

## INFLUENCE OF $^{14}\text{C}$ CONCENTRATION CHANGES IN THE PAST ON STATISTICAL INFERENCE OF TIME INTERVALS

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**ABSTRACT.** The influence of the calibration curve on the statistical inference of time intervals was investigated. For this purpose, the calculation of the summed probability density function was used. Computer simulations were done for batches of 11 samples, each time uniformly covering 200-yr time intervals. The results show that the calibration curve causes the summed probability density function of a group to cover a wider interval than the real-time interval of the phenomenon. Moreover, the estimated time interval may be often shifted in relation to the real-time interval.

### INTRODUCTION

Geologists and archaeologists often face the problem of how to assign time intervals to a phenomenon (geological phase or archaeological culture) based on a group of radiocarbon dates from artifacts associated with this phenomenon. One of the treatment methods for such a group of related  $^{14}\text{C}$  dates is to create summed probability density distributions by summing distributions of the relevant calibrated dates. The 50% confidence interval (interquartile range), which corresponds with the conception of the *floruit* of a culture (Aitchison et al. 1991), or the highest probability confidence intervals of this distribution are usually interpreted as the time limits of the phenomenon. However, the changes of  $^{14}\text{C}$  concentration in the past represented by the calibration curve cause the relation between the  $^{14}\text{C}$  and the calendar (calibrated) date to be ambiguous, and the time intervals estimated by this method may be very different from the actual time intervals in the calendar scale.

The influence of the calibration curve on the statistical inference of time intervals was first mentioned in the late 1980s by Stuiver and Reimer (1989). They showed that in unfavorable circumstances, a large distortion of probability distribution may occur; in their example, the part of the calculated summed probability distribution retained in the original (real) time interval is about 50%. In 1994, McFadgen et al. (1994) discussed the distortion of histograms of calibrated  $^{14}\text{C}$  dates from New Zealand. They emphasized that the “calibrated stochastic distortion (CSD)” results from composition of the statistical spread of measured  $^{14}\text{C}$  ages and the ambiguous character of the calibration curve. This paper presents a more complex diagnosis of the problem.

### METHODS

Analysis was carried out for 200-yr-wide time intervals; i.e., it was assumed that the real timespan of the investigated phenomenon was equal to 200 calendar yr. Because the current work was mainly concerned with the influence of the calibration curve, the following assumptions were made:

- The frequency distribution of the artifacts is uniform for the time interval range of the phenomenon;
- The calendar ages of the collected samples (artifacts) uniformly cover the whole 200 yr of the time interval.

The calculations were performed in the following order:

- It was assumed that 11 samples (artifacts) were collected in the 200-yr time interval under investigation. The calendar ages of the samples were uniformly spread across the interval, with the first one falling at the beginning of the interval and the subsequent ones in 20-yr increments.
- For the calendar date of each sample, the corresponding  $^{14}\text{C}$  age was determined from the calibration curve.
- Each of the  $^{14}\text{C}$  dates was calibrated with the assumption that the error of the  $^{14}\text{C}$  date was equal to 25 yr.
- All probability density distributions obtained as a result of the calibration were calculated in order to discern the summed probability density distribution.
- On the basis of this summed probability density distribution, the following intervals were calculated: interquartile range (*floruit*), the highest probability 50% confidence interval, and the highest probability 95% confidence interval.

Calculations were carried out using the revised and updated calibration module of the Gliwice Radiocarbon Laboratory Calibration Program, GdCALIB (Pazdur and Michczyńska 1989; Michczyńska et al. 1990), and the INTCAL98 calibration curve (Stuiver et al. 1998).

## RESULTS AND DISCUSSION

Figure 1 shows how the calibration curve may influence the summed probability density function and the intervals estimated using this function. In the ideal case (Figure 1a), the obtained distribution should be similar to the uniform distribution. It would present the frequency distribution of the artifacts, and the calculated intervals would be quite a good estimate of the real-time interval. However, when we take into consideration the influence of the calibration curve (Figure 1b), the results become distorted. In Figure 1b, a few types of distortion of calculated intervals can be seen, namely:

- The highest probability intervals are discontinuous (especially the 50% confidence interval);
- All intervals are distinctly longer than in the ideal case; the 95% interval is much longer than the real-time interval;
- A large part of each interval—even the interquartile range and the highest probability 50% confidence interval—is outside of the real-time interval;
- The interquartile interval is clearly shifted in relation to the real-time interval.

