

NEARBY SMALL GROUPS OF GALAXIES

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THE QUESTIONS

To date, our view of the universe has largely been two-dimensional. Velocity data, the basis for a look in the third dimension, have been too incomplete and uneven in quality to provide a clear picture. Nonetheless, the pioneering work by de Vaucouleurs (1975) has given us a rough idea of what the universe is like locally. At least a good fraction of galaxies are improbably close to their nearest neighbours compared with expectations based on statistical fluctuations of a random distribution. Our vocabulary to describe these associations includes the words: binary, group, cloud, cluster and supercluster. Does the real universe indeed have characteristic scales that make these terms meaningful? Or, as Peebles and his co-workers (Davis, Groth & Peebles 1977, and reference therein) would have us believe, is there structure on all scales, at least up to about 15 Mpc? And associated galaxies aside, are there galaxies truly randomly distributed: are there field galaxies? Looking two-dimensionally, it has been possible to arrive at remarkably different conclusions. Turner & Gott (1975) concluded that roughly 40% of all galaxies are randomly distributed while Soneira & Peebles (1977) set an upper limit of 18%. It was roughly this latter figure that de Vaucouleurs (1975) derived with his early look into the third dimension. So we ask: (1) what are the characteristic scales and densities of galaxy associations, and (2) what are the scales and densities of the voids?

The next set of questions concern the dynamical conditions in whatever associations we determine to exist. Do we have evidence that these entities are stable? If they would dissipate in a time short compared with the age of the universe then the answer is yes. Then the follow-up question: is the virial theorem satisfied with "normal" masses assigned to the constituent galaxies, or must we resort to missing mass or anomalous redshift explanations. Some well-known cases bracket the range of possibilities. The rich clusters such as Coma must be bound and there are large discrepancies between luminous and virial masses

(Zwicky 1933). On the other hand, the Local Supercluster is, at least to a first approximation, expanding at the Hubble rate (Sandage & Tammann 1975; Tully & Fisher 1976). Such an entity is hardly bound, unless in a cosmological sense. In between, the small groups, again controversy has been possible arising out of studies of very similar data. Rood, Rothman & Turnrose (1970) found very large virial masses implied for de Vaucouleurs' (1975) groups while Materne & Tammann (1974) found much lower values. Turner (1976) found large mass to light ratios for binaries while Karachentsev (1976) found low ratios.

In this talk we will not be considering the extreme environment of the rich clusters. But we will look at examples covering the gamut of galaxy associations to be found near by. Characteristic crossing times and virial masses will be calculated. These parameters will provide a pretty clear indication of what is going on.

NEW DATA

Radial velocities are needed for a three-dimensional look at the nearby universe and velocities we have. Based on a complete survey of the Palomar Sky Atlas down to $\delta = -45^\circ$, we compiled an extensive list of objects without known redshifts with the intent of observing them in the 21-cm line of neutral hydrogen. The primary selection criterion was one akin to a luminosity classification. Candidates were judged from their structure and size to have redshifts less than 2000 km/s. No such judgement could be made for very early-type systems which, in any event, are weak 21-cm emitters. So our survey was effectively only of types Sbc and later. In addition there were two minor criteria. There was a lower size limit of 1 arc min, and the surface brightness had to exceed the Sky Atlas threshold.

A check has been provided. Subsequently, we have observed all entries without velocities in the Uppsala Catalogue (Nilson 1973) not originally included in our survey and larger than certain limits: for Sdm and later, 2 arc min; for Sd and earlier, 3 arc min. Many of these additional galaxies have been detected with velocities in the range 1000-3000 km/s but only about a dozen have velocities under 1000 km/s. These are mostly unusual cases. Hence we claim to have a high degree of completeness for systems Sc and later, larger than about 1.5 arc min on Nilson's scale, and velocities less than 1000 km/s.

Fortunately we have been complemented by optical observations of mostly early systems. Sandage (private communication) has now obtained velocities for all remaining Shapley-Ames galaxies brighter than 13^m . We now have available some 2000 redshifts over the whole sky out to a cut-off of 3000 km/s. Roughly 60% of these are our own 21-cm redshifts and these have an accuracy of 15-20 km/s. Our observations extend down to $\delta = -45^\circ$, so cover 80% of the sky. Only a small fraction of these observations have been published (Fisher & Tully 1975). In passing, a program also involving M. Goss, U. Mebold and H. van Woerden has begun which will provide consistent coverage of the southern polar region.

To avoid major complications arising from incompleteness, only a restricted volume will be considered today. The boundaries: velocities adjusted for solar motion less than 1100 km/s, galactic latitudes beyond 30° from the plane, exclusion of the Local Group, and (a) in the north, exclusion of a region of 6° radius centered on the Virgo cluster, and (b) in the south, exclusion of the southern half of the hemisphere defined by the great circle lying along $\ell = 30^\circ$ and $\ell = 210^\circ$. This latter criterion is roughly a lower declination limit at $\delta \sim -25^\circ$. This cut eliminates most of the members of our nearest neighbour the Sculptor group from consideration so the entire group has been excluded.

