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# Reconstructing the Primordial Power Spectrum

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**Abstract.** We reconstruct the shape of the primordial power spectrum from the latest cosmic microwave background data, including the new results from the Wilkinson Microwave Anisotropy Probe (WMAP), and large scale structure data from the two degree field galaxy redshift survey (2dFGRS). We discuss two parameterizations taking into account the uncertainties in four cosmological parameters. First we parameterize the initial spectrum by a tilt and a running spectral index, finding marginal evidence for a running spectral index only if the first three WMAP multipoles ($\ell = 2, 3, 4$) are included in the analysis. Secondly, to investigate further the low CMB large scale power, we modify the conventional power-law spectrum by introducing a scale above which there is no power. We find a preferred position of the cut at $k_c \sim 3 \times 10^{-4}$ Mpc$^{-1}$ although $k_c = 0$ (no cut) is not ruled out.

## 1. Introduction

Measurements of the Cosmic Microwave Background (CMB) radiation have taken a leap forward with the recent announcement of the findings of the Wilkinson Microwave Anisotropy Probe (WMAP). With these new data it is possible to set important constraints on the shape of the primordial power spectrum and hence to begin to differentiate between the plethora of models for the early universe. One of the most intriguing results comes from a a combined analysis of the WMAP data with the two degree field galaxy redshift survey (2dFGRS) data (Spergel et al. 2003, Peiris et al. 2003), which indicates that the primordial power spectrum might have curvature. The addition of Lyman-$\alpha$ forest data on smaller scales to strengthen this conclusion is however contentious (Seljak 2003). The low quadrupole and octopole observed in the CMB temperature power spectrum (Spergel et al. 2003) has a low probability in standard models, and may be an indication of some feature in the initial power spectrum on very large scales.

Both inflationary Big-Bang (Guth 1981, Linde 1982, Albrecht & Steinhardt 1982) and more speculative cyclic ekpyrotic (Steinhardt & Turok 2002) models of the early universe predict very nearly Gaussian scalar perturbations in the primordial radiation dominated era. The shape of the perturbation power spectrum depends on the exact model, which typically involves various unknown parameters. The objective of this paper is to constrain the shape of the initial power spectrum directly from observational data with as few assumptions as possible.

A wealth of cosmological information can be obtained from analysing the shape of the cosmic microwave background (CMB) radiation temperature fluctuation power spectrum. However it has been shown that the effect of changing the cosmological parameters can be exactly mimicked by changes in the shape of the primordial power spectrum (Souradeep et al. 1998, Kinney 2001). By including measurements of the CMB polarization and the late time matter power spectrum, as measured for example by galaxy redshift surveys, the degeneracy can be broken because the cosmological parameters affect these data in different ways. In this paper we combine temperature and polarization data from the WMAP observations and other CMB observations on smaller scales with constraints on the matter power spectrum from the 2dFGRS (Percival et al. 2002).

Since the initial power spectrum is an unknown function one is forced to parameterize it. There are numerous possibilities. Wang, Spergel & Strauss (1999), Wang & Matthews (2002) and Mukherjee & Wang (2003c) use a number of bands in wavenumber. Mukherjee & Wang (2003a), Mukherjee & Wang (2003b) and Mukherjee & Wang (2003c) also use a model independent approach but using wavelets. Barriga et al. (2001) test a particular inflationary model, in which a phase transition briefly halts the slow roll of the inflaton.

## 2. Framework

The primordial scalar power spectrum $P_\chi$ is defined by $\langle|\chi|^2\rangle = \int \mathrm{d}\ln k\, \mathcal{P}_\chi(k)$, where $\chi$ is the super-horizon comoving curvature perturbation in the early radiation dominated era. The commonly assumed power-law power spectrum parameterization is then

$$\mathcal{P}_\chi(k) = A_s \left(\frac{k}{k_{s0}}\right)^{n_s-1} . \tag{1}$$

Here $n_s(k) = d\ln\mathcal{P}_\chi(k)/d\ln k + 1$ is the conventional definition of the scalar spectral index, where $n_s = 1$ corresponds to a scale invariant power spectrum (we use $k_{s0} = 0.05\ \mathrm{Mpc}^{-1}$ throughout). The power spectrum amplitude $A_s$ determines the variance of the fluctuations, with $A_s^{1/2} \sim 5 \times 10^{-5}$ to give the observed CMB anisotropy amplitude.