Figure 1c shows a part of calibration curve that corresponds with the time interval considered in Figure 1b. We note that the boundaries of the summed probability distribution presented in Figure 1b are in concordance with the boundaries of the flat and ambiguous part of the calibration curve. This means that the calibration procedure spreads the analyzed time interval and causes the distortion mentioned above. It should be emphasized that these distortions are not a consequence of the calculation method used. Figure 2 shows a comparison of the summed probability density distribution for 11 samples from the time interval 2380–2580 cal BP (the same as in Figure 1b), with the probability distributions of the first and the last event in this group of dates calculated using OxCal v.3.8. It is clearly visible that both methods give the same differences between calculated limits and real limits of the analyzed time interval.

The question remains: How important are these distortions for particular time intervals in the past? In order to answer this question, the 2 factors were calculated for each of the 200-yr time intervals from 0–200 cal BP to 13,800–14,000 cal BP. The factors were defined as follows:

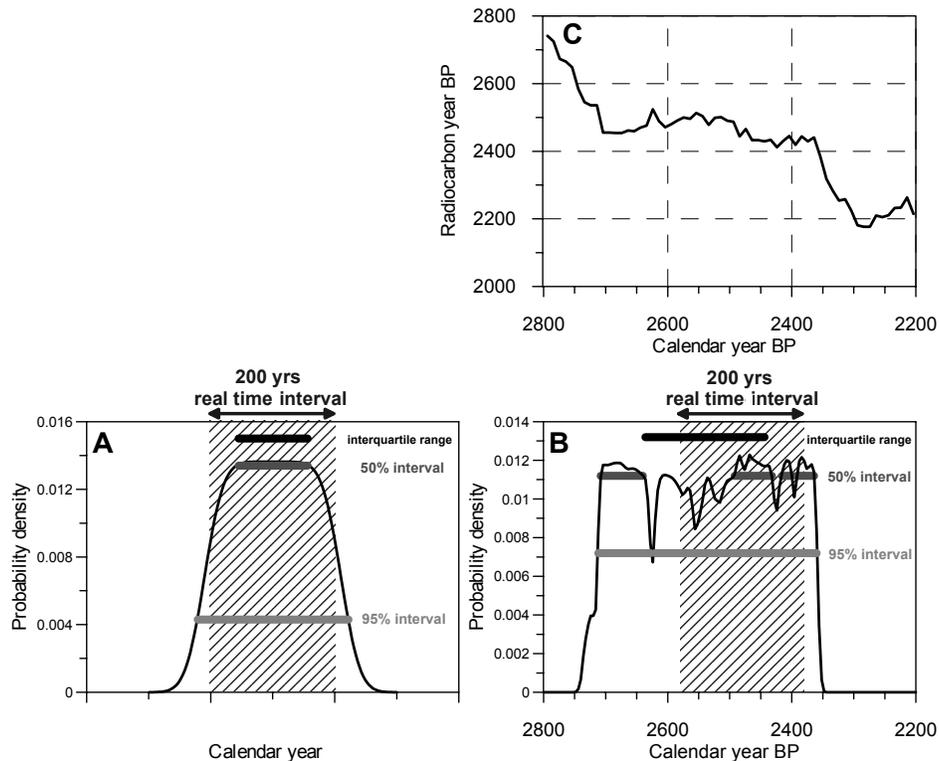


Figure 1 (a) The ideal case: the summed probability density function and the appropriate intervals for 11 samples from the 200-yr time interval obtained by summed Gauss distributions (the influence of the calibration curve was omitted); error of dates = 25 yr; (b) Summed probability density function and appropriate intervals for 11 samples from the time interval 2380–2580 cal BP; error of dates = 25 yr; (c) Part of the calibration curve (Stuiver et al. 1998) that corresponds with the time interval 2380–2580 cal BP.

- *The Common to Input Overlap Factor* (CI Factor) is equal to the length of the common part of the real-time interval (Input Interval) and the calculated interval (Output Interval) divided by the length of the Input Interval (see Figure 3). The CI Factor tells us which part of the real-time interval overlaps with the calculated interval.
- *The Common to Output Overlap Factor* (CO Factor) is equal to the length of the common part of the real-time interval (Input Interval) and the calculated interval (Output Interval) divided by the length of the Output Interval. The CO Factor tells us which part of the calculated interval overlaps with the real interval.

