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Invariant of motion for interstellar dust captured in the solar system

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Abstract. Interstellar dust grains approaching the Sun are influenced mainly by solar gravity, solar electromagnetic radiation and Lorentz force due to the existence of interplanetary magnetic field. These interactions together with the effect of the solar wind on dust grain and the effect of solar cycle are taken into account when modelling behaviour of interstellar dust in the vicinity of the Sun. As a consequence, nonspherical dust grains can be captured and survive in the solar system – they can orbit the Sun in sufficiently large distances from the Sun not to be thermally destroyed. On the other hand, captured spherical dust grains are practically all destroyed. Detailed numerical simulations showed an interesting behaviour of a quantity of the dimension of length cubed divided by time squared. The quantity behaves practically as an invariant of motion: it is a constant during the process when surviving captured interstellar grain is orbiting the Sun. The constancy is fulfilled with an accuracy better than 1%.

Keywords. Dust grain, electromagnetic radiation, Celestial Mechanics

1. Introduction

Particles that enter the Solar System from interstellar space have been identified in the data from their high impact speed and from their impact directions (Grűn *et al.* 1994). The β Pictoris system is assumed to be one of the potential sources of interstellar meteoroids. Baggaley (2000) found that the most significant stream of interstellar meteor particles arrives from a direction that is compatible with being released about 10^6 years ago from β Pictoris at an escape speed of 20 $km.s^{-1}$. Recently, the dust particles within the Solar System were systematically monitored by the Ulysses and Galileo spacecrafts. The typical speed of the detected interstellar dust was found about to be 26 $\mathrm{km\,s^{-1}}$. Frisch *et al.* (1999) reported on the flow of interstellar grains arriving from the upstream direction with the heliocentric ecliptic longitude 259° and latitude of +8°.

The dynamics of dust particles in the Solar System is dominated by the solar gravity, solar electromagnetic radiation and by Lorentz force due to interaction of charged dust grains with the interplanetary magnetic field (Grűn and Landgraf 2001). One of the most recent papers (Jackson 2001) deals with the capture of interstellar dust due to the Poynting-Robertson (P-R) effect. Interstellar particles of the spherical shape vanish in the vicinity of the Sun (they either hit the Sun, or evaporate in the sublimation region close the Sun) (Gulyaev and Shcheglov 1999, Jackson 2001, Kocifaj and Klačka 2003). Morever, from observations of the extinction and polarization by the interstellar medium, it is clear that interstellar particles themselves are not spherical but rather have an elongated structure (Wurm and Schnaiter 2002). Since the P-R effect (Robertson 1937, Klačka 2004) represents only a very special form of equation of motion, which is mainly applicable to particles of spherical shape (see Eq. (120) or Eq. (122) in Klačka 1992; compare Eqs. (40) and (48) in Klačka 2004), we use more general equation of motion. Once the non-sphericity of interstellar particles is established, one can see that their motion considerably differs from the P-R effect (Kocifaj and Klačka 2003, 2004). Trajectories have to be calculated in detail and the final form of the orbit depends on the shape of the particle, on its size and optical properties.

In this paper the motion of interstellar dust particles (ISDPs) in the Solar System is investigated. Gravitational force of the Sun, solar electromagnetic and corpuscular radiation and interplanetary magnetic field are considered. The effect of solar electromagnetic radiation plays an important role in the sense that non-spherical ISDPs can be captured (and survive) much more effectively than spherical particles. It turns out that particles of effective radii $\approx 0.4\ \mu m$, moving initially near the solar equatorial plane and with impact parameter $400\ R_S \lesssim b \lesssim 500\ R_S$ (solar radii) exhibit a high probability of capture and survival in the Solar System (Kocifaj and Klačka 2004). Only a very small number of spherical particles can be captured. An invariant of motion is presented for the motion of non-spherical interstellar grains.

2. Model and equation of motion

All our numerical simulations are based on equation of motion written in the form (see Klačka and Kocifaj 2001, for more details)

$$\begin{aligned} \frac{d\vec{v}}{dt} = &- \frac{4\ \pi^2}{r^2}\ \vec{e}_R\ +\ \frac{q}{m}\,(\vec{v} - \vec{v}_{sw}) \times \vec{B} + \\ &\beta\ \frac{4\ \pi^2}{r^2} \sum_{j=1}^{3} \frac{Q'_{prj}}{Q'_{pr1}} \left\{ \left(1\ -\ 2\ \frac{\vec{v} \cdot \vec{n}_1}{c}\ +\ \frac{\vec{v} \cdot \vec{n}_j}{c} \right) \vec{n}_j\ -\ \frac{\vec{v}}{c} \right\}, \\ \vec{n}_1 \equiv \vec{e}_R \equiv \vec{r}/|\vec{r}|;&\quad \beta = 7.6 \times 10^{-4}\ Q'_{pr1}\ \frac{A'\ \ [m^2]}{m\ \ [kg]}, \end{aligned} \tag{2.1}$$

