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A COMPARISON BETWEEN THE THEORETICAL AND THE MEASURED LONG PROFILE OF THE UNTERAAR GLACIER

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Abstract. The measurements by seismic sounding of the long profile of the Unteraar Glacier reported by Mercanton and Renaud are compared with theoretical calculations. By taking the valley to be a cylinder of parabolic cross-section a theoretical curve for the surface is deduced which depends on one unknown parameter only, the average shear stress on the bed. If this parameter is given the value $0{\cdot}77 \times 10^6$ dynes/cm.^{2} the theoretical curve agrees well with the measurements.

Résumé. Les mesures obtenues par sondages sismiques, présentées par Mercanton et Renaud, concernant le profil en longueur du Glacier d'Unteraar sont comparées aux resultats calculés par la théorie. En assimilant la vallée à un cylindre de section parabolique, on obtient une courbe théorique pour la surface, qui ne dépend que d'un seul paramètre inconnu: la tension tangentielle moyenne exercée par la glace sur le lit du glacier. Si l'on attribue la valeur $0{\cdot}77 \times 10^6$ dynes/cm^2 à ce paramètre, la courbe théorique correspond bien aux mesures citées.

The publication by Mercanton and Renaud [1, 2] of the long profile of the Unteraar Glacier measured by seismic sounding makes possible a rough comparison between theory and observation. The measured profile based on 750 soundings over the glacier surface is reproduced from reference 2 in Fig. 1.

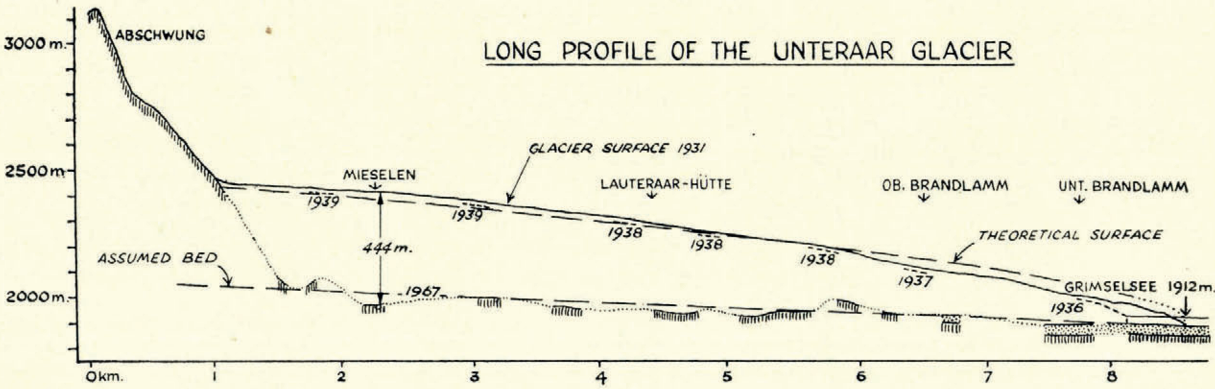

Fig. 1. Long profile of the Unteraar Glacier measured by seismic sounding, reported by Mercanton and Renaud. Vertical scale exaggerated by a factor of two. The observed curve is compared with one calculated by assuming a constant slope of the bed, $\beta = 0{\cdot}0213$ radians

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Suppose a wedge-shaped block of ice, whose surface slope is α, as shown in Fig. 2*b* (below), rests on an inclined rough plane of slope β, where α and β are both small angles. Taking the x axis parallel to the bed and pointing uphill we may consider the equilibrium of the section ABCD, of unit thickness perpendicular to the diagram, bounded by two planes drawn perpendicular to the bed separated by a distance δx. Let us assume that the normal pressure on AB is given to the first approximation by the hydrostatic head, which increases from zero at A to approximately ρgh at B, where $AB=h$, ρ is the density, assumed constant, and g is the acceleration due to gravity. The mean pressure is $\frac{1}{2}\rho gh$, and so the normal force on AB is $\frac{1}{2}\rho gh^2$. The normal force on DC is, therefore, $\frac{1}{2}\rho gh^2+\frac{d}{dx}(\frac{1}{2}\rho gh^2)\delta x$. The other forces acting parallel to the bed are $\tau\delta x$, directed uphill, where τ is the shear force per unit area exerted by the bed on the ice, and $h\delta x \,.\, \rho g \sin\beta$, the component of the weight of the section, directed downhill.

