, in this work, the FAST algorithm is used as a compromise between accuracy, computational cost and reliability (Vuillod et al., 2023).

A sound alternative to perform GSA is the algorithm proposed by Goda (2021), which is based on the Shapley's effect. This concept is derived from the game theory (Shapley, 1953): by considering a team playing to a game to achieve a certain goal, the aim is to quantify the role of the single player in achieving this goal. As discussed by Iooss and Prieur (2019) and by Goda (2021), Shapley's indices can be used for systems characterised by independent or dependent design variables and provide informations complementary to those related to Sobol's indices. For a deeper insight in the matter, the interested reader is addressed to the work by Goda (2021).

In this paper, GSA is applied to a multiple-input-single-output (MISO) system that represents the behaviour of a device which allows producing a signal from a specific acceleration value. Its common use is to trigger airbags in cars: it is constituted of a piston in a chamber oil-fill, blocked by a pre-loaded spring. Depending on the variation of the acceleration, the piston motion triggers a sending of a signal when a certain position is reached. Two different cases will be analysed in this paper, corresponding to two piston architectures. In the first case, the piston has a standard configuration, i.e., it is a cylinder with a centre hole allowing fluid motion inside it. In the second case, the piston is made of a full fill porous material.

One of the main issues of GSA methods is that they require a large number of simulations. In a recent paper, Zhou et al. (2022) propose alternative methods to reduce the computational cost associated with a GSA. However, these source codes are not available because they are still under development or unpublished. Thus, the present work, which deals with open access GSA algorithms, uses another strategy to reduce the computational cost by acting directly on the numerical description of the model: the Model-Based Systems Engineering (MBSE) approach, through the Modelica language, complemented by the Functional Mock-up Interface (FMI) standard. The former allows a lighter physical $0D$ model and the latter a significant reduction in computational costs.

The objective of this paper is to present the GSA - MBSE methodology on simple study cases to get useful information about the influence of the input variables (and on their interaction) on the output responses on the whole design space. Furthermore, the methodology can be extended to more complex systems, where Design of Experiments (DOE) cannot be conveniently applied. Therefore, the goal of this methodology is twofold. On the one hand, GSA based on both Sobol's indices and Shapey's effect will be carried out on both configurations to determine the influence of the design variables on the system output. On the other hand, by exploiting the results of the GSA, the aim is to design the second configuration of the piston to obtain (at least) the same performance of the first one.

The paper is organised as follows. Section 2 provides a brief overview of the theoretical and numerical tools used in this work, i.e., Sobol's indices, Shapley's effect, MBSE approach and the FMI norm.

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Sections 3 and 4 present the classical airbag model and the one including porous material, respectively. Lastly, section 5 ends the paper with conclusions and prospects.

2 THEORETICAL AND NUMERICAL TOOLS

2.1 Global sensitivity analysis: the Sobol's indices

Consider a MISO system whose transfer function is $\mathcal{M} : \zeta \subseteq \mathbb{R}^n \longrightarrow y \subseteq \mathbb{R}^m$, where n is the number of input variables, $\zeta = (\zeta_1, \ldots, \zeta_n)$ is the vector of input variables, y is the considered output. As widely discussed in Saltelli et al. (2010), the generic Sobol's index $S_{\mathbf{u}}$ is defined as:

$$S_{\mathbf{u}} := \frac{\mathrm{Var}\left(\mathcal{M}_{\mathbf{u}}(\zeta_{\mathbf{u}})\right)}{\mathrm{Var}(y)}, \tag{1}$$

which represents the ratio of the variance due to the interaction between the components of $\zeta_{\mathbf{u}}$ (for $\mathbf{u} \in \mathcal{S}$) to the total variance of the output. Sobol's indices can be defined also for orders higher than one. Of course, the Sobol's indices satisfy the following relationship:

$$\sum_{\substack{\mathbf{u}\in\mathcal{S}\\ \mathbf{u}\neq\mathbf{0}}} S_{\mathbf{u}} = \sum_{i=1}^{n} S_i(\zeta_i) + \sum_{1\le i<j\le n} S_{i,j}(\zeta_i,\zeta_j) + \cdots + S_{1,\ldots,n}(\zeta) = 1. \tag{2}$$

