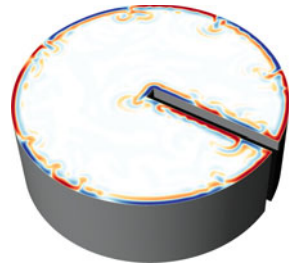


# Tenacious wall states in thermal convection in rapidly rotating containers



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Convection in a container, heated from below, cooled from above and rapidly rotated around a vertical axis, starts from its sidewall. When the imposed vertical temperature gradient is not sufficiently large for bulk modes to set in, thermal convection can start in the form of wall modes, which are observed near the sidewall as pairs of hot ascending and cold descending plumes that drift along the wall. With increasing temperature gradient, different wall and bulk modes occur and interact, leading finally to turbulence. A recent numerical study by Favier & Knobloch (*J. Fluid Mech.*, 895, 2020, R1) reveals an extreme robustness of the wall states. They persist above the onset of bulk modes and turbulence, thereby relating them to the recently discovered boundary zonal flows in highly turbulent rotating thermal convection. More exciting is that the wall modes can be thought of as topologically protected states, as they are robust with respect to the sidewall shape. They stubbornly drift along the wall, following its contour, independent of geometric obstacles.

**Key words:** Bénard convection, rotating flows, waves in rotating fluids

## 1. Introduction

Thermally induced buoyancy, constrained by rapid rotation, governs many astro- and geophysical processes. Examples are thermal convection in the atmosphere, ocean and liquid-metal core of the Earth and convection on the surface of the Sun. A paradigmatic system to study these processes in the laboratory is rotating Rayleigh–Bénard convection (RBC; Ahlers, Grossmann & Lohse 2009) in an upright cylinder with height  $H$ , diameter-to-height aspect ratio  $\Gamma$  and temperature difference  $\Delta$  from the bottom to the top, filled with a fluid of Prandtl number  $Pr = \nu/\kappa$  and rotated uniformly about its axis at rate  $\Omega$ . The strengths of buoyancy and rotation are determined by the Rayleigh number  $Ra \equiv \alpha g H^3 \Delta / (\kappa \nu)$  and the Ekman number  $Ek \equiv \nu / (2\Omega H^2)$ , where  $g$  is the gravitational acceleration and  $\nu$ ,  $\kappa$  and  $\alpha$  are, respectively, the viscosity, thermal diffusivity and isobaric expansion coefficients.

In a laterally unbounded, rotating fluid layer, convection sets in as oscillatory (for small  $Pr$ ) or steady motions, when  $Ra$  exceeds a critical value  $Ra_c^{bulk}$  that scales as

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$\sim Ek^{-4/3}$  in the limit  $Ek \rightarrow 0$  (Chandrasekhar 1961). Experiments in confined containers show, however, that convection can occur for  $Ra < Ra_c^{bulk}$  (Rossby 1969), which means that, in contrast to the non-rotating case, the sidewalls play a destabilizing role in rapidly rotating RBC. The wall modes have the form of  $m$  pairs of hot ascending and cold descending plumes that drift in a retrograde direction along the sidewall (Zhong, Ecke & Steinberg 1991). Asymptotic analysis gives a critical  $Ra$  for the onset of the wall modes, namely,  $Ra_c^{wall} \approx \pi^2 \sqrt{6\sqrt{3}} Ek^{-1} + 46.5 Ek^{-2/3}$ , and the corresponding onset drift frequency  $\omega_c/\Omega \approx Ek Pr^{-1} (4\pi^2 \sqrt{6 + 3\sqrt{3}} - 1464.5 Ek^{1/3})$  and mode number  $m/\Gamma \approx (\pi/2) \sqrt{2 + \sqrt{3}} - 17.5 E^{1/3}$  (Zhang & Liao 2009). The leading terms here were calculated first by Herrmann & Busse (1993). The linear dependences on  $Ra$  of the drift frequency and mean vertical heat transport in the system (Nusselt number  $Nu$ ), near the wall mode onset, indicate a supercritical Hopf bifurcation from the conductive case (Ecke, Zhong & Knobloch 1992). With increasing  $Ra$ , additional wall modes with different  $m$  occur and may be subjected to modulational merging instabilities (Liu & Ecke 1999). The wall modes can further interact with the bulk modes, leading to a very complex nonlinear dynamics (Zhong, Ecke & Steinberg 1993; Bodenschatz, Pesch & Ahlers 2000; Horn & Schmid 2017).

