exponential functions in Chapter II, finishing in the last two chapters with the general theorem of Lindemann and with Gelfond and Schneider's theorem on the transcendence of α^β. There are chapters on Diophantine approximation, continued fractions and normal numbers.

One could hardly expect to exhaust the topic in a monograph of less than 200 pages. Yet the material has been so skilfully and concisely arranged that a very great deal is included. Many of the proofs are new in form and some in essentials also, and the whole book gives the impression of careful work, both in outline and in detail. Much of the book can be understood by a reader with only elementary knowledge, yet it covers a substantial part of what is known. Those chapters (such as the ones dealing with Diophantine approximation and algebraic numbers) which necessarily cover only a part of their topic are supplemented by bibliographical notes.

Professor Niven is to be congratulated on having produced an excellent little book.

A. M. MACBEATH

ZYGMUND, A., *Trigonometric Series* (Cambridge University Press, 2nd ed., 1959). Two Volumes. Volume I, xii+383 pp., Volume II, vii+354 pp., 84s. per volume.

The first book bearing this title and written by Professor Zygmund appeared in 1935, and it has now been replaced by this considerably more comprehensive work. To appreciate fully the labours involved in producing what will certainly be regarded for many years to come as the standard treatise on trigonometric series, it is only necessary to note the amount of space in mathematical periodicals devoted to the subject during the past fifty years. The author has had to synthesise this material, simplify proofs where possible, and present a coherent account without submerging his arguments in tiresome detail. Analysts will agree that he has most successfully carried out this task.

Comprehensive though it is, the book does not, and can hardly be expected to, deal with all the recent advances in the subject. Thus, during the past few years, extended definitions for both absolute and strong summability have appeared, and have produced some interesting results when applied to Fourier Series. Further, to the vast field of multiple Fourier series, the author only devotes one chapter, but he justifies this brief treatment on the grounds that most of the theory of such series is deducible from the corresponding theory of single Fourier series. The reader, therefore, who wishes to use the book as a work of reference may not, on occasions, discover what he is seeking, but he will certainly find his subject matter dealt with from the basic point of view.

It would be pointless in this brief review to present even an outline of the contents of the book, but I feel that the attention of readers should be especially drawn to the first chapter. There the author sets out clearly the results, borrowed from the theory of sets and of integrals, on which the subsequent development rests. A grasp of this chapter, together with a knowledge of the definition and simple properties of summability by the Abel and Cesàro methods, will enable the reader to take up almost any later chapter at will without feeling obliged to become acquainted with the intervening work. Wherever essential groundwork does not appear in this first chapter, a sketch is provided in that chapter where it is to be applied, so that the book may be regarded as being almost completely self-contained.

At the end of each chapter there is a varied set of examples, so that anyone sufficiently interested may be encouraged to acquire, not only a reading, but also a working knowledge of the subject. They range from those which illustrate points not fully dealt with in the main development to some which, if space permitted, could well be classified as theorems. In short, the book will commend itself to analysts on account of the clarity of the exposition, the variety of its subject matter and the elegance in the presentation of what is sometimes rather cumbersome material. Most of them will deem it a privilege to have it on their shelves.

J. M. HYSLOP

SCHWARTZ, L., *Étude des sommes d'exponentielles* (Hermann, Paris, 1959), 152 pp., 1800 F.

This monograph, of considerable interest to the specialist, is a republication, somewhat revised, in three chapters of two works published separately in 1943.

Chapters I and III contain an investigation of the closed linear subspaces of $C(I)$ and $L^p(I)(1 \leqq p \leqq \infty)$ generated by sets of functions $\exp(-2\pi\lambda_\nu z)(\lambda_\nu \varepsilon \Lambda)$. Effectively the numbers λ_ν are real and I is an interval of the real axis (Chapter I) or imaginary axis (Chapter III): in the former case the subspace generated when the system is not total is completely characterised. Functiontheoretic applications are indicated including results on the location of singularities of functions represented in some sense by Dirichlet series.

The treatment involves a combination of transform theory and complex function-theory and is to some extent a development of methods introduced by R. E. A. C. Paley and N. Wiener, N. Levinson and V. Bernstein. Considerable use is made of an inequality for the coefficients, in representations by Dirichlet series, of the same general type as Mandelbrojt's "fundamental inequality". An important innovation is the introduction of some of the "classical" methods and results of functional analysis, for example the representation of linear functionals on $L^p(I)$.

In the short Chapter II the author considers

$$N_p(k, n, \Lambda) = \underset{p}{\text{Max}} \frac{|a_k|}{\| P \|_{L^p(0, \infty)}}$$

where

$$P(x) = a_0 + \dots + a_n e^{-2\pi\lambda_n x}.$$

The case $\lambda_n \equiv n$, $p = \infty$ was considered by S. Bernstein. It is shown that, as $n \to \infty$

$$\log N_p \sim 2\lambda_k \sum_1^n \lambda_p^{-1},$$

and conjectured (proved for $p = 2$) that

$$N_p \sim C_p(k, \Lambda) \exp\left\{2\lambda_k \sum_1^n \lambda_p^{-1}\right\}.$$

Since the original publication important related results have been obtained by the author and by A. F. Leonteev, S. Mandelbrojt, J. P. Kahane and others. Extra references and a regrettably very brief account of recent work have been added in the new edition.

M. E. NOBLE

MICHAL, A. D., *Le Calcul Différentiel dans les Espaces de Banach*, Tome I (Gauthier-Villars, Paris, 1958), translated by E. Mourier, xiv+150 pp., 70 F.

This book is concerned with the extension of the differential calculus to embrace functions which map one Banach space into another, the basic notion of differentiation being that of the Fréchet differential and the associated Fréchet-Michal derivative. It is based primarily on a series of papers published by the author over the last twenty years, and is the first of two volumes of which the English manuscripts were almost