In the volume so defined, 5000 Mpc^3 , we have 412 redshifts. For the analysis which follows, all distances will be taken strictly from redshifts assuming a Hubble constant of 75 km/s/Mpc. The results are not substantially affected by the choice of a Hubble constant or by modest deviations from the Hubble flow.

THE SOUTH GALACTIC HEMISPHERE

It is, of course, known that there are many more nearby galaxies north of the galactic plane than south. De Vaucouleurs (cf 1976) proposes that we are at the outer edge of a supercluster associated with the Virgo cluster. Let us look at the simpler region first then, the relatively empty region away from the supercluster in the southern galactic hemisphere.

A third of the total volume we will consider is in the south, yet there are only 34 redshifts, 8% of the total. Their distribution is shown in Figure 1. Only 7 of these 34 have integrated magnitudes exceeding -19^m . These seven account for 80% of the mass in galaxies in this volume.

a) Correlation Scales

It should be clear from Figure 1 that the galaxies are not randomly distributed. A majority are in one of three....let us call them associations. Moreover these three associations are nearer to one another than could be expected with a random distribution.

We can graph a close kin to the two-point volume covariance function introduced by Peebles (1973). Using each galaxy in the sample in turn as a point of reference, we derive the number of galaxies per unit volume contained within a shell of given radius, then sum and normalize. The parameter, $n(R)$, is related to Peebles' parameter, $\xi(R)$:

$$\xi(R) \sim (n(R) - \bar{n})/\bar{n}$$

where \bar{n} is the mean number density in the sample. Without inspecting a volume large compared with the correlation scales we have no objective way of estimating \bar{n} .

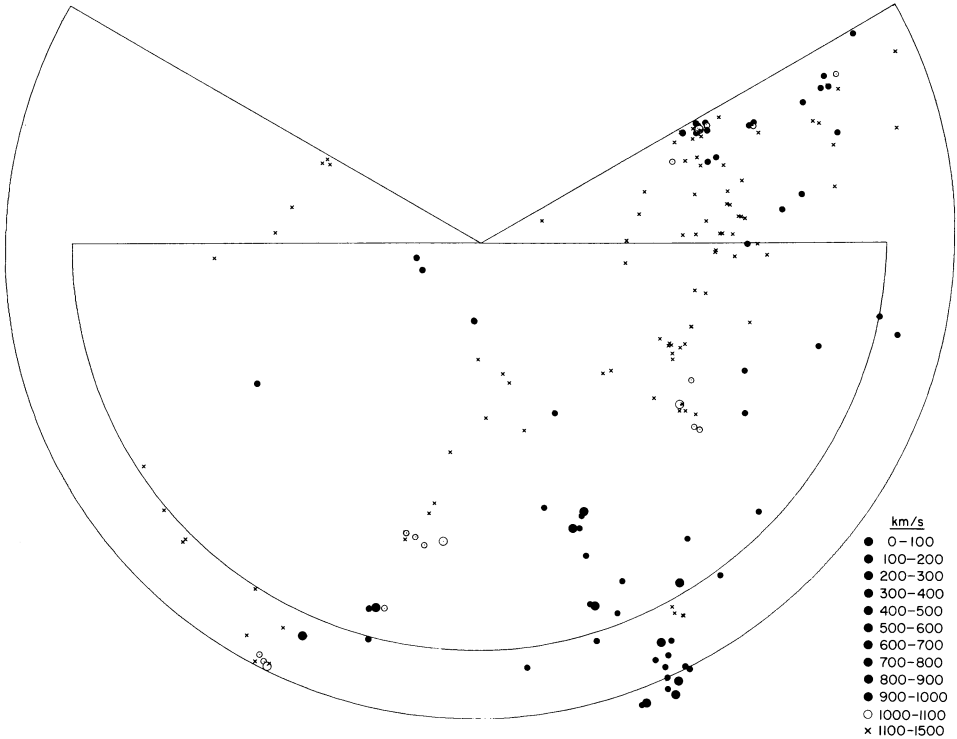


Figure 1. The distribution of nearby galaxies in the southern galactic hemisphere, excluding members of the Local group and the Sculptor group. There are 34 galaxies with $V_0 < 1100$ km/s in the hemiscircle enclosing the area of sky with $b < -30^\circ$ and $30^\circ < \ell < 210^\circ$. Correlations were made between members of this inner volume and all galaxies within a region defined by $V_0 < 1500$ km/s, $b < -20^\circ$ and $0^\circ < \ell < 240^\circ$.

The distribution $n(R)$ obtained from the southern volume is shown in Figure 2. To avoid boundary effects, correlations were made between the 34 galaxies inhabiting the volume defined above with all galaxies in a larger volume, defined by a corrected velocity cut-off of 1500 km/s, $b < -20^\circ$ and $0^\circ < \ell < 240^\circ$. Figure 2 shows that there is a strong correlation between galaxy positions on scales less than 1.5 Mpc and lesser but still significant correlation on scales of 1.5-4 Mpc. Scales larger than this cannot be studied in such a restricted volume.

