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Some results in statistical inference

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This thesis presents two main bodies of work. First a comparison is made of tests of composite statistical hypotheses using the optimal $C(\alpha)$ tests developed in Neyman [4], Bühler and Puri [2] and Bartoo and Puri [1] and the Wald statistic based on maximum likelihood estimators (Wald, [6]). These comparisons are carried out in the case where the parameter under test is interior to open sets in parameter space. It is also shown that the two test procedures are asymptotically equivalent in this case. In particular the problem of constructing tests associated with a mixture of two normal components with one component known is treated in detail. The problem arises out of studies of Down's Syndrome considered in Penrose and Smith [5] and Moran [3].

Second, in the case where parameter space is bounded, a study is made of the joint asymptotic distribution function of the maximum likelihood estimator when the parameter under test lies on the boundary of this parameter space, which is taken for simplicity as a subset of s dimensional euclidean space, Θ, say, given by $0 \le \theta_i < a_i$, $i = 1, \ldots, t$, say, $(a_i > 0)$; $-\infty < \theta_i < \infty$, $i = t+1, \ldots, s$. The hypothesis considered is $H_0 : \theta_i = 0$, $i = 1, \ldots, p \le t$; $\theta_{p+1}, \ldots, \theta_t, \ldots, \theta_s$ fixed and unknown in a closed subset of Θ. We take $p \le t$ as we are more likely to be interested in problems where a subset of $(\theta_1, \ldots, \theta_t)$ lies on a boundary of Θ, rather than fixing $(\theta_1, \ldots, \theta_t)$ in a "corner" of Θ, in which case $p = t$.

If θ_{nj} denotes the maximum likelihood estimator of θ_j,

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$j = 1, \ldots, s$, based on a sample of size n , it is shown that the joint asymptotic distribution function of $Z_{nj} = n^{\frac{1}{2}}(\theta_{nj}-\theta_n)$, $j = 1, \ldots, s$, is a mixture of normal components, the means vector and covariance matrix of each component being determined by setting certain components of the maximum likelihood estimator to zero. The correspondence between the optimal $C(\alpha)$ tests and tests based on maximum likelihood estimators is now lost apart from the case of scalar parameter under test with one sided alternative hypotheses.

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