Thus, to the first approximation,

$$\frac{d}{dx}(\tfrac{1}{2}\rho gh^2)\,\delta x+\rho gh\beta\,\delta x=\tau\,\delta x$$

Hence, carrying out the differentiation and dividing by $\rho gh\,\delta x$, we have

$$\frac{dh}{dx}+\beta=\frac{\tau}{\rho gh} \qquad (1)$$

But $\left(\frac{dh}{dx}+\beta\right)$ is the slope of the upper surface, α. Hence,

$$\tau=\rho gh\alpha \qquad (2)$$

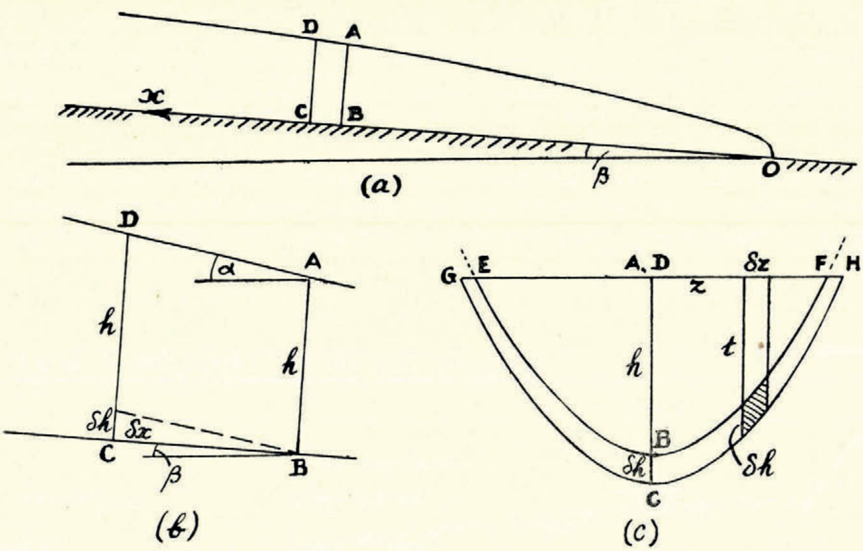

Fig. 2

To this approximation, therefore, the shear stress at the bed is numerically the same as if the bed were of slope α and the block were parallel-sided rather than wedge-shaped. It is essentially the slope of the surface rather than that of the bed which, together with the local depth, decides the shear stress on the bed. A derivation of formula (1) starting from a different assumption is given in reference 3 (equation 16).

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If it is assumed that τ is constant over the bed and that β is also constant (Fig. 2*a*), so that we have a block of ice of varying thickness resting on a plane slope, equation (1) may be integrated to give

$$x=\frac{h_0}{\beta^2}\log_e\frac{h_0}{h_0-\beta h}-\frac{h}{\beta},\ (h<h_0/\beta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where h_0 is written in place of $\tau/\rho g$, and $h=o$ when $x=o$. This is the equation of the surface profile of the block. It does not hold near the end of the block where h is small, because there dh/dx, and hence α, become numerically large.