Shown in Figure 3 is the same type of covariance diagram but restricted to the 7 more massive systems and their correlation with massive galaxies only. There are no close pairs of large galaxies in this small sample, but there is a significant enhancement in the covariance function between 2 and 3.5 Mpc. Note that in a homogeneous cubic close packed universe, a large galaxy in this volume would expect his nearest large neighbour to be 6 Mpc removed.

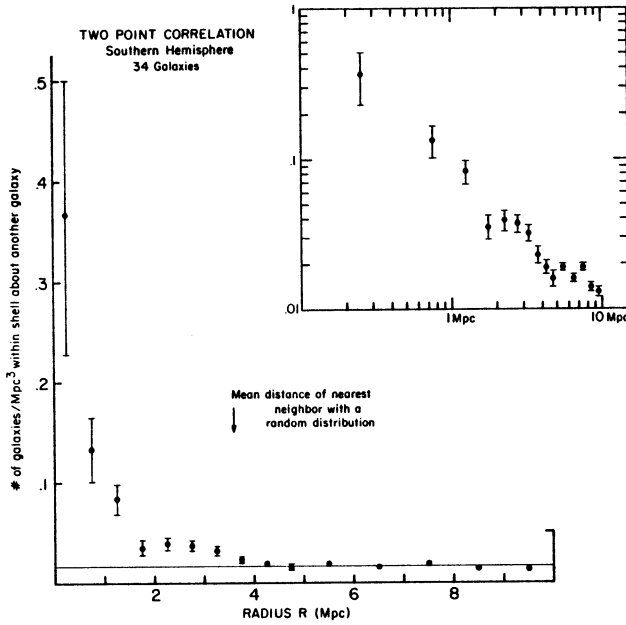


Figure 2. Unnormalized two-point volume covariance function for all galaxies in the southern hemisphere volume defined in Figure 1. We plot $n(R)$ versus R where $n(R)$ is related to Peeble's (1973) volume covariant function: $\xi(R) \sim n(R)/n-1$. Error bars are \sqrt{N} .

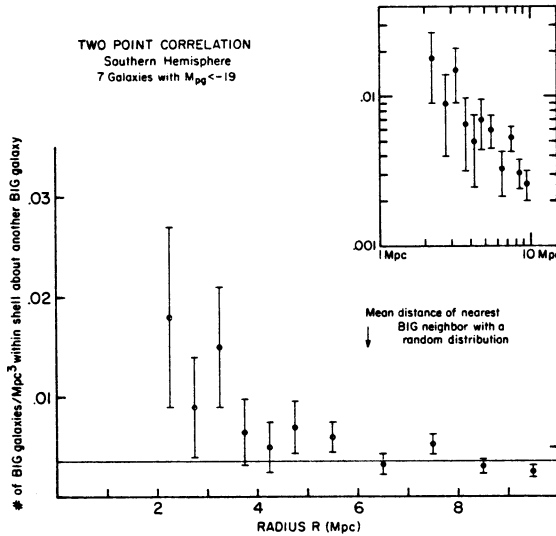


Figure 3. Two-point volume covariance function for only those galaxies with $M_{pg} < -19^m$ in the southern volume.

To return to the sample of 34 again, the case for associations between galaxies can be put differently. Turner & Gott (1976) have, in the two-dimensional case, used an algorithm which purportedly would pick out ten-fold density enhancements above a mean density. We can do similarly. The mean number density of galaxies in our southern volume is 0.02 galaxies/Mpc³. A radius is defined such that the volume number density within this radius about an isolated galaxy is 10 times the above mean number density. Then any galaxies with such spheres which abut form pairs or larger units with ten-fold the mean density. In other words, any galaxy with a neighbour within twice this radius, i.e. within 2.1 Mpc, is in a region of at least ten-fold the mean density. Of 34 galaxies, 29 fulfill this criterion. If the criterion were relaxed to 3 times the mean density then it is satisfied by fully 33 of the 34.

Yes, the rule is proven with one exception; one "field" galaxy. This system, DDO 215, is a remarkable 7 Mpc from any other with a known redshift. It is the sole galaxy of any significance in a volume of 1440 Mpc³; if "significant" is an appropriate word to describe an irregular with $M \sim -16^m$! DDO 215 is, indeed, at about the center of the most conspicuous void in the southern volume. Its presence provides us with something other than an upper limit to the density of this vast emptiness: 7×10^{-4} galaxies/Mpc³. Yet it is possible to form dwarf galaxies in utter isolation!

b) *Crossing Times and Virial Masses:*

We would like to examine the dynamical properties of galaxy associations. Although the above discussion leads to the conclusion that almost all galaxies are positively correlated in position with other galaxies, here only the more obvious clusterings will be considered. Entities which stand apart as enhancements on the local mean density will be referred to as groups, with no a priori restrictions on dimensions, internal dispersions or space densities.