## 2. The exciting findings by Favier & Knobloch (2020)

The onset of wall modes has been studied in detail (Goldstein *et al.* 1993), but important questions remained unanswered: How robust are they? Do they appear only in specific regular geometries? The recent study by Favier & Knobloch (2020) provides evidence of the robustness of the wall modes: the introduction of an extreme radial barrier into the cylinder (cf. figure 1*a,b*) has basically no effect on the wall mode properties. Warm uprising and cold downwelling thermal plumes drift with the same frequency (figure 1*c,d*) along the sidewall, no matter what particular shape it has. They just follow any provided geometry. The drift frequency remains linearly dependent on  $Ra$  throughout the whole purely wall-mode regime (grey region in figure 1*e*), which in the case of  $Ek = 10^{-6}$  is approximately two decades of  $Ra$ .

Favier & Knobloch (2020) also show that the wall modes (already slightly above their onset, at lower  $Ra$ ) have many qualitatively similar features with the boundary zonal flows (BZF), recently discovered in measurements and simulations of highly turbulent (high- $Ra$ ) rotating RBC (see Zhang *et al.* (2020) and de Wit *et al.* (2020), and figure 1*e* in this paper). In particular, the flow patterns near the sidewall in both cases drift anticyclonically, while the mean azimuthal flows there are cyclonic. Although the characteristics (e.g. drift speeds, the scalings of the heat and momentum transport and layer thicknesses associated with the zonal flows, etc.) of the wall modes near their onset and of the BZF in turbulent regimes are quantitatively different, the study by Favier & Knobloch (2020) provides a clear link between these two important wall states, assuming a complex nonlinear transformation between them.

## 3. Future

Thanks to Favier & Knobloch (2020), we now know that the wall states are very robust and this gives rise to further questions, motivating further investigations of these tenacious wall states. As the authors speculate, the wall modes can be interpreted as topologically protected states due to their robustness with respect to the shape of the container sidewall. Can one identify similar states in other three-dimensional dissipative nonlinear systems? How exactly do the different wall and turbulent bulk

