Probability, Statistics and Truth by Richard von Mises. George Allen and Unwin, London, 1957. ix + 244 pp. 28/-.

This is the second English edition of von Mises' well-known lectures on probability and statistics. It has been largely retranslated from the third German edition of 1951 which was extensively revised by the author, two years before his death. The present edition has been prepared by his widow, Dr. Hilda Geiringer.

Richard von Mises was one of the most ardent advocates of the frequency definition of probability and contended that this was the only definition that made any real sense. It is easy to see the inadequacies of the Laplacian classical definition in terms of "equally likely" possibilities, one which never would have gained wide currency had it not served as a fairly adequate mathematical model for common games of chance like tossing coins, rolling dice and dealing cards. With standard patterns of coins, dice and decks of cards, and with customary techniques of handling them, it seems quite reasonable to assume that all the possible individual cases in a given situation are in fact "equally likely", - if we can interpret these words in a sense which does not require us to argue in a circle. The definition obviously fails with a biased coin or die, which is so constructed that all possibilities are not equally likely, and more important, it has no relevance to such things as the probability that a white Canadian male, aged 30, will live to the age of 65, which is of practical interest to an insurance company.

A totally different idea of probability is that it measures the intensity or degree of rational belief in a proposition, as, for example, that the first five books of the Bible were actually written by Moses. This notion is certainly behind many common uses of the word "probable", and has been championed by Keynes, Jeffreys, and others, but the difficulty is to link it in some reasonable way with the mathematical calculus of probability. Some principle such as the so-called "Principle of Indifference" is required, allowing us to say that if there is no known reason for preferring one alternative to another, then, relative to the present state of our knowledge, the two are equally probable. The alternatives must not be capable of being further split up ("red" and "not-red" are not alternatives in this sense) and there must be no relevant evidence relating to one alternative unless

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there is corresponding evidence relating to the other. In some such way as this, the proponents of subjective probability manage to arrive at the customary rules of the calculus, but it is not at all evident what are the equi-ranking alternatives in such a case as that mentioned above of Moses and the Pentateuch.

Von Mises argued that the term probability can be applied only to events which form part of an infinite sequence of events, and indeed only when this sequence (then called a "collective") has some very special properties. One property is that the relative frequency of occurrence of the particular attribute in question must tend to a definite limit (which is the probability of this attribute) as the number of events tends to infinity. Another is that this relative frequency must in the limit be the same for all sub-sequences picked out from the original sequence by any rule whatsoever (provided of course that the choice of any item for the sub-sequence does not depend on the attribute of that particular item). This sounds like a very serious set of restrictions and it may well be asked whether such a thing as a collective actually exists. Von Mises' answer is that a collective is an ideal concept like the concept of a "particle" or a "rigid body" in dynamics, and that in the real world there are long series of events, such as the record of roulette plays at Monte Carlo, which exhibit a close approximation to the properties of a collective.

Some writers, including the most sophisticated, avoid the difficulty of defining probability by treating it as a mathematical concept. It is introduced as a special kind of "measure", non-negative, normed, and completely additive, defined on a σ -field of sets of points in an abstract space. This "definition" leads to the usual rules for calculating probabilities, but is hardly satisfactory to the man who wants to know what he is really talking about. Von Mises has little sympathy with those who would view probability as merely a part of the theory of sets. One might just as well say that quantum theory is merely a part of the theory of groups. Probability for him is a natural science, dealing with certain kinds of observable phenomena which more or less resemble ideal collectives. Set theory (and in particular the theory of integration) is a tool for the solution of problems in probability.

The discussion of all these matters, and the application of probability to statistics and to physics is the subject matter of this book. The six lectures which compose it are almost entirely non-mathematical in character, but contain closely-reasoned and very illuminating treatments of such difficult matters as the meaning of randomness, Bernoulli's theorem, Bayes's theorem, the Laws of Large Numbers, and Fisher's concept of "likelihood". Any student of probability and statistics is likely to find his understanding of these topics improved by reading von Mises' discussion. The section on the Laws of Large Numbers is particularly rewarding.

Since, in von Mises' scheme, probabilities relate only to collectives, it is necessary in order to carry out the usual calculations to be able to form new collectives out of given ones. Four fundamental operations are described, called "selection", "mixing", "partition", and "combination". Roughly speaking, the first corresponds to the requirement of randomness, the second to the addition rule for mutually exclusive events. The solution of the historic Chevalier de Mere's problem (on the chance of seeing 6 at least once in four throws of a die) is carried out in detail by means of these operations - a solution which takes about five pages and strikes anyone familiar with the customary treatment as rather long and cumbrous. It does, however, bring out the meaning of the probabilities involved.

In the last sentence of the book, the author justifies his title: "Starting from a logically clear concept of probability, based on experience, using arguments which are usually called statistical, we can discover truth in wide domains of human interest." The quest for final completely deterministic theories of the world, which might satisfy our desire for causality, is, in von Mises' view, nothing but a prejudice. The sooner it disappears entirely, the better. Let us accept the fact that probability is the basis of a great part of the physical world.

E.S. Keeping, University of Alberta