For the groups that will be considered, characteristic crossing times and the virial relationship are calculated. The moment of inertia radius (Jackson 1975) is used:

$$R_I = \left(\frac{\sum m_i r_i^2}{\sum m_i} \right)^{1/2}$$

where m_i , r_i are individual masses and linear distances from the group center of mass. Projected R_I are listed in Table I. A deprojection adjustment of $\sqrt{1.5}$ is included in deriving crossing times as a fraction of the age of the universe. The kinetic and potential energy are calculated assuming mass to luminosity ratios of 7 for spirals and irregulars and 15 for ellipticals and lenticulars. The ratio M_{VT}/M_L gives the discrepancy between the mass required to fulfill the virial theorem and the assumed mass from the group luminosity.

In the volume south of the galactic plane, unfortunately, there are no clean groups of significant proportions. Perhaps the most in-

teresting, Cetus I (group 15 in de Vaucouleurs 1975; hereafter DV15), spills across our volume cut-off of 1100 km/s and careful attention is required of nearby groupings at yet higher velocity. Indeed, there is a rich field of galaxies extending some 12 Mpc from Cetus I to the Fornax cluster that we are only glimpsing the edge of in the volume we are considering.

The kind of dilemma we regularly face is shown up in a small group near NGC 7814. There are four galaxies very close to NGC 7814 with a velocity dispersion among themselves of only 19 km/s. However NGC 7814 has a larger redshift than their mean by 200 km/s! But this is an optical redshift of low quality, while the rest are 21-cm measurements. Is NGC 7814 really at a different distance along the line-of-sight (though undoubtedly positively correlated in the sense of the covariance function)? Is it at the same distance and the virial theorem is applicable (large masses implied)? Or do we just have a bad velocity?

NGC 628. Then there is an association in which the largest galaxy is NGC 628. There are 6 galaxies within a projected radius of 1 Mpc with 21-cm redshifts measured. As can be seen from the data compiled in Table I, there is a severe discrepancy between the virial mass and the luminous mass if the entity is assumed to be bound. With standard statistical projection factors for radial velocities and radii on the plane of the sky, the crossing time is less than the age of the universe by a factor of five. We have two choices: (1) there is a case for hidden mass or anomalous redshifts, or (2) although surely correlated in the sense of the covariance function, the high velocity pair (NGC 600 and companion) are removed in the line-of-sight from the others, and the crossing time is comparable with the age of the universe.

Obviously, in a given instance this ambiguity cannot be resolved. Were we to consider many such groups we might apply a statistical test. Assuming line-of-sight distances from redshifts, we could ask if these associations are elongated or flattened in the line-of-sight or are they spherical. Elongation recalls the analogy of structure in our own Galaxy noted to point toward the sun which Bart Bok has called the Fingers of God telling us we are doing something wrong. In our case, the implication would be high velocity dispersions and the reality of large virial masses. However in the north and south volumes under consideration, given the exclusion of the Virgo cluster, there are no very obvious fingers pointing at us.

NGC 1023. The pickings are slim in the southern galactic hemisphere, but there is one of the cleanest galaxy groups to be found locally excluded from our volume by its proximity to the galactic plane: the NGC 1023 group (DV7). In spite of the low galactic latitude ($18^\circ < |b| < 25^\circ$) we have included it in our study (see Table I). Materne (1974) has pronounced this group stable on the basis of 5 redshifts. We now have 13 and all of the original velocities have been improved. The

TABLE I

Group	DV#, K# TG#	# Gal.	Mass 10 ⁴ M _⊙	V _B km/s	σ km/s	R _I kpc	T _H T _I °	PE 10 ⁵⁸ ergs	KE	KE/ PE	M _V M _L	Note
NGC 628		6	3	825	66	240	0.2	0.3	4.	14.	28.	1
		1	74%									
NGC 1023	DV 7	13	8	726	59	570	0.5	2.0	8.	4.	8.	2
		3	78%									
Leo	DV9 K31	4	6	661	45	70	0.09	9.	4.	0.4	0.9	
-M66	TG 38	4	Similar									
Leo	DV11 K27	6	6	691	60	120	0.1	12.	7.	0.6	1.1	
-M96	TG 27	6	Similar									
Leo combined		16	15	675	66	640	0.5	26.	20.	0.8	1.6	
-all in vicinity		12	Similar									
M81	DV2 K22	6	2	164	116	30	0.02	0.9	7.	7.	15.	3
-restricted	TG 16	2	87%									
M81+M82 coupled		(6)	2	164	(6)	30	0.3	0.3	0.02	0.07	0.1	4
-restricted												
M81+M82 coupled		(14)	3	191	43	41	0.5	0.4	1.4	3.5	7.	4
-all in vicinity		3	76%									
M101	DV 5 K46	4	2	389	(18)	20	0.05	0.3	0.2	0.7	1.5	
-restricted	TG 82	1	91%									
M101		9	3	391	26	150	0.3	0.4	0.5	1.4	2.8	
-all in vicinity		1	81%									
N5005		12	5	992	83	270	0.2	1.7	9.	5.	11.	5
+N5033	TG 67	2	75%									
N5907	DV30 K54	7	6	871	31	200	0.3	2.2	1.9	0.8	1.6	
	TG 97	2	87%									
N3184	DV12 +	12	3	613	68	1,180	0.9	0.2	4.	21.	42.	
		6	90%									
DDO 168		7	.05	296	23	290	0.7	3x10 ⁻⁴	8x10 ⁻³	32.	64.	6
dwarfs		1	55%									
Centaurus	DV4 K47	9	8	325	28	470	0.9	1.9	2.0	1.0	2.	
		4	98%									