In slow roll inflationary models, it is expected that the spectrum varies only very slowly and that $|n_s - 1|$ is much smaller than unity (Lyth & Riotto 1999). In general there is a direct relation between the potential of the inflaton field and the spectral index. As the potential evolves during inflation the spectral index can vary slightly as different modes leave the horizon. This can be characterized by including a second order term in the logarithmic expansion of the power spectrum $n_{\rm run}$, defined by

$$\ln \mathcal{P}_\chi(k) = \ln A_s + (n_s - 1)\ln\left(\frac{k}{k_{s0}}\right) + \frac{n_{\rm run}}{2}\left(\ln\left(\frac{k}{k_{s0}}\right)\right)^2 . \qquad (2)$$

The value of $n_s$ therefore depends on the pivot scale used. For example, to convert to a new pivot scale the relation is $n_{\rm s}(k'_{s0}) = n_{\rm s}(k_{s0}) + n_{\rm run}\log(k'_{s0}/k_{s0})$. More generally double inflation models or multiple field inflation can lead to breaks and spikes in the primordial power spectrum, see eg. Linde (1990). This motivates a more general parameterization of the primordial power spectrum in terms of amplitudes over discrete bands in wavenumber.

In this paper we vary four cosmological parameters, using flat priors on the baryon density $\Omega_{\rm b}h^2$, the cold dark matter density $\Omega_{\rm c}h^2$, the Hubble constant $h = H_0/(100\ {\rm km\ s^{-1}Mpc^{-1}})$, and the redshift of reionization $z_{\rm re}$ (we assume $6 < z_{\rm re} < 30$). Throughout we assume that the universe is flat with a cosmological constant. We assume purely Gaussian adiabatic scalar perturbations and ignore tensor modes in this paper.

We use the latest WMAP[1] (Verde et al. 2003, Hinshaw et al. 2003, Kogut et al. 2003) temperature and temperature-polarization cross-correlation anisotropy data. We also include almost independent bandpowers on smaller linear scales ($800 < \ell < 2000$) from ACBAR[2] (Kuo et al 2004), CBI (Pearson et al. 2003) and the VSA (Grainge et al. 2003).

We constrain the matter power spectrum at low redshift by using the galaxy power spectrum measurements of the 2dFGRS (Percival et al. 2002) over the range $0.02 < k/(h\ {\rm Mpc}^{-1}) < 0.15$. We assume that the galaxy power spectrum measured by the 2dFGRS is a simple unknown multiple of the underlying matter power spectrum (linear bias), so in our analysis the 2dFGRS data serves to constrain the shape but not (directly) the amplitude of the matter power spectrum.

To evaluate the posterior distributions of parameters given the data we use the Markov-Chain Monte Carlo method to generate a list of samples (coordinates in parameter space) such that the number density of samples is proportional to the probability density. We use a modified version of the CosmoMC[3] code, using CAMB (Lewis, Challinor & Lasenby 2000) (based on CMBFAST (Seljak & Zaldarriaga 1996)) to generate the CMB and matter power spectrum transfer functions. For further details see Lewis & Bridle (2002), Christensen et al. (2001), Verde et al. (2003) and references therein.

## 3. Power Law Spectra With and Without a Running Spectral Index

In most parameter studies, the scalar initial power spectrum is parameterized by a constant spectral index, $n_s$ and an amplitude. Even with only two parameters

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[1] http://lambda.gsfc.nasa.gov/

