

THE  
JOURNAL  
OF  
SYMBOLIC LOGIC

EDITED BY

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### ERRATA

#### VOLUME 12

- Page 63, line 10 from the bottom. For « p. 86 », read « pp. 83–86 ».
- Page 63, line 5 from the bottom. Add at the end of the line: « [This note appears in German, and without title; the title which is here given was supplied by the author.] ».

#### VOLUME 13

- Page 112, line 16. For « 1946 », read « 1947 ».
- Page 115, line 25. For « a », read «  $a$  ».
- Page 122, line 12 from the bottom. For « pp. 261–266 », read « pp. 271–276 ».
- Page 124, line 12. Add: « [As stated in the appendix, the work originally came out in parts: pp. 1–188 in 1941, pp. 189–328, 329–380 in 1942, pp. 381–492 in 1943, and the rest in 1945.] ».
- Page 125, line 6 from the bottom. For « XII 55 », read « XII 52 ».
- Page 149, line 19 from the bottom. The first word in the line should be « lead ».
- Page 166, line 37 from the bottom. For  $u$ , read  $U$ .
- Page 167, line 19 from the bottom. For « XIII 000 », read « XIII 166(1) ».

#### VOLUME 14

- Page ii, line 5. For « January, 1950 », read « February 2, 1950 ».
- Page 57, line 5 from the bottom; page 58, line 3 from the bottom; and page 279, first column, line 24. For « YRJÖÖ », read « YRJÖ ».
- Page 61, line 23. For « pp. 248–260 », read « pp. 258–270 »; and last line, for « pp. 311–346 », read « pp. 331–346 ».

- Page 87, line 6. For  $u$ , read  $U$ .  
 Page 104, line 4. Add  $\wedge$  at the end of the line.  
 Page 109. In the last displayed formula, add the condition  $\sim Y = 0$ , so that the part of the formula in brackets becomes:

$$[\text{Int}(X) \wedge \sim X = 0 \wedge \sim Y = 0 \wedge (A = X^2 \vee X^2 = 1 + AY^2)].$$

- Page 111, line 6. For « A6 », read « A3 ».  
 Page 114. In [8], for « *Vollständigkeit* », read « *Vollständigkeit* ».  
 Page 141, line 14. For « *notwendig* », read « *nicht notwendig* ».  
 Page 143, line 3. The parentheses are part of the original title of the article, hence should appear as italic parentheses.  
 Page 193, line 21. For « vol. 222 », read « vol. 223 ».  
 Page 195, line 11. For « Amsterdam », read « Bussum ».  
 Page 211, line 11. For « o-transform », read « o-transform ».  
 Page 233, line 2. For the main implication sign in the formula, read a sign of disjunction,  $\vee$ .  
 Page 243, line 16. For  $e$ , read  $\acute{e}$ .  
 Page 244, line 9. For « o », read « of ».  
 Page 249, line 18. For « Kaufman », read « Kaufmann ».  
 Page 257, line 24. Insert a comma before « Bd. ».  
 Page 268, first column, line 26 from the bottom. For « Alfred », read « Alfred Jules ».  
 Page 269, second column, line 15. For « de », read « des ».

#### VOLUME 15

- Page 58, line 15. For « *Scientarum* », read « *Scientiarum* ».  
 Page 62, line 29. After « again », insert « (in connection with the definition of symmetry of a relation) ».  
 Page 64, line 39. For « *Pryczynek* », read « *Przyczynek* ».  
 Page 71, line 20. For « *existance* », read « *existence* ».  
 Page 73, line 30. For the second « ' » , read « ' ».  
 Page 77, line 10. After  $Y$ , insert a parenthesis,  $)$ .  
 Page 96, line 24. For « UI and EG », read « UG and EI ».  
 Page 120, line 21. For  $F_n^n$ , read  $F_n^n$ .  
 Page 121, line 5. After the first « type », insert « 1 ».  
 Page 123, line 23. For  $L''$ , read  $L'''$ .  
 Page 129, line 24. For « *philosophical* », read « *philosophica* ».  
 Page 133, line 9 from the bottom. For  $O_r$ , read  $O_r$ ; and for  $1_r$ , read  $1_r$ .  
 Page 133, line 8 from the bottom. For  $0_r$ , read  $0_r$ .  
 Page 143, line 19 from bottom. For « CARRUCIO », read « CARRUCCIO ».  
 Page 153, line 10. After the colon, insert « ' ».  
 Page 159, line 6. The parentheses are part of the original title of the article, hence should appear as italic parentheses.  
 Page 177, line 15 and line 16. For  $\equiv$ , read  $\sim$  (twice).  
 Page 180. In the next to last truth-table, for «  $\mathfrak{B.C.}$  », read «  $\mathfrak{B} \cdot \mathfrak{C}$  ».  
 Page 184. In the upper left corner, instead of « 84 », the page number should be « 184 ».  
 Page 188, line 6. For  $\equiv$ , read  $\neq$ .  
 Page 192, line 23. For  $(x; y)$  and  $(w; 0)$ , read  $(x; y; z)$  and  $(w; 0; z)$ , respectively.  
 Page 193, footnote 46. For « (1935) », read « (1936) ».  
 Page 198. In the first line of the footnote, for « remarks », read « remarks ».  
 Page 202, line 4 from the bottom. Insert « & » at the end of the line.  
 Page 218, line 21. For the first « of », read « on ».  
 Page 223, line 12. For « HELEN », read « HELENA ».  
 Page 226, line 3 from the bottom. For « vol. 14 no. 4 », read « vol. 14 no. 11 ».  
 Page 228, line 9. In the fifth formula in the line, supply a bar over  $\alpha$ .  
 Page 228. In the displayed formula, for  $\Lambda_x$ , read  $\Lambda_x$ .  
 Page 234, line 12 from the bottom. The parentheses are part of the original title of the article, hence should appear as italic parentheses.  
 Page 239, line 26. For « *Lattices* », read « *lattices* ».  
 Page 240, line 13. For « XV 156(1) », read « XV 156(2) ».