Table I - Description and Notes

Column 2: DV = de Vaucouleurs (1975); K = Karachentsev (1970);
TG = Turner & Gott (1976)

Column 4: $M = \sum_i m_i$

Column 5: $V_B = \sum_i m_i V_i / \sum_i m_i$

Column 6: $\sigma = \left(\sum_i m_i (V_i - V_B)^2 / \sum_i m_i \right)^{1/2}$

Column 7: $R_I = \left(\sum_i m_i r_i^2 / \sum_i m_i \right)^{1/2}$

Column 8: $T_I H_0 = (\sqrt{1.5} R_I / \sqrt{3} \sigma) H_0$

Column 9: $PE = -(2/\pi) G \sum_{\text{pairs}} m_i m_j / r_{ij}$

Column 10: $KE = (3/2) \sum_i m_i (V_i - V_B)^2$

Column 12: $M_{VT}/M_L = 2 KE/|PE|$

Notes

1. NGC 600 + companion only in line-of-sight?
2. 60% of KE in NGC 891.
3. Almost all KE and PE in M81-M82 pair.
4. M81 and M82 considered as a single object with combined mass and barycentric velocity.
5. NGC 5005 velocity should be checked. Dominates KE.
6. Near CVn I region.

case for stability has been diminished: the ratio $KE/|PE|$ is 4. This value is probably not significantly different from unity given our uncertainties in distance, mass, velocity and projection factors. But as important, the crossing time, at 0.7×10^{10} years, is half the age of the universe.

THE NORTH GALACTIC HEMISPHERE

The differences between north and south are not subtle. In Figure 4 there are plotted 378 galaxies with $b > 30^\circ$ and $V < 1100$ km/s (i.e., out to but excluding the Virgo cluster). There are 99 galaxies with $M_{pg} < -19^m$. The major regions of concentration are in Leo, in Virgo (southern extension) and, especially, in Canus Venatici-Ursa Major. There is also a tremendously large region where there are no galaxies at all.

a) Correlation Scales

The two-point covariance distribution is shown in Figure 5 for the northern material. In the logarithmic plot a featureless power spectrum is seen on scales up to 6 Mpc. Larger scales cannot be tested in such a restricted volume. The correlation between big galaxies only, shown in Figure 6, is qualitatively similar. In the logarithmic plot there is a suggestion of a dearth of pairings of large systems on scales less