According to Equation 1, the first-order Sobol's index $S_i(\zeta_i)$, also referred to as elementary Sobol's index provides a measure of the influence of the single input variable ζ_i on the output y. However, the elementary Sobol's index does not provide any information about the influence of the variable ζ_i on the output y when interacting with other input variables ζ_k, $k \in \mathcal{S}$, $k \neq i$.
Thus, the $2^n - 1$ Sobol's indices can provide precious informations for the GSA, but their computation can be prohibitive when a large number of variables is considered. To this end, a measure often referred to as the "total Sobol's index", S_{T_i}, is used (Saltelli, 2002). This index provides a measure of the contribution to the output variance of ζ_i, including all variance caused by its interactions, of any order, with the other input variables. The total Sobol's index related to the input variable ζ_i can be defined as:

$$S_{T_i} := \sum_{\substack{\mathbf{u}\in\mathcal{S}\\ i\in\mathbf{u}\neq\mathbf{0}}} S_{\mathbf{u}}. \tag{3}$$

Due to its definition, unlike the elementary indices S_i, the sum of the total Sobol's indices can be greater than or equal to one, i.e., $\sum_{i=1}^{n} S_{T_i} \geq 1$. This is due to the fact that the interaction between two variables, e.g., ζ_i and ζ_j, is counted in both the associated total indices, i.e., S_{T_i} and S_{T_j}. The sum of the total indices is equal to one only when the model is purely additive. According to Eq. (3), if $S_{T_i} \approx 0$, one can state that the variable ζ_i does not influence at all the considered output.
In the following of this paper the elementary Sobol's indices are indicated as S_i. For a deeper insight in the matter the interested reader is addressed to (Saltelli et al., 2010).

2.2 Global sensitivity analysis: the Shapley's effect

Recently, Owen (2014) has proposed a GSA method for systems characterised by dependent input variables based on Shapley's effect, a concept taken from the game theory (Shapley, 1953). Nevertheless, since in this work only non-linear systems characterised by independent input variables are considered, only the formulation of the Shapley's indices for this type of systems is briefly recalled here below.
According to (Iooss and Prieur, 2019), in the case of independent input variables, the Shapley's index related to the generic input ζ_i is defined as:

$$S_{H_i} := \sum_{\substack{\mathbf{u}\in\mathcal{S}\\ i\in\mathbf{u}\neq\mathbf{0}}} \frac{S_{\mathbf{u}}}{n_u}, \tag{4}$$

where n_u is the cardinality of the array $\mathbf{u}$ collecting the indices $u_k \in \mathcal{S}$ used to compute the elementary index $S_{\mathbf{u}}$, i.e., $n_u = 1$ if $\mathbf{u}^{\mathrm{T}} = (1)$, $n_u = 2$ if $\mathbf{u}^{\mathrm{T}} = (1,2)$, etc. Shapley's indices are characterised by

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two main properties: they are positive semidefinite and their sum is equal to one (Iooss and Prieur, 2019). Moreover, the Shapley's index related to the generic input variable ζ_i always falls between the elementary and total Sobol's indices associated to the same variable, i.e., $S_i \leq S_{H_i} \leq S_{T_i}$.

2.3 Modelica and global sensitivity analysis co-simulation

In the present work, the number of simulations required to obtain accurate results for the GSA respect the convergence criteria of the GSA method. Therefore, in order to minimise the global computational cost of the GSA, a modelling strategy based on the MBSE approach, via the Modelica language, is adopted. The $0D$ model is then built by linking different physical components. Each of these components behaves according to its own system of Differential Algebraic Equations (DAE). Finally, the software is able to build up the global DEA system before solving it.
Once the Modelica model has been created and validated, a Functional Mock-up Unit (FMU) file is exported and run in co-simulation (CS) mode, defined by the FMI standard. One of the main advantages of this format, is the computational cost: a Modelica simulation of the studied cases requires about 5 s, while an FMU simulation requires about 10^{-2} s. Figure 1 illustrates the interactions between the different tools.