The calculation may now be modified so as to take account of the sides of the glacier valley. Let Fig. 2*a* and *b* now represent the central longitudinal section of a symmetrical valley. The cross-section perpendicular to the bed at B is shown by EBF in Fig. 2*c*. EF is assumed to be a straight horizontal line. The cross-section at C is shown by GCH, drawn so that GH coincides with EF. Let it be assumed that the cross-section of the valley remains the same at all relevant places; in the figure, then, EBF is at a constant vertical distance δh from GCH. The forces acting parallel to the bed on the ice between these two sections are due, as before, to the pressure difference on the sections, the weight, and the drag of the bed. The force due to the pressure difference is the force acting on the strip shown in Fig. 2*c* between GCH and EBF. This is

$$\int_E^F(\rho g t\,\delta h)dz$$

where t is the depth at a distance z from AB. The difference between taking the limits of integration from E to F and G to H is small. The force is thus

$$\rho g A\,\delta h$$

where A is the area of the section EBF.

The downhill force due to the weight is

$$\rho g A\,\delta x\,.\,\sin\beta$$

The uphill force due to the drag of the bed is

$$\tau p\,\delta x$$

where p is the perimeter EBF and τ is the average shear traction over the perimeter. Hence equating the forces, to the first approximation,

$$\rho g A\,\delta h+\rho g A\,\delta x\,.\,\beta=\tau p\,\delta x$$

$$\text{i.e.}\quad \frac{dh}{dx}+\beta=\frac{\tau}{\rho g R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $R=A/p$, the hydraulic radius of the section.

It will be noted that

$$\tau=\rho g R\left(\frac{dh}{dx}+\beta\right)=\rho g R\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus once again it is α, the slope of the surface, which, together with R, gives the average shear stress on the bed. In fact formulae (2) and (5) are both special cases of the general relation

$$\tau=\rho g R\sin\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This relation is exactly true for a glacier of constant cross-section and slope α, as may be proved by simple resolution of forces.[4] It is also approximately true in a glacier of slowly varying cross-section if the current values of R and the surface slope α are used. This may be proved as follows. Formula (6) gives, approximately, the average shear stress acting on an imaginary cylindrical surface whose cross-section is the same as that of the glacier bed but whose generators are of slope α. This surface makes everywhere only a small angle with the actual bed; a small rotation only of the stress tensor is therefore needed at each point to give the shear stress on the bed. The formula is thus still a valid approximation. *Formula* (6) *applies equally well to an ice sheet and a glacier and there is no restriction on the value of* α. *The only proviso is that the area of cross-section should not change rapidly.*

This reasoning shows that the derivation of formulae (4) and (5) illustrated in Fig. 2*b* and *c* is unnecessarily restrictive in that the cross-section of the valley was assumed to remain constant down its length. A derivation on the same lines in which the cross-section is allowed to change slowly would be slightly more complicated because an extra term would enter the equations due to the longitudinal component of the normal pressure exerted by the valley on the ice.

In principle, if the thickness of the glacier at any one cross-section is known, if the shape of the valley is known and if τ is assumed to be constant, equation (4) can be integrated numerically, step by step, to give the complete surface profile of the glacier. For the present application it is reasonable, as shown below, to simplify the procedure by assuming that β is constant and that R is proportional to h over the range to be considered. Thus

$$R=ch$$

where c is a constant. Then equation (4) becomes

$$\frac{dh}{dx}+\beta=\frac{\tau}{\rho gch}=\frac{\tau'}{\rho gh} \qquad (7)$$

where $\tau'=\tau/c$.

Equation (7) is the same as equation (1) with the effective value of τ changed. For constant β it therefore integrates in the same way to give

$$x=\frac{h'_0}{\beta^2}\log_e\frac{h'_0}{h'_0-\beta h}-\frac{h}{\beta},\ (h<h'_0/\beta) \qquad (8)$$

where $h'_0=\tau'/\rho g$ and $h=0$ when $x=0$.

To apply this equation to the Unteraar Glacier we first need to know the value of c. If the cross-section of the valley is approximated by a parabola we find that using Koechlin's figures [5] the best shape of parabola is given by

$$z^2=1073\,(h-t),\ \text{where } z \text{ and } (h-t) \text{ are in metres.}$$

Further calculation shows that $R=220$ m. when $h=400$ m. i.e. $c=0{\cdot}55$,

and $R=62$ m. when $h=100$ m. i.e. $c=0{\cdot}62$.