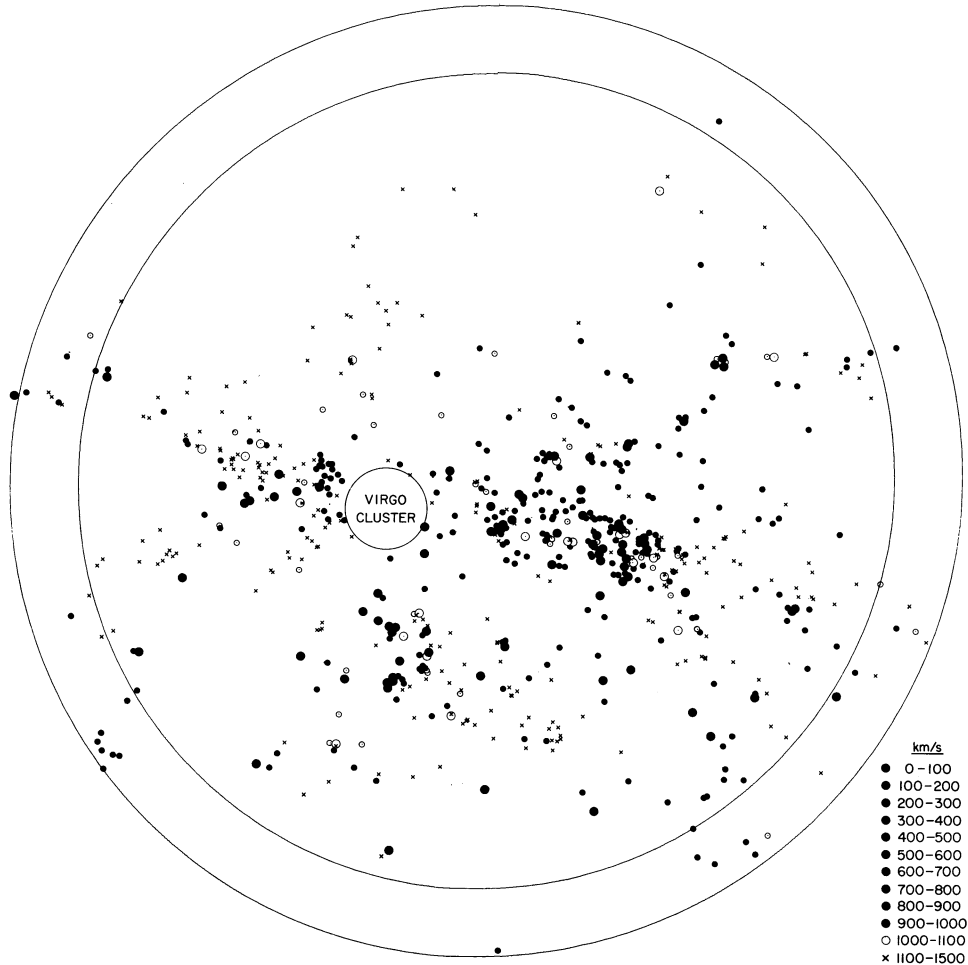


Figure 4. The distribution of nearby galaxies in the northern galactic hemisphere. There are 378 galaxies with $V_r < 1100$ km/s and $b > 30^\circ$. Correlations were made between these and all galaxies with $V_r < 1500$ km/s and $b > 20^\circ$, but excluding those within a 6° radius of the center of the Virgo cluster.

than 1 Mpc compared with a power law distribution or the distribution of all galaxies. This may be an artifact given that line-of-sight distances come from velocities and dispersions may be high for close pairs.

As for in the south, we can determine the fraction of galaxies which meet the ten times mean density criterion. Of 378 galaxies, all but 29 (92%) meet the demanding requirement that their nearest neighbour lie within 1.2 Mpc implying a group of two or more with a local number density 10 times the mean in the northern volume. However the mean density in the northern volume must be unusually high, and a more

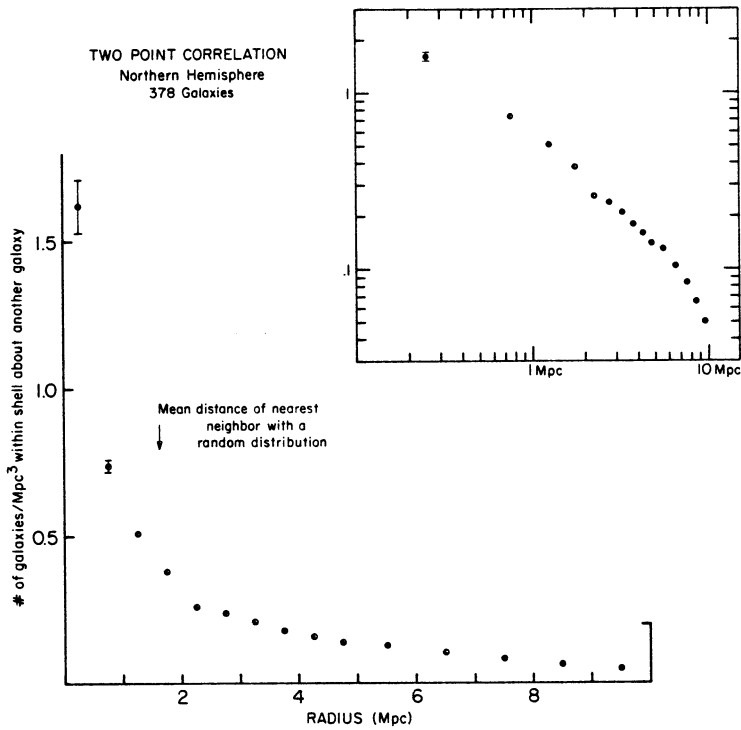
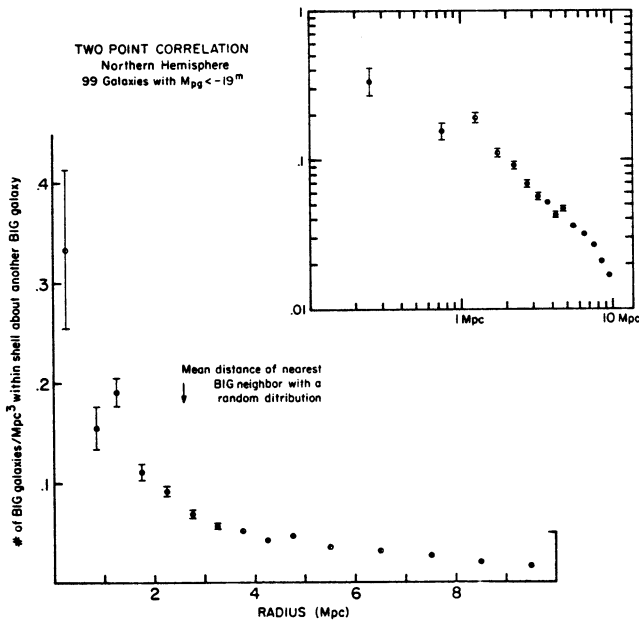


Figure 5. Two-point volume covariance function for the northern galactic hemisphere region defined in Figure 4.