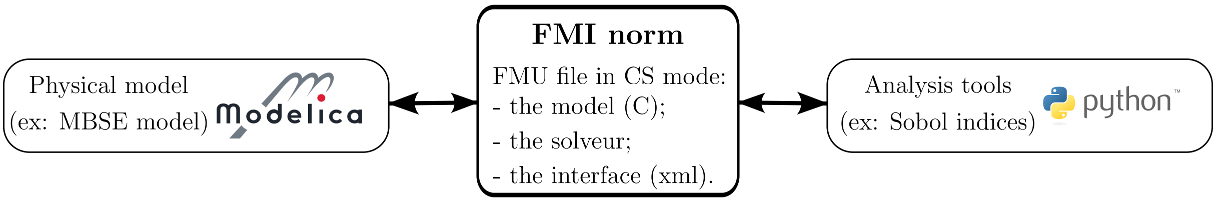

Figure 1. Illustration of the interactions between the Modelica model, FMI norm and GSA codes.

3 CLASSICAL AIRBAG TEST CASE

3.1 Model description

The reference test case is a device for measuring acceleration mainly used to trigger airbags, illustrated in Figure 2a. The system is composed of a piston with a centre hole, in motion without gap in a chamber oil-fill. At $t = 0$ s, the piston displacement is $x = 0$ m and it is constrained by a pre-loaded spring. At $t = 0^+$ s, the piston motion is initiated by a sudden acceleration. This latter is due to a sudden stop of the device, for example a car crash against an obstacle. As a consequence, the device external deceleration induces an acceleration to the piston. The functional scheme, shown in Figure 2b, illustrates the different forces and motions involved in this model: its behaviour can be assimilated to a mass linked to a support through a spring in parallel of a singular pressure drop.

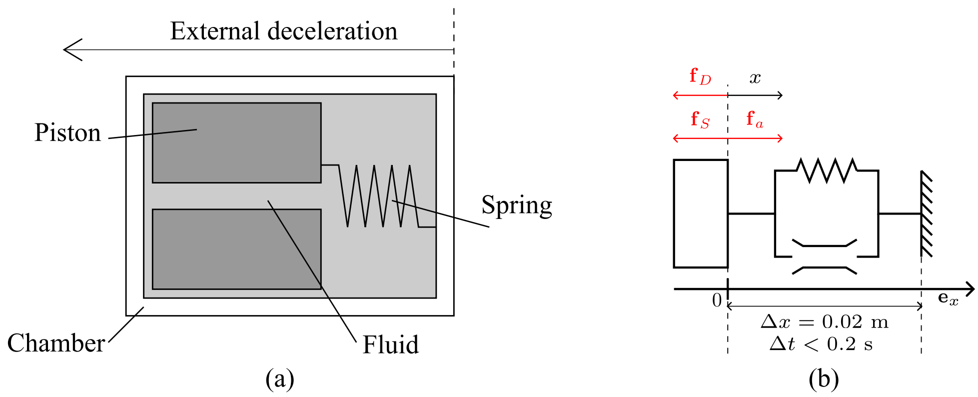

Figure 2. Initial airbag model: (a) its representative scheme and in (b) its functional scheme.

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The behaviour of such system can be determined by solving the second Newton's law along the $\mathbf{e}_x$ axis:

$$M\mathbf{a} = \sum_{i=1}^{q} \mathbf{f}_{\text{ext},i}, \tag{5}$$

where M is the piston mass, $\mathbf{a}$ its acceleration, whose component along the x axis is equal to $\ddot{x}(t)$, $\mathbf{f}_{\text{ext},i}$ is the generic i-th external force applied to the system and q is the total number of applied forces. The first considered force is the one if the spring, expressed as:

$$\mathbf{f}_S = -k(x - L_0)\mathbf{e}_x, \tag{6}$$

where k is the spring constant, L_0 is its unstreched length and $\mathbf{e}_x$ is the unit vector of the x axis (Figure 2b). At $t = 0$ s, the spring is pre-loaded. The second external load is the drag force that reads:

$$\mathbf{f}_D = \text{sign}\,(v_{\text{int}})\,A_p\,\Delta P_{\text{Sing}}\mathbf{e}_x, \text{ with } \Delta P_{\text{Sing}} = \frac{1}{2}\xi\rho_{\text{oil}}v_{\text{int}}^2, \tag{7}$$

where v_{int} is the oil velocity inside the piston, ΔP_{Sing} is the singular pressure loss and A_p is the area of the piston section expressed by $A_p = \pi\left(R^2 - r_{\text{hole}}^2\right)$ with R the external piston radius and r_{hole} the piston hole radius. Regarding the singular pressure loss expression, $\xi = 0.5$ is the singular pressure loss coefficient in a regular pipe and ρ_{oil} is the oil density. The last load is the added-mass force whose expression is:

$$\mathbf{f}_a = aM\left(1 - \frac{\rho_{\text{oil}}}{\rho_{\text{steel}}}\right)\mathbf{e}_x, \tag{8}$$

where a is the external deceleration occurring in the interval Δt_a and ρ_{steel} the steel density composing the piston.
Moreover, we assume that inside flow is equal to equivalent flow of the piston motion. Thus, it is possible to deduce that $v_{\text{int}} = \frac{A_p}{A_{\text{int}}}v_p$ with A_{int} the internal section area.
To model this system, the MBSE approach is used via the Modelica language. The final model is composed of four elementary blocks, corresponding to the functional scheme shown in Figure 2b (a mass, a spring, a singular pressure loss and a support), subject to a fifth one used to apply the input force $\mathbf{f}_a$.

3.2 Study context

In this application, the output of interest is the time Δt needed by the piston to reach the electrical contact from its starting position, to trigger the inflation of the airbag. This distance Δx is fixed to 0.02 m and corresponds to the end of the chamber in the scheme of Figure 2a. An admissible value of Δt to achieve this goal is lower than 0.2 s (to save car passengers without hitting them). The input variables (with the related ranges of variation) and the constant parameters together with the reference values (that allow satisfying the requirement on Δt) are listed in Table 1. To perform the GSA, the distribution of each design variable in the related range follows a uniform law $\mathcal{U}$.

Table 1. Constant parameters and input variables: reference values and intervals of variation.

Constant parameter	Reference value	Input variable	Reference value	Range of variation
R [m]	0.025	r_{hole} [m]	0.008	$\mathcal{U}([0.006, 0.01])$
ξ [-]	0.5	L_p [m]	0.03	$\mathcal{U}([0.02, 0.04])$
ρ_{oil} [kgm^{-3}]	900	k [Nm^{-1}]	150	$\mathcal{U}([90, 210])$
ρ_{steel} [kgm^{-3}]	8000	L_0 [m]	0.03	$\mathcal{U}([0.02, 0.04])$
a [ms^{-2}]	200			
Δt_a [s]	0.05			

During the FMU export step, it is necessary to define not only the inputs, outputs and constant parameters of the model, but also the solver type that allows the co-simulation run to be performed. In this study, a fourth-order Runge-Kutta solver is chosen, with integration and communication time steps with the GSA algorithm set to 10^{-4} s.

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3.3 Global sensitivity analysis results

The results of the GSA, considering the FAST method to compute the Sobol's indices, are computed with $N = 2^{12}$ and compared with reference values obtained with $N = 2^{22}$. The Shapley's indices values, computed with the Python version of the algorithm proposed by Goda (2021) with $N = 2^{14}$ are compared to the results obtained by the same algorithm by taking $N = 2^{22}$ samples (Vuillod et al., 2023). These results are shown in Figure 3 where one can read also the confidence interval with a 95 % confidence level.

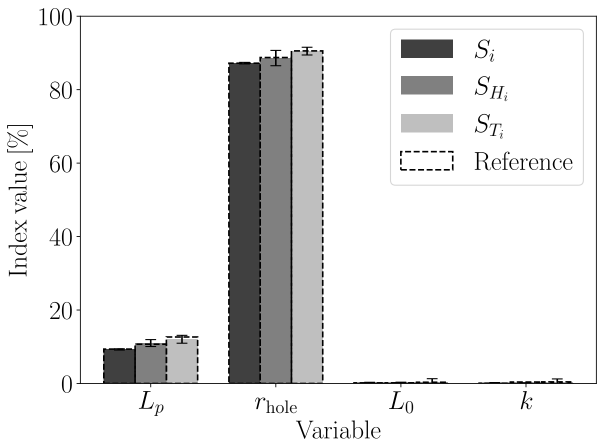

Figure 3. GSA results on the airbag model. The global variance of the considered output Δt is $\text{Var}(\Delta t) = 1.08 \cdot 10^{-5}$.