[2] http://cosmology.berkeley.edu/group/swlh/acbar/data

[3] http://cosmologist.info/cosmomc/

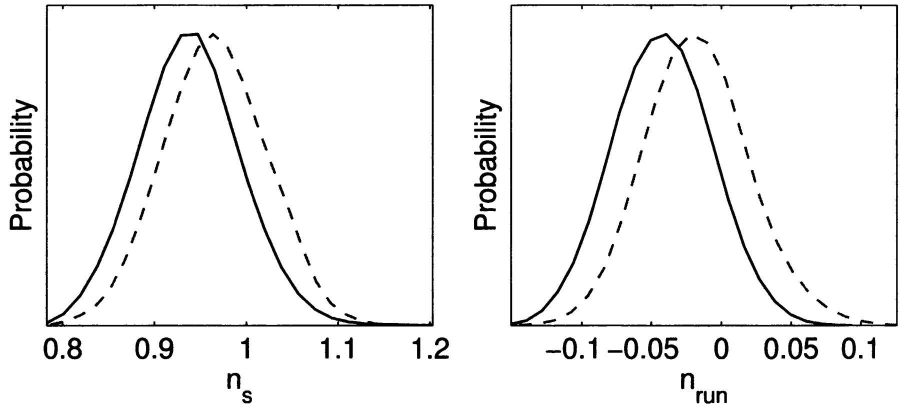

Figure 1. Marginalized distributions of the running spectral index slope parameters including (solid) and without (dashed) the WMAP temperature data at $\ell < 5$.

defining the primordial power spectrum there are large degeneracies between these and the other cosmological parameters. For example, larger baryon densities decrease the height of the CMB second acoustic peak thereby mimicking the effect of a red spectral tilt.

Using the WMAP (Hinshaw et al. 2003,; Kogut et al. 2003) power spectrum results alone we find a tight marginalized constraint $n_s - 30.4(\Omega_b h^2 - 0.025) = 1.04 \pm 0.02$ (68% confidence). However the orthogonal direction is very poorly constrained with[4] $n_s + 30.4(\Omega_b h^2 - 0.025) = 1 \pm 0.14$. The best fit model to the WMAP data (Spergel et al. 2003) with $n_s = 0.97$ can be closely approximated by completely different models with $n_s > 1.1$, higher reionization redshifts, rapid Hubble expansion and high power spectrum amplitude. On integrating out the value of $\Omega_b h^2$ the constraint on the spectral index weakens to $n_s = 1.05 \pm 0.08$. Similarly the amplitude of the primordial curvature fluctuations on 0.05Mpc$^{-1}$ scales is constrained by WMAP to be $A_s^{1/2} = (5.5 \pm 0.8) \times 10^{-5}$, whereas the parameter combination $A_s^{1/2} e^{-\tau} = (4.3 \pm 0.1) \times 10^{-5}$ is much more tightly constrained since it comes directly from the observed temperature anisotropy amplitude. To partially break these degeneracies the WMAP team adopt a prior on the reionization optical depth of $\tau < 0.3$. By adding the 2dFGRS and additional CMB data at $\ell > 800$ we find the parameter constraints which broadly agree with the analysis reported by the WMAP team (Spergel et al. 2003), see Figure 1.

By adding a running spectral index we find the marginalized 1-sigma result $n_{\rm run} = -0.04 \pm 0.03$ shown by the solid line in the right hand panel of Figure 1 in rough agreement with the WMAP team. For the pivot scale used ($k_{s0} = 0.05$ Mpc$^{-1}$) a red tilt is preferred (left hand panel of Figure 1. As expected, the

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[4] This wide spread may be partly due to approximations used in the WMAP likelihood parameterization.

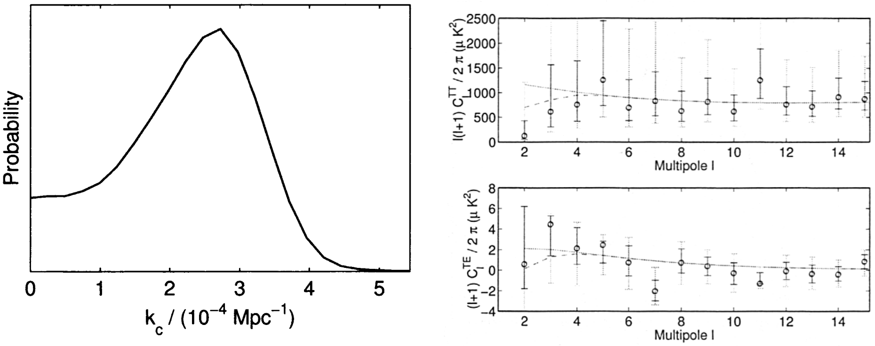

Figure 2. Left: Marginalized probability distribution of a large scale power cut-off parameter $k_c$ for which $\mathcal{P}_\chi(k < k_c) = 0$. Right panels: a concordance model with $k_c = 0$ (solid lines) and with $k_c = 2.7 \times 10^{-4}$ (dashed lines) compared to the WMAP data. Dark and light error bars show the 68 and 95 confidence limits on the theoretical value at each multipole (for the TE error bars we use the Gaussian contribution to the likelihood given the observed TT estimator, taking into account the correlation between the TT and TE estimators, assuming the WMAP best fit model for $C_\ell^{\rm TT}$ and $C_\ell^{\rm EE}$, where $(C_\ell^{\rm TE})^2 < C_\ell^{\rm TT} C_\ell^{\rm EE}$).

effect of adding $n_{\rm run}$ as a free parameter is to increase the uncertainties on the cosmological parameters but only within the original uncertainties.