Figure 6. Two-point volume covariance function between galaxies with $M_{pg} < -19^m$ in the northern volume.



realistic mean that is more universally characteristic might be that derived from the southern volume. Remarkably, only 3 galaxies in the north are in a lower density region than ten-fold the mean southern density. For all of these, the local density exceeds five-fold the mean southern density. The most isolated galaxy in the northern hemisphere has a nearest neighbour at 2.6 Mpc! In the regions considered in both hemispheres only one of 412 galaxies is negatively correlated!

In spite of the large number of galaxies in the northern sky within 1100 km/s, it is still possible to find a contiguous region devoid of galaxies of more than 1000 Mpc³. That is roughly one-third of the total volume we are considering here! In Table II, some characteristic number densities from our sample are given:

	TABLE II <u>Volume</u>	<u>Galaxies/Mpc³</u>
Virgo cluster	15 Mpc ³	10
C Vn - U Ma	150 "	1
The void	> 1000 "	< 10 ⁻³

b) *Crossing Times and Virial Masses*

Let us turn quickly to the details of several individual groups. There can be no attempt at completion. Instead we will try to explore the range of possibilities by picking examples that are as clean as possible. The very difficult CVn-UMa region will be ignored.

Leo. In Leo are two tight groups near to each other which are well known (DV9 and DV11; in the catalog by Karachentsev, 1970, they are K31 and K27; in that by Turner and Gott, 1976, they are TG38 and TG 27). Materne and Tammann (1974) found these two groups considered as a single entity to be stable assuming conventional masses. Our analysis is summarized in Table I. For both tight groups, crossing times are short but the virial theorem is satisfied assuming only conventional masses. Combined together, the single large entity is still stable and now crossing times compare with the age of the universe. Including five more nearby systems, two rather deviant in velocity and three removed spatially, the situation is not substantially changed (as all five are low mass objects).

These two groups are reminiscent of perhaps the most interesting of the entities catalogued by Turner and Gott (1976; see Gott and Turner 1977): condensed groups with several massive galaxies within a radius of roughly 100 kpc, often early morphological types, and group crossing times substantially less than the age of the universe. Inevitably Gott and Turner found severe virial mass discrepancies. The most noteworthy exceptions were these cases in Leo. So we agree that

in these nearest examples of tight groups with several massive members there is no evidence for large masses. Unfortunately, there are no other clean groups of this type within the volume we are considering.

M81. Although we would put as many as 14 galaxies into an extended M81 groups, both the kinetic and the potential energy are dominated by the M81-M82 pair. On this basis alone there is a large virial discrepancy with our assumed masses. An individual case can be rationalized: for example, suppose M82 is near perigalacticon on an eccentric orbit. Considering M81 and M82 as a single unit, the remaining close neighbours would be easily bound and for the extended group the crossing time begins to compare with the age of the Universe.

M101. In this group, M101 is by far the dominant galaxy and as a result the mass weighted velocity dispersion and moment of inertia radius are unrealistically low. However the virial analysis should apply. Stability is implied for both the restricted and most general group.

NGC 5005 + NGC 5033. This group contains two galaxies of comparable mass, plus a host of small companions. The problem is that NGC 5005 has a reported velocity which deviates considerably from most of the rest in the group and dominates the kinetic energy term. This velocity, which we have drawn from the Second Reference Catalogue (de Vaucouleurs, de Vaucouleurs and Corwin 1976), should be checked before drawing any firm conclusions.

NGC 5907. Here again is a group dominated by two members. In this case, however, there is no virial discrepancy. There are several other more distant galaxies which must be associated in the sense of the covariance function. However, they could not be bound.

NGC 3184. We now consider a very different kind of group. A portion of this entity was catalogued by de Vaucouleurs (DV12). However it can now be followed considerably further south with roughly constant density on the plane of the sky. Not too far away to the north is the group DV6 and to the south are the two Leo groups, DV9 and 11 already discussed. It is seen in Table I that the moment of inertia radius is much larger than for any of the other groups considered and the consequence is that the crossing time becomes comparable with the age of the universe. Since there is a large virial mass discrepancy, the implication is that the group is not bound.

DDO 168 It may not be fair to consider this association of dwarf irregular galaxies to be an independent group, as they only stand apart from the Canis Venatici I region (DV 3) by 1 Mpc. However, the Table I data show that in themselves they are certainly not bound. There is no problem with the dissipation time-scale.

Centaurus A. We again leave the strict confines of our working volume for one last very nice example. Galaxies in the Centaurus group (DV4)

are in the galactic latitude range $13^\circ < b < 32^\circ$, and there is the danger that some are hiding further south. But this group is nicely isolated and it contains four significant galaxies. We see in Table I that the crossing time is comparable with the age of the universe and there is no appreciable mass discrepancy.