where $\vec{v}$ (measured in units $[AU year^{-1}]$) is the dust grain's heliocentric velocity, $\vec{r}$ (measured in AU) is the position vector of the particle with respect to the Sun, A' is the geometrical cross section of a sphere of volume identical to the volume of the particle ($A' = \pi a'^2_{eff}$, where a'_{eff} is the "effective radius"), m is the mass of the particle, Q'_{pr1}, Q'_{pr2}, Q'_{pr3} are components of the radiation pressure efficiency factor vector; see Eq. (27) in Klačka and Kocifaj (2001). As for the Lorentz acceleration, the charge of the grain is $q = 4\pi\varepsilon_0 U a'_{eff}$; ε_0 is the permitivity of vacuum, U is the surface potential given by Kimura and Mann (1998), radially expanding solar wind is given by

$$\vec{v}_{sw} = v_{sw}\hat{\vec{e}}_R, \tag{2.2}$$

magnetic field is considered in the form (Parker 1958, Grűn *et al.* 1994)

$$\begin{aligned} \vec{B} &= B_R \hat{\vec{e}}_R + B_T \hat{\vec{e}}_T, \\ B_R &= B_{R0}\,(r_0/r)^2 \cos(\pi * t[years]/11 + \varphi_0), \\ B_T &= B_{T0}\,(r_0/r) \cos\vartheta \cos(\pi * t[years]/11 + \varphi_0), \\ B_{R0} &= B_{T0} = 3nT, \\ r_0 &= 1AU,\ \ \hat{\vec{e}}_T = \hat{\vec{\omega}} \times \hat{\vec{e}}_R, \end{aligned} \tag{2.3}$$

where $\hat{\vec{\omega}}$ defines magnetic axis of the Sun, ϑ is the altitude from the solar equatorial plane; different orientations of magnetic field for northern and southern hemispheres are also considered.

We simulated the continuous flow of interstellar dust by the particle stream with time-independent volume concentration and random distribution of tilts of particle rotation axes (uniform in solid angles) at large distances from the Sun. Particles enter the Solar System from interstellar space from their upstream direction (Grűn *et al.* 1994). We initially consider the particle to approach the Sun from infinity with velocity v_∞ = 26 km/s (Landgraf *et al.* 1999, Jackson 2001) with an impact parameter b. We assume spatial dependency of dust surface potential according to Kimura and Mann (1998) with value +3V in the central part of the Solar System. The slow solar wind component of 400 $km.s^{-1}$ is taken into account. The phase angle $\varphi_0 = 0$ of magnetic field is assumed.

As for the model of the cosmic dust particle we have used particle U2015B10 (Clanton *et al.* 1984; Kocifaj *et al.* 1999). The particle shape appears to be representative of cosmic dust, since its aspect aspect ratio (equal to 1.4) coincides with the results obtained by means of the mid-infrared spectropolarimetry (Hildebrand and Dragovan 1995): the interstellar grains were identified to be oblate and the best fit of the aspect ratio lies between 4/3 and 3/2. The characteristics of radiation scattered by the cosmic dust particle were calculated using the discrete dipole approximation (DDA; Draine 1988). The advantage of the DDA method is its easy application to various geometries and material configurations. Our calculations are limited to magnesium-rich silicate particles, as these could be representative for interstellar dust grains (Dorschner *et al.* 1995). We are interested in particles of the radius 0.4 μm moving from the infinity in plane $\alpha = 90°$, where α is the particle position angle within dust stream (Kocifaj and Klačka 2004). The slope of particle rotation axis is fixed during simulation. Impact parameters vary between 150 and 500 solar radii. Such input parameters corespond to particles which survive in the Solar System with high probability: it was shown in Kocifaj and Klačka (2004) that charged grains can be captured only in a very narrow belt near ecliptic plane and surviving charged grains (grains are not sublimated, i.e. their perihelion distances from the Sun are large enough) exist mainly for impact parameters $b > 150$ solar radii $\equiv$ 0.7 AU.