The mean value of c in this range is thus

$$c=0{\cdot}585$$

The bed of the Unteraar as drawn in Fig. 1 may be approximated reasonably well by a straight line of slope 0·0213 radians, labelled "assumed bed." With this value for β, equation (8) gives the best fit with the observed profile when $h'_0=15{\cdot}0$ m. This gives $\tau'=1{\cdot}32$ bars (1 bar$=10^6$ dynes/cm.^{2}) and hence $\tau=0{\cdot}585\times1{\cdot}32=0{\cdot}77$ bars. The curve so calculated is drawn in Fig. 1 and labelled "theoretical surface", so that the agreement may be judged.

As a check on the value of τ used, it may be mentioned that starting with Mercanton's figures [6] for the width, depth and surface slope at three different places on the Unteraar and assuming parabolic sections in order to calculate R, the values of the average shear stress on the bed at these three places come out to be 0·566, 0·557 and 0·485 bars. The mean value is 0·54 bars, which differs from 0·77 bars by 30 per cent. Another check on the value is the fact that shear stresses of about these magnitudes have been found by Glen [7] in laboratory experiments on ice to produce the rates of shear observed in glaciers.[4]

Conclusion

By assuming the valley of the Unteraar to be a cylinder of parabolic cross-section and taking the shear stress on the bed to be constant and equal to 0·77 bars, the calculated and measured

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profiles agree quite well. In view of the various assumptions that have been made, the observed irregularities of the glacier, and the fact that the theoretical curve depends on only one disposable parameter τ, which takes a reasonable value, the agreement between theory and observation seems quite satisfactory.

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A NOTE ON THE USE OF MARGINAL DRAINAGE IN THE RECOGNITION OF UNGLACIATED ENCLAVES*

By S. E. Hollingworth

A recent interesting note by Professor D. L. Linton mentioning several bases for the delimitation of unglaciated enclaves would appear to invite some qualifying comment on one of the methods mentioned. The criterion of marginal drainage and overflow channels as marking the limit of extraneous ice against rising ice-free ground is legitimate, especially when their true marginal or subaerial character is supported by evidence of contemporaneous deposits in glacier-dammed lakes. They then mark the ice margin at their time of operation, but it is quite another matter to consider that in general the highest channels mark the maximum encroachment of the ice. The latter is certainly the view generally held for the Cleveland Hills area of north-east Yorkshire since Kendall's classic paper of 1902. In this example, cited by Professor Linton, the difference in the positions of the ice margin at the maximum and that during the initiation of the earliest marginal drainage may have been slight, but there seems some justification for envisaging a considerable change in climatic conditions in the interval.

It is generally accepted that marginal drainage channels will only operate below the snow line, for above that a level cross section of the land-ice contact will be concave upwards due to marginal snow accumulation and to a more rapid outflow of the thicker ice some distance from the ice margin. While assignment of a particular altitude to the snow line can only be a vague generalization at the best, it is perhaps not unreasonable to assume a severe climate with perhaps a snow line in the Cleveland area at the time of the newer drift glaciation of, say 1000 to 1200 ft. O.D. (305–366 m.) with little permanent snow on the exposed wind-swept plateau summits. Thick drifts of snow would survive the winter in the margins of the enveloping ice and the valleys within the hills might well be choked with permanent snow fields. If this were the case, an appreciable, possibly major, change of climatic conditions resulting in the raising of the snow line would be needed before marginal drainage in glacier lakes comes into being on the scale of the early stages of Kendall's sequence.

Independently of this speculation as to snow line, recent development of the additions to our

* Comment on Professor D. L. Linton's note on unglaciated enclaves in glaciated regions (*Journal of Glaciology*, Vol. 1, No. 8, 1951, p. 451–52.)