The values of the Overlap Factors for the analyzed interval (CO Factor, CI Factor) should be compared with the values for the ideal case (CO Factor<sub>ideal case</sub>, CI Factor<sub>ideal case</sub>) in order to draw conclusions concerning the distortion of the interval. The most important conclusions are the following:

- When the CO Factor < CO Factor<sub>ideal case</sub>, the calculated interval is wider than the real interval or shifted in relation to the real one;
- When the CI Factor > CI Factor<sub>ideal case</sub>, the calculated interval is wider than the real interval;
- When the CI Factor < CI Factor<sub>ideal case</sub>, poor agreement between the calculated and the real interval exists, and the calculated interval may be shifted in relation to the real one.

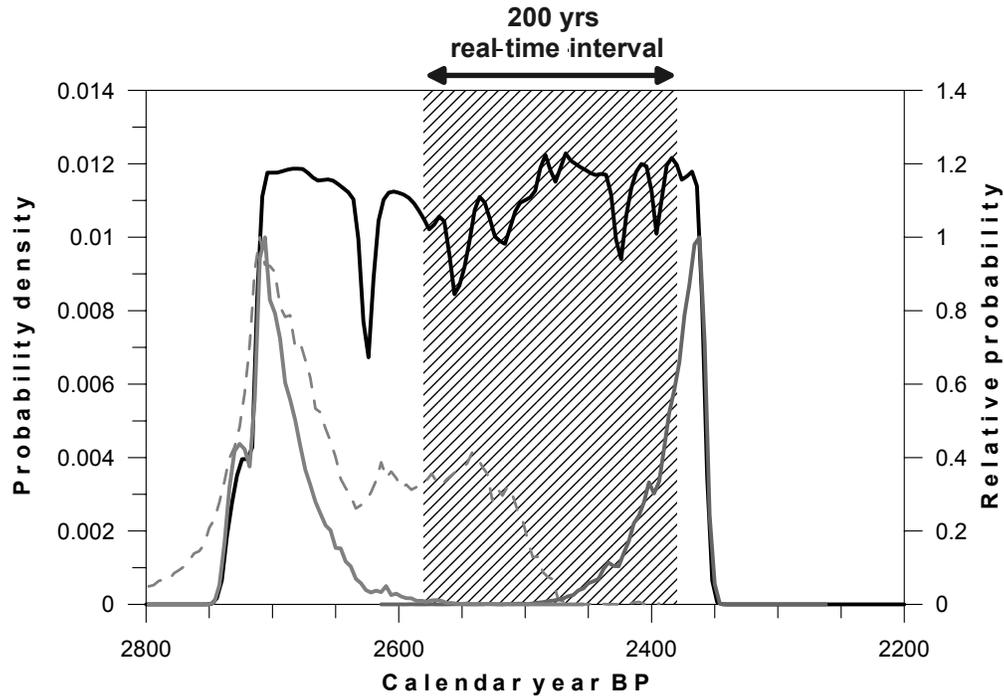


Figure 2 Comparison of the summed probability density distribution for 11 samples from the time interval 2380–2580 cal BP (black line), with the probability distributions of the first and the last event in this group of dates calculated using OxCal v.3.8 (grey, continuous lines). The grey dashed line shows the probability distribution of the last event in the group calculated using the “boundary” option of OxCal v.3.8. It should be emphasized that this distribution comprises the limit of the real-time interval, but the maximum of the distribution is still shifted.

