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Marcus Vinícius Midena Ramos, ***Formalization of Context-Free Language Theory***, Universidade Federal de Pernambuco, Brazil, 2016. Supervised by Ruy J. G. B. de Queiroz. MSC: 03B35, 03D05, 03F65, 68Q42, 68Q45, 68T15. Keywords: proof assistants, Coq, formalization, language theory, context-free languages, context-free grammars.

Abstract

Proof assistants are software-based tools that are used in the mechanization of proof construction and validation in mathematics and computer science, and also in certified program development. Different such tools are being increasingly used in order to accelerate and simplify proof checking, and the Coq proof assistant is one of the most well known and used in large-scale projects. Language and automata theory is a well-established area of mathematics, relevant to computer science foundations and information technology. In particular, context-free language theory is of fundamental importance in the analysis, design, and implementation of computer programming languages. This work describes a formalization effort, using the Coq proof assistant, of fundamental results of the classical theory of context-free grammars and languages. These include closure properties (union, concatenation, and Kleene star), grammar simplification (elimination of useless symbols, inaccessible symbols, empty rules, and unit rules), the existence of a Chomsky Normal Form for context-free grammars and the Pumping Lemma for context-free languages. The result is an important set of libraries covering the main results of context-free language theory, with more than 500 lemmas and theorems fully proved and checked. As it turns out, this is a comprehensive formalization of the classical context-free language theory in the Coq proof assistant and includes the formalization of the Pumping Lemma for context-free languages. The perspectives for the further development of this work are diverse and can be grouped in three different areas: inclusion of new devices and results, code extraction, and general enhancements of its libraries.

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Douglas Ulrich, ***Some Applications of Set Theory to Model Theory***, University of Maryland, College Park, USA, 2018. Supervised by Michael C. Laskowski. MSC: 03C55. Keywords: Keisler's order, Borel complexity.

Abstract

We investigate set-theoretic dividing lines in model theory. In particular, we are interested in Keisler's order and Borel complexity.

Keisler's order $\trianglelefteq$ is a pre-order on complete countable theories T, measuring the saturation of ultrapowers of models of T. In Chapter 3, we present a self-contained survey on Keisler's order, cast in terms of full Boolean-valued models and pseudosaturation. The cornerstone of our development is a compactness theorem for full Boolean-valued models. As an application, we show that if T is a complete countable theory and $\mathcal{B}$ is a complete Boolean algebra, then λ^+-saturated $\mathcal{B}$-valued models of T exist. Moreover, if $\mathcal{U}$ is an ultrafilter on T and $\mathbf{M}$ is a λ^+-saturated $\mathbf{B}$-valued model of T, then whether or not $\mathbf{M}/\mathcal{U}$ is λ^+-saturated just depends on $\mathcal{U}$ and T; we say that $\mathcal{U}$ λ^+-saturates T in this case. We show that Keisler's order can be formulated as follows: $T_0 \trianglelefteq T_1$ if and only if for every cardinal λ, for every complete Boolean algebra $\mathcal{B}$ with the λ^+-c.c., and for every ultrafilter $\mathcal{U}$ on $\mathcal{B}$, if $\mathcal{U}$ λ^+-saturates T_1, then $\mathcal{U}$ λ^+-saturates T_0. We also prove that if $\mathcal{B}$ is a complete Boolean algebra with an antichain of size λ, then there is a λ^+-good ultrafilter on $\mathcal{B}$; conversely, if $\mathcal{B}$ is a complete Boolean algebra with the λ-c.c., then then no nonprincipal ultrafilter on $\mathcal{U}$ λ^+-saturates any unsimple theory, and no $\aleph_1$-incomplete ultrafilter on $\mathcal{U}$ λ^+-saturates any nonlow theory. Finally, we

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streamline treatments of the interpretability orders $\trianglelefteq^*_\kappa$ of Shelah, the key new notion being that of pseudosaturation.

In Chapter 4, we uniformize many ultrafilter constructions in Keisler's order. As a particular application, we prove that for all $3 \leq k < k'$, $T_{k+1,k} \ntrianglelefteq T_{k'+1,k'}$, where $T_{n,k}$ is the theory of the random k-ary n-clique free hypergraph. This improves the previous result of Malliaris and Shelah that $T_{k+1,k} \ntrianglelefteq T_{k'+1,k'}$ for all $k < k' - 1$.