From these results, one can infer that the influence of the interaction among the design variables on the output of the system is negligible. Indeed, the elementary and total Sobol's indices are very close. Quantitatively, the influence of the interaction among input variables on the output can be assessed from Equation 2 and it is equal to 3.30 %.

From Figure 3, the most influential variable is the radius of the hole of the piston with $S_{T_i} = 90$ %, which is eight times greater than the second most influential variable, i.e., the piston length L_p. Conversely, the spring constant k and its unstreched length L_0 are characterised by a total index that goes to zero. Accordingly, the behaviour of the system is strongly influenced by the singular pressure loss definition, which is the only force involving the variable r_{hole}. Regarding the Shapley's indices, they fall always between elementary and total Sobol's indices, but the reliability offered by Goda's algorithm is lower than that characterising the FAST method for the computation of Sobol's indices.

Moreover, the global variance of the considered output Δt is $\text{Var}(\Delta t) = 1.08 \cdot 10^{-5}$ s. A new solution to reduce the sensitivity of the system linked to solely one parameter, r_{hole}, is studied in the following section.

4 AIRBAG INCLUDING POROUS MATERIAL

4.1 Model description

To reduce the dependence of the piston motion to the singular pressure loss, one can imagine to change the architecture of the piston by using a solution made of a porous material, without centre hole. Of course, in this case one has to conceive the piston to obtain the same (or better) behaviour, in terms of stability of the motion, of the classic configuration. Such a piston can be fabricated by AM technology to full-fill physical requirement (porosity, void volume, etc.). To this end, this test case, schematically shown in Figure 4a, has been modelled in Modelica environment according to functional scheme illustrated in Figure 4b. The external forces applied to the piston are the same as the first case, except the singular pressure loss that is replaced by a pressure loss of a porous material. The goal is to keep the same behaviour of the first solution, while being less sensitive to the pressure losses.

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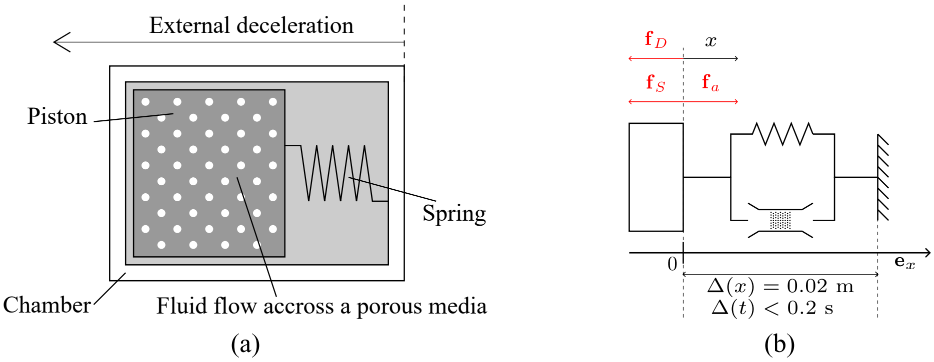

Figure 4. Airbag model including porous material with in (a) its representative scheme and in (b) its functional scheme.

To achieve this goal, the first step is to design the porous material. Therefore, in this work, the Ergün fluid law is considered to characterise the fluid behaviour through the porous material, taking into account the following hypotheses (Gjengedal et al., 2020):

- the gravity does not affect the fluid behaviour;
- the Reynold number is sufficiently high. Indeed, $\mathrm{Re} = \frac{\rho_{\mathrm{oil}} v_{\mathrm{c}} L_{\mathrm{c}}}{\mu_{\mathrm{oil}}} \approx 450$, where $v_{\mathrm{c}} \approx 1$ [ms^{-1}] is the characteristic oil velocity, $L_{\mathrm{c}} \approx 10^{-2}$ [m] the characteristic length and $\mu_{\mathrm{oil}} \approx 0.02$ [Pas] the oil dynamic viscosity;
- the porous material is made of uniform distribution of pores.