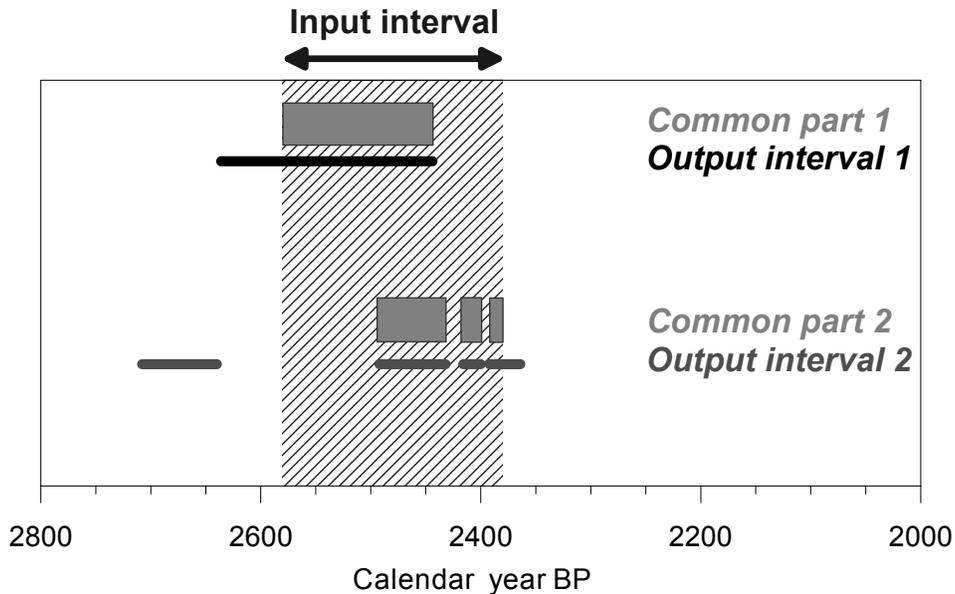


Figure 3 An example of the relationship between the real interval (Input Interval), the interval calculated on the basis of the summed probability density function (Output Interval), and the common part of these intervals.