Borel complexity is a pre-order on sentences of $\mathcal{L}_{\omega_1\omega}$ measuring the complexity of countable models. In Chapter 5, we describe joint work with Richard Rast and Chris Laskowski on this order. They key idea is the following: suppose Φ is a sentence of $\mathcal{L}_{\omega_1\omega}$. Define $\mathrm{css}(\Phi)_{\mathrm{ptl}}$ to be the set of all sentences $\phi \in \mathcal{L}_{\infty\omega}$, such that in some forcing extension $\mathbb{V}[G]$, ϕ becomes the canonical Scott sentence of some model of Φ. Define $\|\Phi\|$ to be the cardinality of $\mathrm{css}(\Phi)_{\mathrm{ptl}}$ (possibly ∞). We show that if $\Phi \leq_B \Psi$ then this induces an injection from $\mathrm{css}(\Phi)_{\mathrm{ptl}}$ to $\mathrm{css}(\Psi)_{\mathrm{ptl}}$, whence $\|\Phi\| \leq \|\Psi\|$. This is a potent new method for proving nonreducibilities in $\leq_B$, and we give several applications, including the first example of a complete first order theory T with non-Borel isomorphism relation, but which is not Borel complete.

In Chapter 6, we introduce the notion of thickness. The motivation is as follows: suppose Φ is a sentence of $\mathcal{L}_{\omega_1\omega}$ with $\|\Phi\| = \infty$; we wish to still be able to apply counting arguments to $\mathrm{css}(\Phi)_{\mathrm{ptl}}$. The thickness spectrum $\tau(\Phi,\kappa)$ of Φ accomplishes this; roughly, $\tau(\Phi,\kappa) \approx |\mathrm{CSS}(\Phi)_{\mathrm{ptl}} \cap \mathbb{V}_{\kappa^+}|$, although care must be taken to ensure that thickness is a $\leq_B$-reducibility invariant. We present several applications of the notion of thickness; in particular, we show that all the Friedman-Stanley jumps of torsion abelian groups are non-Borel complete. We also show that if Φ has the Schröder-Bernstein property (that is, whenever two countable models of Φ are biembeddable, then they are isomorphic), then under large cardinals, Φ is not Borel complete.

In Chapter 7, we describe joint work with Saharon Shelah on the complexity of countable torsion-free abelian groups. In particular, we show that if certain large cardinals fail, then torsion-free abelian groups are $a\Delta^1_2$-complete, where $\leq_{a\Delta^1_2}$ is a well-known coarsening of $\leq_B$.

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Victoria Noquez, ***Vaught's Two-Cardinal Theorem and Notions of Minimality in Continuous Logic***, University of Illinois at Chicago, USA, 2017. Supervised by David Marker. MSC: 03C65. Keywords: continuous logic, model theory.

Abstract

Much of the work in this thesis was motivated by an effort to prove a continuous analogue of the Baldwin-Lachlan characterization of uncountable categoricity: a theory T in a countable language is uncountably categorical (has one model up to isomorphism of size κ for some uncountable κ) if and only if T has no Vaughtian pairs and T is ω-stable.

As in the classical setting, we approach the forward direction by proving a continuous version of Vaught's Two-Cardinal theorem. A continuous theory T has a (κ,λ)-model if there is $\mathcal{M} \models T$ with density character κ which has a definable subset with density character λ. We show that if T has a (κ,λ)-model for infinite cardinals $\kappa > \lambda$, then T has an $(\aleph_1,\aleph_0)$-model. We also show that with the additional assumption that T is ω-stable, if T has an $(\aleph_1,\aleph_0)$-model, then for any uncountable κ, T has a $(\kappa,\aleph_0)$-model. This provides us with the tools necessary to prove the forward direction of the Baldwin-Lachlan characterization of uncountable categoricity for continuous logic.

Towards the reverse direction, we introduce a continuous notion of strong minimality, saying that a set is minimal if every subset which is the zero set of a definable predicate is totally bounded, or its approximate complements are all totally bounded. We show that this characterization is equivalent to the classical definition of strong minimality, and that it allows us to use algebraic closure to define a notion of dimension which determines models up to isomorphism. However, the only known examples of strongly minimal theories in the