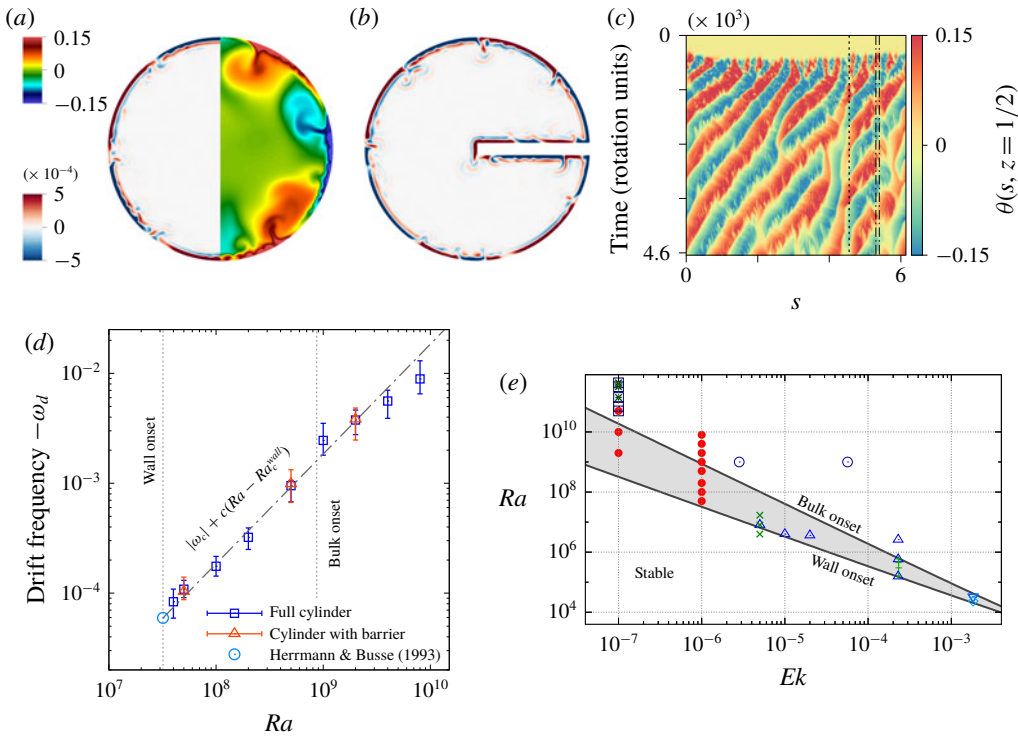


FIGURE 1. For panels (a–d)  $Ek = 10^{-6}$ ,  $Pr = 1$ ,  $\Gamma = 1.5$ , the cylinder mid-height, and for panels (a–c)  $Ra = 5 \times 10^8$ . (a) Instantaneous vertical velocity (left) and temperature (right) in a full cylinder; (b) vertical velocity; and (c) temporal evolution of the temperature along the arclength  $s$  at a distance  $10^{-2}H$  from the wall in a cylinder with a barrier, whose corners are shown with vertical lines. (d) Drift frequency,  $-\omega_d \equiv \omega/(2\Omega)$ , as a function of  $Ra$ , in a full cylinder (blue squares) and in a cylinder with a barrier (red triangles), together with the frequency  $\omega_d = \omega_c \approx -59Ek/Pr$  (Herrmann & Busse 1993) for the onset of the instability (blue circle). (e) Onsets of the wall (Zhang & Liao 2009) and bulk (Chandrasekhar 1961) modes with the parameter range studied by Favier & Knobloch (2020) (filled red circles), Zhang *et al.* (2020) (blue circles), de Wit *et al.* (2020) (black squares and stars) and others. Figures adopted from Favier & Knobloch (2020).

modes interact and evolve with growing  $Ra$ ? What is the role of the sidewall zonal flow in transitional and turbulent regimes? What is its contribution to the global heat transport (Zhong *et al.* 2009), energy balances (Horn & Shishkina 2015) and reorganization of the global flow structure (Kunnen *et al.* 2011; Weiss & Ahlers 2011)? Can one predict the BZF properties, for any given  $Ra$ ,  $Ek$ ,  $Pr$  and  $\Gamma$ ?

The relationship between the wall modes at their onset and the turbulent BZF in the rotating system is similar to that between the bulk modes at their onset and the turbulent large-scale circulation (LSC) in the problem with no rotation. The BZF can also be interpreted as the remains of the LSC that is suppressed by rotation and confined to the near-sidewall region, or as the superstructure that substitutes the LSC in turbulent, rapidly rotating RBC. Deeper investigations of these relationships are desired. Further questions are related to the geostrophic regime, which is probably the most relevant in geo- and astrophysical studies (Cheng *et al.* 2018). Measurements and simulations for very fast rotation (low  $Ek$ ) and strong thermal driving (high  $Ra$ )

show that the heat transport in the system is highly influenced by the BZF (Zhang *et al.* 2020). If the wall states are that robust and their role is that important, then how should modern experiments be designed, to gain the relevant knowledge on geo- and astrophysical flows in the laboratory?

### Declaration of interests

The author reports no conflict of interest.

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