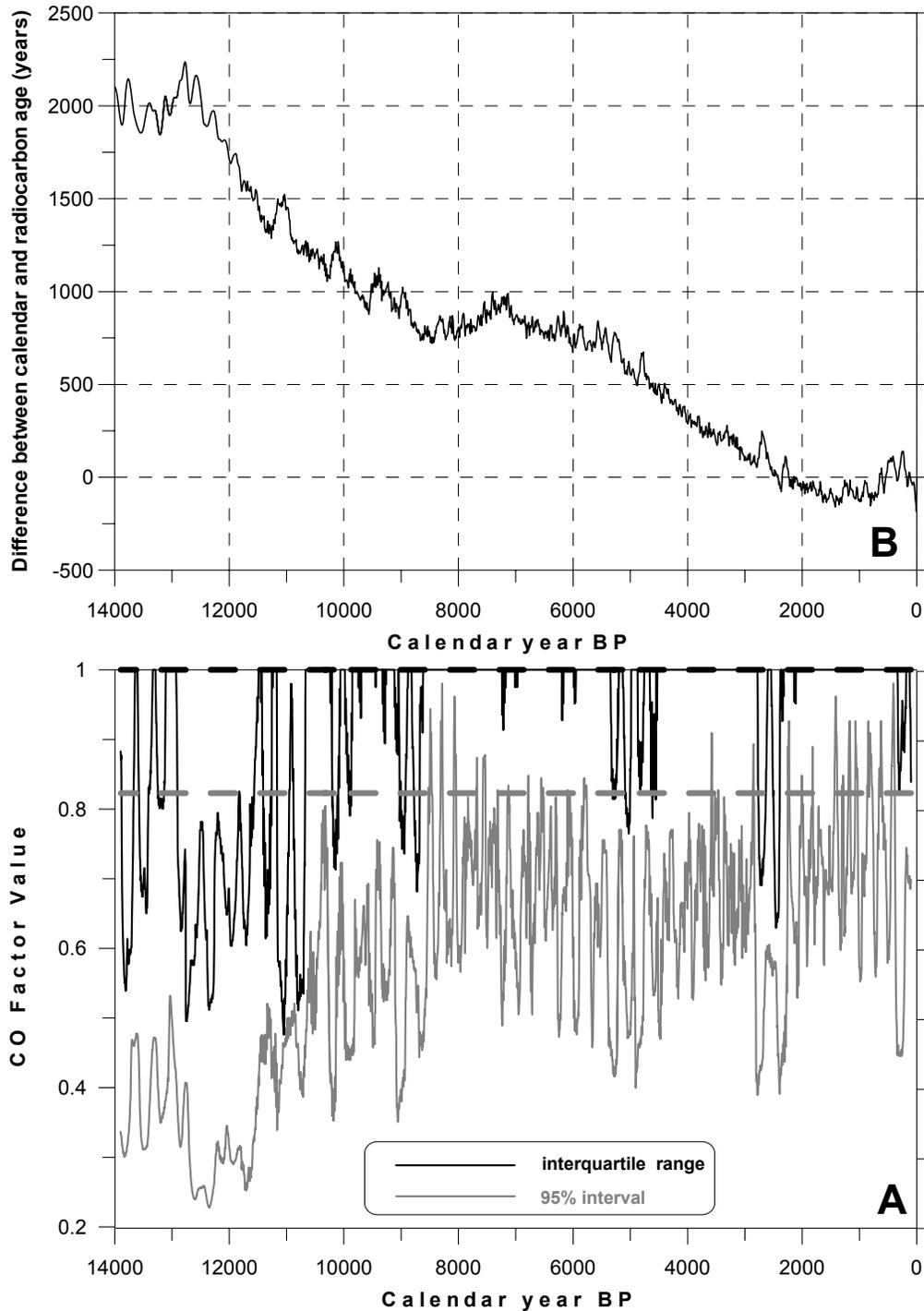


Figure 4 (a) Common to Output Factors for the interquartile range and the highest probability 95% confidence interval calculated for each 200-yr time period from 0–200 cal BP to 13,800–14,000 cal BP. The black and the grey dashed lines show the values of the CO Factor in the ideal case for the interquartile interval and for the highest probability 95% confidence interval, respectively; (b) Values of the difference between calendar age and corresponding  $^{14}\text{C}$  age calculated on the basis of the calibration curve.

The results of calculations of Overlap Factors are shown in Figure 4 and Figure 5. The calculated values of the factors for a particular interval were assigned to the middle of that interval—i.e., values assigned, for example, to 2000 cal BP, correspond with time interval 1900–2100 cal BP. Figure 4a shows that for almost the entire 14,000-yr period, the value of the CO Factor for the highest probability 95% confidence interval is lower than in the ideal case. Likewise, the value of the CO Factor for the interquartile interval is often lower than 1 (CO Factor value in the ideal case). It allows us to draw the conclusion that the calibration curve causes the summed probability density function to cover a wider interval than the real-time interval of the phenomenon. Moreover, the low values of the CO Factor for the interquartile range suggest that the calculated intervals are shifted in relation to the real intervals. The relation between the shape of the calibration curve and the value of the CO Factor can be clearly seen when we compare Figures 4a and 4b. Figure 4b presents values for the difference between calendar age and corresponding  $^{14}\text{C}$  age calculated on the basis of the calibration curve. We note that the lowest values of the CO Factor correspond with the major wiggles which are characteristic of the ambiguous region of the calibration curve.

In Figure 5, we observe that the values of the CI Factor for the interquartile range are almost always greater than the value of this factor in the ideal case. It confirms that the calibration curve extends the interval comprised by the probability density distribution, and thereby, the intervals calculated on the base of this distribution. On the other hand, the values of the CI Factor for the highest probability 50% confidence interval are, in most cases, greater than the value in the ideal case, and, in some cases, less than this value. Thus, the calculated intervals may be shifted in relation to the real-time interval. It should be emphasized that the CI Factor values for the interquartile range are always greater than for the highest probability 50% confidence interval.