Then, as demonstrated in Gjengedal et al. (2020), the Ergün law reads:

$$\frac{\Delta P_{\mathrm{porous}}}{L_{\mathrm{porous}}} = \frac{2\alpha A_{\mathrm{spe}}^2}{\Phi^3} \mu_{\mathrm{oil}} v_s + \frac{\beta}{8} \frac{A_{\mathrm{spe}}}{\Phi^3} \rho_{\mathrm{oil}} v_s^2, \tag{9}$$

where $\Delta P_{\mathrm{porous}}$ is the pressure loss through a porous medium, L_{porous} is the characteristic length of the porous medium, α is a geometric coefficient used to fit the porous material behaviour as the first case, Φ is the porosity volume fraction, $\beta_0 \in [1.1, 5.6]$, a factor depending on the geometric relation used to fit the first case behaviour, and v_s, the surface velocity corresponding to the flow without any disturbances. A_{spe} is the specific porous area, expressed as follows by Gjengedal et al. (2020):

$$A_{\mathrm{spe}} = \frac{6(1-\Phi)}{d\phi}, \tag{10}$$

where d is the average pore diameter and ϕ is a coefficient that translates the porous discontinuity. In this case the pores are considered to be perfectly ordered and spherical. Therefore $\phi = 1$.

Considering both the oil flow behaviour of the initial system and Equation 9 adapted to this case, the drag force is:

$$\mathbf{f}_D = -\mathrm{sign}\left(v_p\right) A_p \Delta P_{\mathrm{Porous}} \mathbf{e}_x, \text{ with } \Delta P = 72 L_p \alpha \frac{(1-\Phi)^2}{d^2 \Phi^3} \mu_{\mathrm{oil}} v_p + 0.75 L_p \beta \frac{(1-\Phi)}{d\Phi^3} \rho_{\mathrm{oil}} v_p^2, \tag{11}$$

where v_p is the piston velocity.

The use of a porous material also changes the expression for the added mass force. Indeed, the initial piston mass value M is now reduced by the volume fraction of the porosity, i.e., $M_{\mathrm{Porous}} = (1-\Phi)M$. Consequently, the added-mass force become: $\mathbf{f}_a = a M_{\mathrm{Porous}} \left(1 - \frac{\rho_{\mathrm{oil}}}{\rho_{\mathrm{steel}}}\right) \mathbf{e}_x$.

4.2 Study context

Also in this case, the output of the system is the time Δt needed by the piston to cover the distance $\Delta x = 0.02$ m. The input variables (with the related ranges of variation) and the constant parameters together with the reference values (that allow satisfying the requirement on Δt) are listed in Table 2. To perform the GSA, the distribution of each design variable in the related range follows a uniform law

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$\mathcal{U}$. Thus, the pores of the porous media do not follow a specific organisation, but merely a statistical distribution.

Table 2. Constant parameters and input variables: reference values and intervals of variation.

Constant parameter	Value	Input variable	Reference value	Range of variation
R [m]	0.025	L_p [m]	0.03	$\mathcal{U}([0.02, 0.04])$
ρ_{oil} [kgm^{-3}]	900	k [Nm^{-1}]	150	$\mathcal{U}([90, 210])$
ρ_{steel} [kgm^{-3}]	8000	L_0 [m]	0.03	$\mathcal{U}([0.02, 0.04])$
a [ms^{-2}]	200	Φ [%]	55	$\mathcal{U}([50, 60])$
Δt_a [s]	0.05	d_{pore} [m]	0.00175	$\mathcal{U}([0.0015, 0.002])$
μ_{oil} [Pas]	0.02			

As in the first test case, the Modelica has been exported as an FMU file with the same parameters used for the fourth order Runge Kutta solver.

