

Redundant Calibration: breaking the constraints of limited sky information

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Abstract. The latest generation of low frequency radio interferometers, e.g. LOFAR, MWA, PAPER, has been pushing down the detection limits on the hydrogen signal from the Epoch of Reionisation. However, due to the challenges posed by foregrounds and instrumental systematics the signal has eluded detection thus far. To overcome these challenges we require a detailed understanding of the calibration of these relatively new telescopes. This led to a renewed interest in redundant calibration. Classical calibration schemes depend on sky models based on limited knowledge of the low frequency sky. Redundant calibration, however, allows us to escape our ignorance as it is sky model independent. We will review the field of redundant calibration, and present work we have undertaken to understand the limitations of this calibration method.

Keywords. instrumentation: interferometers, methods: numerical.

1. Introduction

The detection of the 21 cm signal from the Epoch of Reionisation (EoR) is a challenging experiment; we have to deal with large radiometric noise, bright foregrounds, and complex signal chains. The noise and the foregrounds hide the signal, and the signal chain leaves an instrumental imprint on the radio signals we are trying to measure. Only when we fully understand the impact of the instrument, we can mitigate it. We can then proceed to average over larger sets of data, building up to the required 1000 hours of observations, to achieve the desired signal to noise ratio. Accurate calibration of each observation is therefore a key element in the EoR experiment, but also in general to achieve the full potential of these new telescopes. Calibration solves for the unknown complex antenna gains g_i describing the antenna imprints on the signal. Currently, we use the measured signal correlations c_{ij} between antennas i and j forming a baseline pair, and models of the true correlations or visibilities v_{ij} to solve for the antenna gains. These are related to each other through

$$c_{ij} = g_i^* g_j v_{ij} + n_{ij}, \quad (1.1)$$

where the asterisk denotes complex conjugation and n_{ij} is complex Gaussian noise. These model based calibration methods rely on the completeness of our sky knowledge, which is lacking in this low frequency regime. Missing sources in source model catalogues impact our calibration solutions (Wijnholds *et al.* 2016), and these effects are enhanced in redundant arrays (e.g. PAPER, MWA, HERA), which contain many of the same baselines (Grobler *et al.* 2014). However, these redundant arrays do have extra sensitivity on scales probed by the redundant baselines. This makes them excellent for power spectrum measurements. This need for redundancy in the detection of the 21 cm power spectrum led to a resurgence of redundant calibration. As part of the MWA EoR experiment we are currently also undertaking research to study the applicability of redundant calibration.

2. Redundant Calibration Framework

Redundant calibration relies on the presence of redundant baselines, i.e. pairs of antennas that measure the same visibility. Because a group of redundant baselines sample the same visibility v , the number of unknowns, e.g. gains g and visibilities v , are lower in redundant arrays compared to non-redundant arrays. In theory, a sufficient amount of redundant baselines allows for a model-independent calibration of the antennas in the array. The concept is described in detail in Wieringa (1991) and has been further developed in Liu *et al.* (2010), Noorishad *et al.* (2012), Marthi & Chengalur (2013), Zheng *et al.* (2014).

Currently, two linear implementations of redundant calibration, `logcal` and `lincal`, have gathered most of the attention within the community due their simplicity. They form the backbone of Omnical, which was developed as part of the MIT Epoch of Reionization experiment (MITEoR) (Zheng *et al.* 2014). It has been used to calibrate PAPER (Parsons *et al.* 2010) leading to their limits on the 21 cm signal (Ali *et al.* 2015). We are currently investigating the applicability of redundant calibration in the MWA (Tingay *et al.* 2013) and similar work is done for HERA (Dillon *et al.* 2016). Non-linear calibration has also been explored in (Marthi *et al.* 2013). Using steepest descent they find that non-linear least squares (NLS) can be faster than the linear methods, and that NLS produces noise unbiased results, similar to `lincal`. We will discuss noise bias in Section 3.2.

In `logcal` (Wieringa 1992) we take the logarithm of the complex quantities in Equation 1.1. This creates a system of equations in which the logarithm of the amplitudes and phases of each quantity, e.g. $\ln|g_i|$ and $\arg|g_j|$ respectively, are separated. Equation 2.1 shows the resulting two equations for the amplitudes and phases, when we ignore the noise term n_{ij} .

$$\begin{aligned} \ln|c_{ij}| &= \ln|g_i| + \ln|g_j| + \ln|v_{ij}| \\ \arg|c_{ij}| &= \arg|g_j| - \arg|g_i| + \arg|v_{ij}| \end{aligned} \tag{2.1}$$

Both systems of equations can be written in matrix form. We then solve the system using the general least square solution for a matrix equation of the form $\mathbf{Ax} = \mathbf{b}$. The solution is given by $\hat{\mathbf{x}} = [\mathbf{A}^T \mathbf{A}]^{-1} \mathbf{A}^T \mathbf{b}$.

In `lincal` (Liu *et al.* 2010) we Taylor expand Equation 1.1 around an initial guess for the gains g_i^0 and v_{ij}^0 , leading to

$$c_{ij} = g_i^0 g_j^{0*} v_{ij}^0 + g_j^{0*} v_{ij}^0 \Delta g_i + g_i^0 v_{ij}^0 \Delta g_j + g_i^0 g_j^{0*} \Delta v_{ij}, \tag{2.2}$$

where Δg_i and Δv_{ij} are the required corrections to our guesses. Analogous to the system of equations for `logcal`, we can write a similar matrix for the linearized framework, this time containing the real and imaginary components of our initial guesses g_i^0 and v_{ij}^0 . But now we solve for Δg_i and Δv_{ij} to correct our guesses. This matrix has to be updated and inverted for each iteration, making this method quite computationally expensive.