3. Invariant of motion

In spite of the fact that real motion of ISDPs in the Solar System is very complicated, we have succeeded in finding a quantity which can be considered to be a constant of motion for the case when the ISDPs revolve around the Sun. The motivation comes from Kepler's third law which holds for two-body problem: $(a[AU])^3/(T[years])^2 = 1 - \beta$, where a is semi-major axis, T is period of revolution around the Sun and β is a nondimensional quantity and is often called the ratio of radiation pressure force (it's radial component independent of velocity) to the gravitational force. However, our physical situation is much more complicated and it is evident that the semi-major axis is not conserved during the revolution. Fig. 1 depicts two possibilities which are of the same dimension as the third Kepler's law: $< (r[AU])^3 > /(T[years])^2$ and $< r[AU] >^3 /(T[years])^2$, where r is the actual distance of the ISDP from the center of the Sun and the symbol $< x >$ denotes time averaging of the quantity x over the period T. One can see from Fig. 1 that $< r[AU] >^3 /(T[years])^2$ exhibits a large dispersion (from 0.15 up to 0.36) during the motion of the ISDP. However, surprisingly, $< (r[AU])^3 > /(T[years])^2$ is approximately constant (the values vary from 0.64 up to 0.69), i. e. it behaves as an

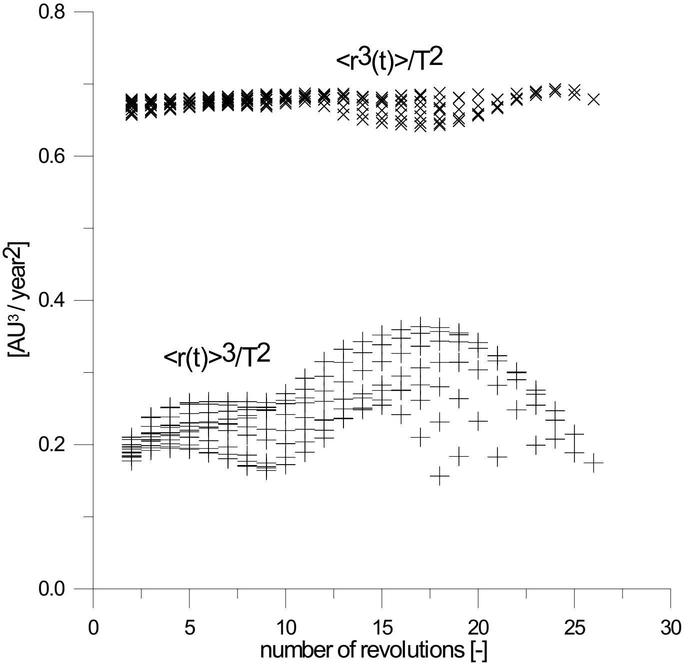

Figure 1. Dependency of quantities $\langle r^3(t)\rangle/T^2$ and $\langle r(t)\rangle^3/T^2$ on number of revolutions around the Sun. The distribution contains averaged data over each revolution for particles moving in region $b \in (300, 500)$ solar radii. Model characteristics: particle radius: $a'_{eff} = 0.4\ \mu m$, incident slope of particle rotation axis with respect to the orbital plane: $\theta = 40°$, $\phi = 0°$.

approximate invariant. Indeed, the upper part of Fig. 2 confirms this statement, and, moreover, $< (r[AU])^3 > /(T[years])^2 = 0.673 \pm 0.002$; the values lie in the interval 0.671 to 0.675. However, this numerical value is not consistent with the value $1 - \beta$, as it is in the case of exact Kepler's third law: in our case $\beta = 0.4847$. Moreover, while the obtained value of $< r^3 > /T^2$ corresponds to Keplerian motion for eccentricity $e = 0.318$, the small values of $< r >^3 /T^2$ cannot be understood in terms of Keplerian motion:

$$\frac{\langle r^3\rangle}{T^2} = (1-\beta)\ (1-e^2)^{9/2}\ \frac{1}{2\ \pi}\ \int_0^{2\pi} \frac{dx}{(1+e\cos x)^5},$$
$$\frac{\langle r\rangle^3}{T^2} = (1-\beta)\ (1-e^2)^{15/2}\ \left\{\frac{1}{2\ \pi}\ \int_0^{2\pi} \frac{dx}{(1+e\cos x)^3}\right\}^3. \tag{3.1}$$

(Central acceleration $-\ 4\pi^2(1-\beta)\ \vec{e}_R/r^2$ is used for calculations of orbital elements.)

The question how the approximate invariant depends on impact parameter is answered in Fig. 2: the relation $< (r[AU])^3 > /(T[years])^2 \approx 0.673$ may be considered as an expression independent of impact parameter. Relative errors of the considered quantities are presented in Fig. 3. The reduction of error with an increase of initial impact parameter b for non-spherical particles is evident. Such behaviour can be observed for both functions: $< r(t)^3 > /T^2$, and $< r(t) >^3 /T^2$. Moreover, there is a significant gap between the curves (one order in magnitude). In general, the relative error has a decreasing trend with b and seems to be independent of number of revolutions of ISDP around the Sun.