4.3 Global sensitivity analysis results

The results of the GSA, considering the FAST method to compute the Sobol's indices, are computed with $N = 2^{12}$ and compared with reference values obtained with $N = 2^{22}$. The Shapley's indices values, computed with the Python version of the algorithm proposed by Goda (2021) with $N = 2^{14}$ are compared to the results obtained with the same algorithm by taking $N = 2^{22}$ samples (Vuillod et al., 2023). These results are shown in Figure 5 where one can see also the confidence interval with a 95 % confidence level.

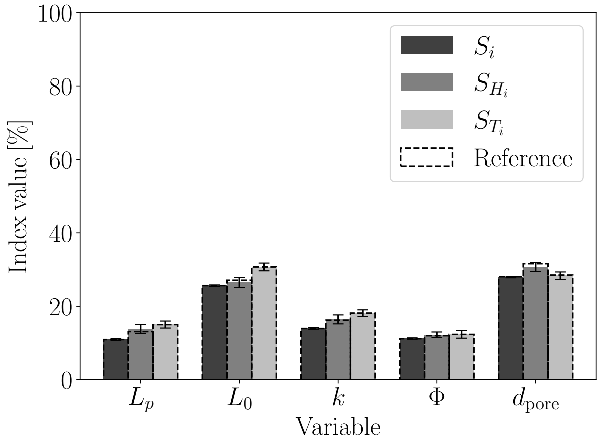

Figure 5. GSA results on the airbag model including porous material. The global variance of the considered output Δt is $\text{Var}(\Delta t) = 1.45 \cdot 10^{-5}$ s.

As already seen in the previous section, the values of S_i and S_{T_i} are very close. According to Equation 2, the influence of the interaction between the input variables is about 10.24 %. This is slightly higher than the first configuration of the system.

Figure 5 highlights that all variables are characterised by indices S_i, S_{T_i} and S_{H_i} included between 10 % and 30 %, and the global system variance is approximately equal to $1.45 \cdot 10^{-5}$ s. Moreover, with the second system, sources of uncertainty could be filtered out from the system behaviour by properly acting on the multiple design variables. Finally, since the number of design variables is higher than the classical configuration, the design space of this second case is wider than that of the former one. Of course, the design problems should be formulated by adding more design requirements according to the physics of the problem at hand. In any case, the main design requirement to be included in the problem formulation is the same of the previous formulation, i.e., $\Delta t < 0.2$ s.

As far as the Shapley indices are concerned, their values are between the elementary Sobol indices and the total ones, except for the variable d_{pore}, which is higher the total index. Since the proportion of interactions is very low, i.e., the elementary and total indices are very close, this error may be due to the issue related to the convergence of the Shapley's code (Vuillod et al., 2023).

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5 CONCLUSIONS

In this paper, a GSA is used to study two configurations of a device reproducing the behaviour of an airbag: a classical piston-spring-mass device and another including a porous material. A MBSE approach, implemented via the Modelica language and exported in FMU format, is adopted as modelling strategy. The objective is to study and to understand the influence of different input variables on the device behaviour, and to point out the importance of this modelling approach when addressing new design solutions.

The results show that, for the classical configuration, the centre hole radius is the variable with the strongest influence on the system output. Indeed, its total Sobol's index value is about 90 %, more than 8 times greater than the second influential variable, the piston length. For the porous configuration, all indices, i.e., the Sobol's and Shapley's ones, are between 10 % and 30 %. Moreover, for both cases, the global output variance is about 10^{-5} s and the interactions are negligible. Two conclusions can be drawn from these results:

- The behaviour of the first device is essentially driven by one variable, the centre hole radius.
- In the second device, the desired behaviour can be obtained by different sets of parameters. This indicates that a larger design domain can be considered to identify, if needed, an optimal solution, or to reduce the effects of sources of uncertainty.

This work has demonstrated the potential of the MBSE approach coupled with the use of the FMU format to perform an efficient GSA on relatively simple non-linear systems used in preliminary design. Ongoing studies address the use of this numerical framework to perform GSA on complex multi-physics problems where the MBSE approach is enriched with metamodels reproducing efficiently the behaviour of those phenomena for which accurate (high-fidelity) models are needed.

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