3. Implementation issues

Omnical and software under development for HERA combine both linear implementations. They use `logcal` to get an initial guess and using `lincal` to further improve the solutions (Zheng *et al.* 2014). However, this has not completely resolved the issues associated with redundant calibration. In this section we will highlight some of these issues, and show examples which we encountered while investigating the applicability to the MWA.

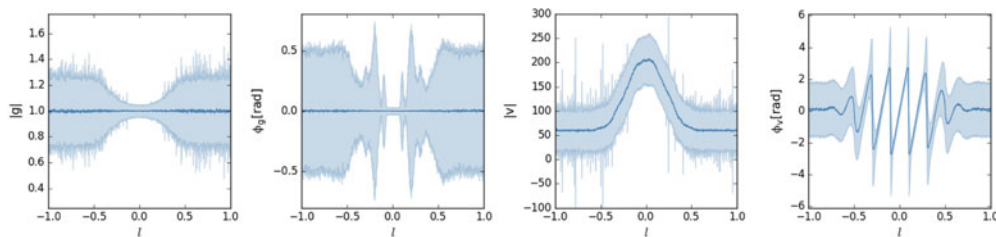


Figure 1. The `logcal` solutions for a simplistic 5 element interferometer with Gaussian-beams calibrated on a sky containing a 200 *Jy* source and randomized background sky drawn from a source count distribution. *From left to right:* the amplitude gain $|g|$, the phase gain ϕ_g of a single antenna, the visibility amplitude $|v|$ and the visibility phase ϕ_v for a redundant visibility as a function of source position l . We show the solution mean (dark blue), and the 1-sigma solution variance (shaded blue). The mean of the amplitude solutions equals the input gain $|g| = 1$, the amplitude solution variance inversely follows the shape of the beam, i.e. the variance increases when the beam response decreases. The mean of the phase solutions fluctuates around $\phi = 0$ and jumps along with the variance at so-called phase wrapping points, which are further explained in the text. The visibility amplitude mean follows the shape of the Gaussian beam, caused by the source population variance and outliers caused by the noise variance. Similarly we see the visibility phase becoming more defined as the strong source moves into the beam, and we see its phase wrapping.

3.1. Absolute Calibration

Redundant calibration only yields relative solutions for the antenna gains. We can multiply the antenna gain amplitudes with some arbitrary factor α and simultaneously divide the visibility amplitude by α while keeping the system internally consistent. Similarly we can add an overall phase to each antenna gains, which will cancel out due to complex conjugation. Additionally we can also add an overall phase gradient to our antenna gains, as long as we subtract this from our visibilities. This final degeneracy corresponds to a rotation of the sky, i.e. redundant calibration can not determine the pointing of our telescope. (Wieringa 1992, Liu *et al.* 2010). To break these degeneracies we need some form of external calibration.

3.2. Noise Bias

Because `logcal` relies on the logarithm of Equation 1.1 it is noise biased. The real and imaginary components of the complex noise term are Gaussian-distributed, however, this is not true for the amplitude and the phase. This will lead to a systematic bias in the `logcal` solutions, and is also the reason why `lincal` was developed. We can work around the noise by averaging data over time scales on which the gain does not change much, but sufficient enough to lower the noise. (Liu *et al.* 2010)

3.3. Phase Wrapping

Phase wrapping adds an additional degeneracy to our complex solutions. We can add and subtract multiples of 2π to our gains g_i and visibilities v_{ij} without changing the measured correlation c_{ij} , see Equation 1.1. This manifests itself in different ways for each linear implementation. In `logcal` phase wrapping shows itself by causing a large solution variance when certain redundant baseline groups measure a visibility phase of π . In `lincal` this will manifest itself by converging to a false minimum. In Figure 1 we show the solution variance of `logcal` in simulations. We attempt to calibrate a 5 element linear array on a randomised point source sky drawn from a source count distribution and a strong point source of 200 *Jy*. We show the solution variance as a function of the location of that strong source on the sky and we see that the variance for the phase

increases when the measured phase wraps around. The amplitude variance depends on the S/N of the amplitude, which is higher when the source is at phase centre. We need to unwrap the phases to some degree, before we apply `logcal`. `Omnical` uses the median phase within a redundant group. (Zheng *et al.* 2014) the HERA software currently uses products of visibilities.

3.4. Convergence

We have already mentioned that `lincal` can converge to the wrong minimum due to phase wrapping. But another reason why `lincal` might converge to a wrong solution are bad fiducial guesses. Combining both the logarithmic and linearised implementation was proposed to circumvent this. However, the aforementioned problem of phase wrapping makes `logcal` unstable on certain sky configurations, which gives `lincal` bad initial guesses. On top of that `logcal` is noise biased, causing additional issues.

3.5. Non-Redundancy

Finally, all radio telescopes are inherently non-redundant to some degree. There will be positions errors, and the antenna response patterns vary from antenna to antenna. The impact of these non-redundancy is currently being researched by the HERA and MWA collaborations.

4. Solutions and Outlook

The EoR community has been undertaking work to understand redundant calibration and its limiting factors. Here we have summarised literature to date and shown some initial results of the impact of the structure of the sky signal. The next step will be the inclusion of position offsets, and understanding the effects of beam variations. But the need for some external calibration has motivated us to look at ways to combine sky models and redundancy. Some initial work into such a hybrid method has been mentioned in Sievers (2017), and we see this as the way forward in the era of the Square Kilometre Array.

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