We find that the evidence for running comes predominantly from the very largest scale multipoles. When we exclude $\ell < 5$ multipoles from the WMAP temperature (TT) likelihood the running spectral index distribution shifts to becomes highly consistent with $n_{\rm run} = 0$, as shown in Figure 1. The constraints on all cosmological parameters except for $n_{\rm s}$ and $n_{\rm run}$ are changed very little on removing the lowest multipoles. The running parameterization is therefore not ideally suited to the data: WMAP provides some evidence for low power on the very largest scales, but this is only crudely fit by using a running spectral index. The WMAP analysis relies on Lyman-$\alpha$ forest at wavenumbers greater than $k \sim 0.1\text{Mpc}^{-1}$ to give evidence for more red tilt on small scales consistent with a running index, but the validity of this analysis is in serious doubt (Seljak 2003). We conclude that the marginal preference for a running spectral index from our CMB + 2dFGRS analysis is primarily driven by the first three CMB multipoles.

## 4. Power Spectrum on the Largest Observable Scales

The quadrupole and octopole estimators observed by WMAP are low compared to the other large scale multipoles. In a given model, the low multipole estimators have a wide $\chi^2$-like distribution (which has the peak below the ensemble mean), so the low values could just be chance. However any model that predicts small values for the low multipoles would be favoured by the data, by a factor of up to about forty, so this could be a hint that there is a sharp fall in power on the largest scales. Tegmark et al. (2003) find that the quadrupole and octopole appear to be aligned, perhaps indicative of some anisotropic effect on on very large spatial scales which would not be well modeled by a statistically isotropic power spectrum model. Here we assume that the alignment is a coincidence, and proceed to consider whether the shape of the initial power spectrum could help to explain the low large scale signal.

There is only a limited amount of information on the largest scales due to cosmic variance, so we cannot hope to fit many extra parameters. We choose to assume there is a sharp total cut-off in the primordial power on scales larger than $k = k_c$ (for a discussion of an exponential cut-off in closed models see Efstathiou 2003; see Contaldi et al. 2003 for possible motivation for a cut-off). As discussed in the previous section a constant spectral index is a good fit on smaller scales, so we parameterize the primordial spectrum as

$$\mathcal{P}_\chi(k) = \begin{cases} 0 & k < k_c \\ A_s \left(\frac{k}{k_{s0}}\right)^{n_s - 1} & k \geq k_c \ . \end{cases} \tag{3}$$

Marginalizing over the other parameters we find the constraint on $k_c$ shown in Figure 2. We find a preference for a cut-off at $k_c = (2.7^{+0.5}_{-1.6}) \times 10^{-4}$ Mpc$^{-1}$, a scale which can give a significantly lower quadrupole and octopole without significantly affecting the higher multipoles (Figure 2). However, according to this model a spectrum with no cut-off is not strongly excluded by the data. Our parameterization cannot achieve values for the quadrupole as low as observed because there is a significant Integrated-Sachs Wolfe contribution to the quadrupole from scales smaller than the cut. The best-fit cut-off model has a very similar probability to the best-fit running model, although the cut-off model is marginally preferred. The constraints on the cosmological parameters are virtually unaffected by adding this free cut-off scale.

A comparison with Spergel et al. (2003)'s assessment of the significance of the low CMB power at small multipoles is not straightforward because the statistical tests differ. A more detailed analysis of this important issue is required, but given the results of Figure 2 it would seem premature to discard simple continuous power law models.

## 5. Conclusions

We have explored two parameterizations of the primordial power spectrum and found that in each case a simple scale invariant spectrum is an acceptable fit to the data. Some deviation towards a red tilt is preferred and there is a marginal indication of a cut-off in power on the largest observable scales. The unexpectedly small power on superhorizon scales observed by WMAP, if real, provides marginal evidence for a drop in the primordial power spectrum on the largest observable scales See Bridle et al. (2003) for more details.

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