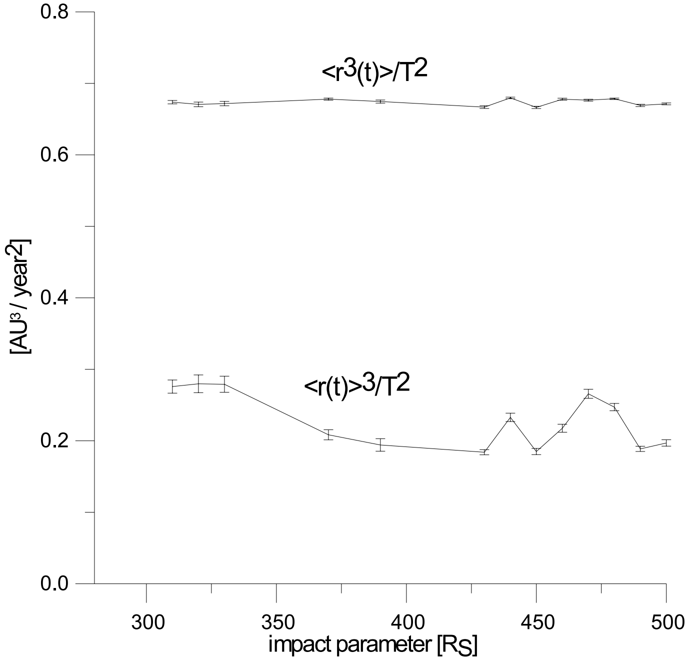

Figure 2. Dependency of quantities $\langle r^3(t)\rangle/T^2$ and $\langle r(t)\rangle^3/T^2$ (averaged over total number of revolutions) on initial impact parameter b. The error bars define mean square deviations. Model characteristics are the same as in Fig. 1.

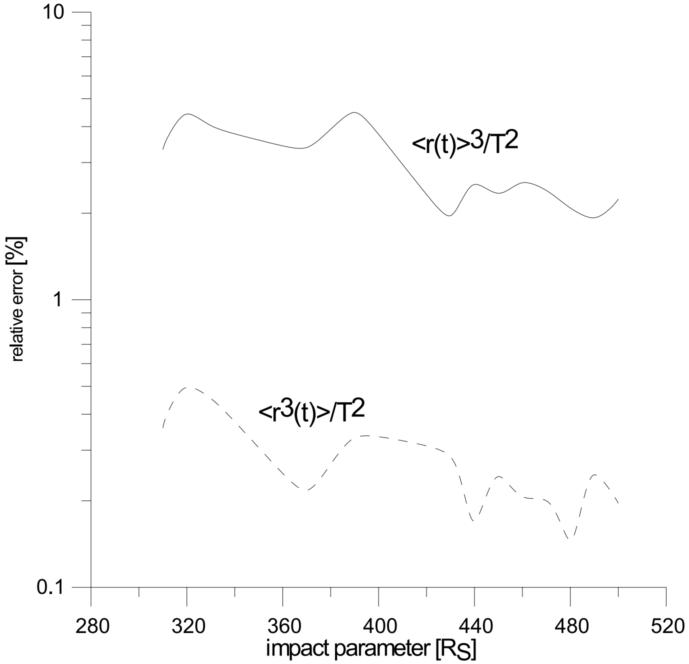

Figure 3. Relative error of $\langle r^3(t)\rangle/T^2$ and $\langle r(t)\rangle^3/T^2$ as a function of initial impact parameter b. Model characteristics are the same as in Fig. 1.

4. Conclusion

The characteristics of motion of the interstellar dust particles (ISDPs) in the Solar System were investigated. Gravitational force of the Sun, solar electromagnetic and corpuscular radiation and interplanetary magnetic field were considered. The effect of solar electromagnetic radiation plays an important role in the sense that non-spherical ISDPs can be captured (and survive) much more effectively than spherical particles. Surviving non-spherical ISDPs orbiting around the Sun are characterized by a nearly constant value of $< r^3 > /T^2$, where T is orbital period and $< r^3 >$ is time average of cubed distance from the Sun – time average over the period T: $< r^3 > /T^2 = 0.673 \pm 0.002$. The validity of quasi-constant profile of $< (r[AU])^3 > /(T[years])^2$ needs to be analysed for different types of cosmic dust particles in the future works. In any case, such a fact could be efficiently utilized in modelling of ISDPs dynamics in the circumsolar region.

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