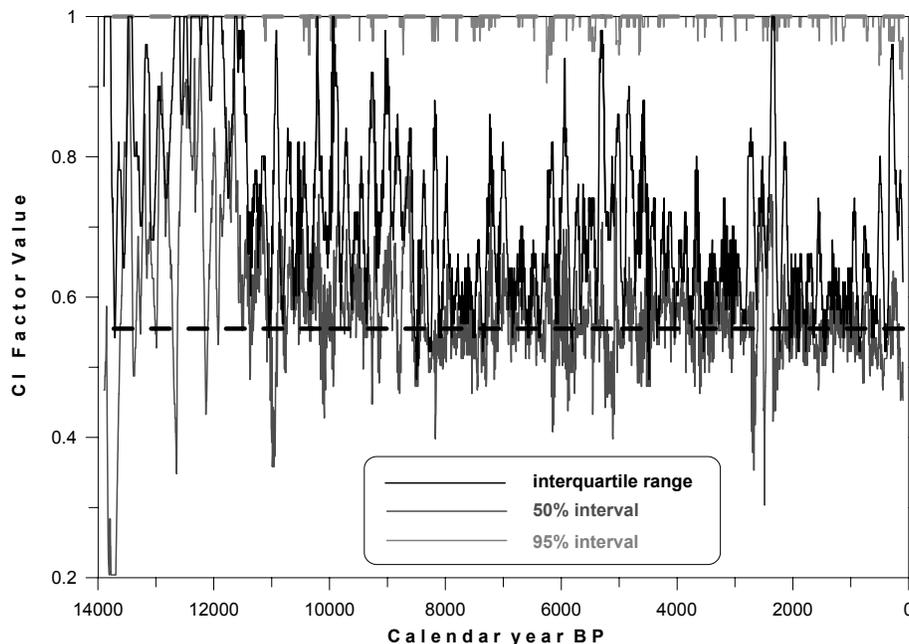


Figure 5 *Common to Input Factors* for the interquartile range, the highest probability 50% confidence interval and the highest probability 95% confidence interval calculated for each 200-yr time period from 0–200 cal BP to 13,800–14,000 cal BP. The black and the grey dashed lines show the values of the CI Factor in the ideal case for the interquartile interval (and the highest probability 50% confidence interval) and for the highest probability 95% confidence interval, respectively.

In order to assign the periods when calculated intervals are shifted in relation to the real-time intervals, the end and beginning values of the interquartile interval were calculated in relation to the beginning of the real interval. Figure 6 presents these values calculated for each of the 200-yr time intervals from 0–200 cal BP to 13,800–14,000 cal BP. It can be seen that for the greater part of the analyzed 14,000-yr period, the interquartile interval is contained in the real interval; therefore, the first may be used as an estimate of the latter. However, for about one-third of the 14,000 yr, we may observe a distinct shift of the interquartile interval (and consequently, the probability density function) in relation to the real interval. This shift is extremely large for the period past 10,500 cal BP.

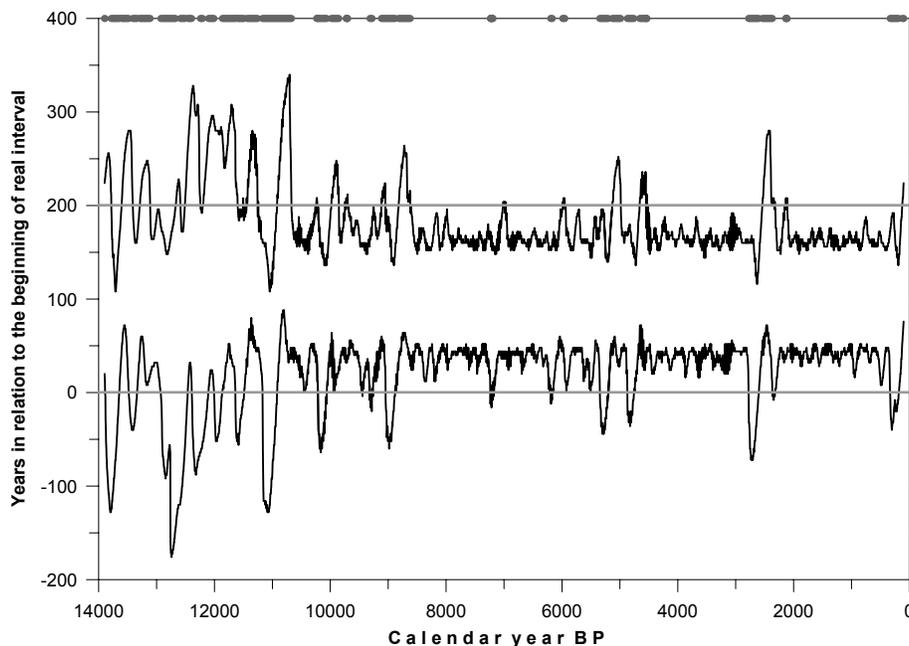


Figure 6 The values of the beginning and the end of the interquartile range shown in relation to the beginning of the real interval calculated for each 200-yr time interval from 0–200 cal BP to 13,800–14,000 cal BP. Grey continuous lines show the limits of the real-time interval. Grey dashed lines on the top of the figure indicate periods when a large shift of the interquartile interval in relation to the real interval is observed.

## CONCLUSIONS

This study shows that the influence of the calibration curve on the statistical inference of time intervals is very important. First of all, the calibration curve always causes the summed probability density function of a group of  $^{14}\text{C}$  dates associated with a phenomenon to cover a wider interval than the real time period of this phenomenon. Moreover, we may observe a distinct shift of the summed probability density function in relation to the real-time interval for about one-third of the 14,000 yr. The results also show that the interquartile interval estimates the length of the real time period of the phenomenon better than the highest probability 50% confidence interval and the highest probability 95% confidence interval.

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