So we have been able to show the existence of groups of galaxies which satisfy the stability criterion of the virial theorem assuming only masses typically associated with galaxies. These groups characteristically have radii R_T less than 300 kpc. Groups that are larger usually have crossing times greater than $0.5 H_0^{-1}$. For the few exceptions that turn up (NGC 628 + NGC 600, M81 + M82, NGC 5005 + NGC 5033) it is possible to suggest plausible scenerios which do not require high mass to light ratios. We feel that the evidence weights strongly against the existence of a lot of unseen matter distributed like the galaxies in small groups.

Clearly, the analysis can be taken a lot further. There has been no sensitivity to the possibility that a fraction of a group may be bound while the rest is expanding. We are not making effective use of our velocity data through application of the virial theorem in those common instances where there are only one or two massive systems but ten or so "test particles" of insignificant mass making up the group. These small galaxies offer the means of weighing the groups to substantial radii.

Finally, the importance of obtaining good redshifts (accuracies < 20 km/s) for all nearby galaxies must be stressed. One may have a dozen good velocities in a group but a single bad value associated with a large galaxy can compromise the results.

CONCLUSIONS

1. In agreement with Peebles and co-workers, the volume two-point covariance function shows a featureless power law spectrum over those scales that we can meaningfully examine.

2. There is no evidence for a significant number of uncorrelated "field" galaxies.

3. There are two large voids even in the restricted region we have surveyed, which includes much of the Local Supercluster. These voids encompass about half the total volume under discussion and between them contain all of one galaxy.

4. The number density of galaxies averaged over a fairly large region in the plane of the Local Supercluster is at least 10^3 times the number density in the voids.

5. The moderately compact small groups that have been studied show little or no virial discrepancy with the optical masses we have

accepted. Our results suggest virial mass to luminosity ratios in the range 5–30.

6. The loosely correlated groups have crossing times which are a large fraction of the age of the universe: typically $0.5 - 1.0 H_0^{-1}$.

7. In the volume we have considered there are no convincing examples of groups such as those identified by Gott & Turner with both short crossing times and large virial mass discrepancies. If real, such groups are rare.

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DISCUSSION

Davis: I believe there is an inconsistency in your low estimate of the peculiar RMS velocity of galaxies ($\sigma \sim 50 \text{ km s}^{-1}$) and the covariance analysis. Your covariance coordinate s is the redshift separation of the pairs, which is the physical separation added to the relative peculiar velocity of the pair. This coordinate produces bias in the covariance function slope, flattening it from $\gamma \sim 2$ to roughly $\gamma \sim 1$. The covariance function will have the slope $\gamma \sim 2$ only for redshift

separations $s \geq 6/H_0$. Therefore if the slope $\gamma \sim 1$ corresponds to redshift separation $s \geq 10$ Mpc (as shown in your data in the Northern sky), then the true peculiar velocity dispersion σ must be of the order of 500 km s^{-1} ($H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$), i.e. by a factor of 10 higher than you state. Your group selection procedure unfortunately guarantees that your estimate of σ will be much less than 500 km s^{-1} .

Tully: Indeed, the slope on the covariance function is $\gamma = 0.9$ which is quite different from the $\gamma = 1.77$ derived by the Princeton group. A very high velocity dispersion would explain this discrepancy. However, I feel that such a large dispersion is completely excluded by the observational material. We are aware of the dangers which could lead to an under-estimation of σ and our group selection criteria do not guarantee such an under-estimation.

Gott: One way to test the membership of groups is to plot a 3 dimensional map of the galaxies assuming they all lie exactly at their redshift distances. If the groups in such a "redshift space" look spherical then the internal velocity dispersions of the groups are small; if however the groups in the redshift space appear as "fingers" pointing at the Earth, then this shows that the internal velocity dispersions are large.

Tully: I agree but I have not yet completed this test. My impression is that I do not find much evidence for "fingers".

Ostriker: A point concerning nomenclature. It might help in understanding if one did not use the term M_{Light} since that depends on an assumed mass-to-light ratio. It would be clearer if the observed quantities (M_{VT}/L) were plotted directly.

Tully: I agree. I just used a method which conveniently allowed for a difference in M/L values between ellipticals and spirals.

Ozernoy: Is there a difference between the mass discrepancy in loose groups and compact groups?

Tully: The only compact groups with several massive members in our sample are the two in Leo, for which there is no mass discrepancy. The rest of our discussion has been based on groups which contain only a couple of massive members or are very loose.

Silk: Have you derived the multiplicity function of your groups?

Tully: We have not derived the multiplicity function, but we intend to do so.

Silk: It may be worth pointing out that a different shape would be expected for this function as compared to that obtained by Gott and Turner because of the different definitions of groups: in particular the use of a volume as opposed to a surface density enhancement.

Van Woerden: Dr Tully's observations taken in the northern hemisphere provide only partial coverage of the sky. We should soon be able to fill in the remainder of the sky from observations at Parkes (by Van Woerden, Goss, Mebold and Siegman), where we have measured about 500 galaxies in the 21-cm line.