CERTAIN TRANSFORMATIONS OF NEARLY-POISED BILATERAL HYPERGEOMETRIC SERIES OF SPECIAL TYPE

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1. Introduction. A few years ago Bailey **(1)** gave certain transformations of both terminating and non-terminating nearly-poised hypergeometric series of the ordinary type and later on he also deduced basic analogues of some of his transformations. Recently, **(3)** I gave certain transformations of both ordinary and basic terminating nearly-poised bilateral hypergeometric series which generalized Bailey's results. Since transformations of nearly-poised series have not been systematically studied so far, I deduced in another paper **(4)** certain relations of both ordinary and basic bilateral series which involved either only nearly-poised series or both terminating well-poised and non-terminating nearly-poised series. In this paper I obtain certain transformations of non-terminating nearly-poised bilateral series of special types ${}_4H_4$ and ${}_5H_5$ and these transformations are generalizations of Bailey's known results. In the sequel the sum of a particular ${}_3H_3$ is also given and is believed to be new.

The following notation is used throughout the paper:

$$(a)_n = a(a+1)\dots(a+n-1);\ (a)_0 = 1;\ (a)_{-n} = (-1)^n/(1-a)n;$$

$${}_rH_r\begin{bmatrix} a_1, a_2, \dots, a_r; z \\ b_1, b_2, \dots, b_r \end{bmatrix} = \sum_{n=-\infty}^{\infty} \frac{(a_1)_n(a_2)_n \dots (a_r)_n}{(b_1)_n(b_2)_n \dots (b_r)_n} z^n;$$

$$\Gamma\begin{bmatrix} a_1, a_2, \dots, a_r; \\ b_1, b_2, \dots, b_r \end{bmatrix} = \frac{\Gamma(a_1)\Gamma(a_2)\dots\Gamma(a_r)}{\Gamma(b_1)\Gamma(b_2)\dots\Gamma(b_r)}.$$

Also, idem $(a; b)$ means that the preceding expression is repeated with a and b interchanged.

2. In a recent paper **(4)** I have deduced the following relation between M nearly-poised hypergeometric series of the type ${}_MH_M$ with unit argument:

$$\Gamma\begin{bmatrix} a_2, a_3, \dots, a_M, 1-a_2, 1-a_3, \dots, 1-a_M; \\ b_1, c_1+b_1-c_2, c_1+b_1-c_3, \dots, c_1+b_1-c_{M-1}, b_M, 1-c_1, \dots, 1-c_M \end{bmatrix}$$
$$\times\ {}_MH_M\begin{bmatrix} c_1, c_2, \dots, c_{M-1}, c_M; \\ b_1, c_1+b_1-c_2, \dots, c_1+b_1-c_{M-1}, b_M \end{bmatrix}$$
$$(2.1)\qquad +\ \Gamma\begin{bmatrix} a_2-1, 2-a_2, a_2-a_3, \dots, a_2-a_M, 1+a_3-a_2, \dots, 1+a_M-a_2; \\ 1+b_1-a_2, 1+c_1+b_1-c_2-a_2, \dots, 1+c_1+b_1-c_{M-1}-a_2, 1+b_M-a_2, a_2-c_1, \dots, a_2-c_M \end{bmatrix}$$

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$$\times\, {}_MH_M\begin{bmatrix} 1+c_1-a_2, 1+c_2-a_2, \ldots, 1+c_{M-1}-a_2, 1+c_M-a_2; \\ 1+b_1-a_2, 1+c_1+b_1-c_2-a_2, \ldots, \\ 1+c_1+b_1-c_{M-1}-a_2, 1+b_M-a_2 \end{bmatrix}$$

$$+ \text{ idem } (a_2; a_3, a_4, \ldots, a_M) = 0.$$

The transformation (2.1) can be deduced directly from Slater's transformation (**2**, (10)).

If we take $M = 4$ in (2.1) and then reverse the first ${}_4H_4$ series on the left and put $c_4 = 0$ we get the following relation between a nearly-poised ${}_4F_3$ series of the first kind and three nearly-poised ${}_4H_4$ series:

$$\Gamma\begin{bmatrix} a_2, a_3, a_4, 1-a_2, 1-a_3, 1-a_4; \\ b_1, c_1+b_1-c_2, c_1+b_1-c_3, b_4, 1-c_1, 1-c_2, 1-c_3 \end{bmatrix}$$

$$\times\, {}_4F_3\begin{bmatrix} 1-b_4, 1-b_1, 1+c_2-c_1-b_1, 1+c_3-c_1-b_1; \\ 1-c_1, \quad 1-c_2, \quad 1-c_3 \end{bmatrix}$$

$$(2.2) \quad + \Gamma\begin{bmatrix} a_2-1, 2-a_2, 1+a_3-a_2, 1+a_4-a_2, a_2-a_3, a_2-a_4; \\ 1+b_1-a_2, 1+c_1+b_1-c_2-a_2, 1+c_1+b_1-c_3-a_2, \\ 1+b_4-a_2, a_2-c_1, a_2-c_2, a_2-c_3, a_2 \end{bmatrix}$$

$$\times\, {}_4H_4\begin{bmatrix} 1+c_1-a_2, 1+c_2-a_2, 1+c_3-a_2, 1-a_2; \\ 1+b_1-a_2, 1+c_1+b_1-c_2-a_2, 1+c_1+b_1-c_3-a_2, \\ 1+b_4-a_2 \end{bmatrix}$$

$$+ \text{ idem } (a_2; a_3, a_4) = 0.$$

Since the nearly-poised ${}_4F_3$ series of the first kind on the left of (2.2) can be expressed in terms of two Saalschützian ${}_5F_4$ series (**1**, § 6.5 (1)), we get the following relation between two Saalschützian ${}_5F_4$ and three nearly-poised ${}_4H_4$ series:

$$\Gamma\begin{bmatrix} a_2, a_3, a_4, 1-a_2, 1-a_3, 1-a_4, 2b_1+c_1-c_2-c_3-1; \\ b_1, c_1+b_1-c_2, c_1+b_1-c_3, b_4, c_1+b_1-c_2-c_3, b_1-c_2, b_1-c_3, \\ 2-b_1-c_1 \end{bmatrix}$$

$$\times\, {}_5F_4\begin{bmatrix} 1-b_1, 1+c_2-c_1-b_1, 1+c_3-c_1-b_1, \\ 1+\tfrac{1}{2}(b_4-c_1-b_1), \tfrac{1}{2}(1+b_4-c_1-b_1); \\ 1+b_4-c_1-b_1, 1-\tfrac{1}{2}(c_1+b_1), \tfrac{1}{2}(3-c_1-b_1), \\ 2+c_2+c_3-2b_1-c_1 \end{bmatrix}$$

$$+ \Gamma\begin{bmatrix} a_2, a_3, a_4, 1-a_2, 1-a_3, 1-a_4, 1+c_2+c_3-c_1-2b_1, \\ 3b_1+b_4+c_1-2c_2-2c_3-1; \\ b_1, c_1+b_1-c_2, c_1+b_1-c_3, b_4, 1-b_1, 1+c_2-c_1-b_1, \\ 1+c_3-c_1-b_1, b_1+b_4-c_2-c_3, 3b_1+c_1-2c_2-2c_3 \end{bmatrix}$$

$$(2.3) \quad \times\, {}_5F_4\begin{bmatrix} c_1+b_1-c_2-c_3, b_1-c_2, b_1-c_3, \\ \tfrac{1}{2}(3b_1+b_4+c_1-2c_2-2c_3-1), \tfrac{1}{2}(3b_1+b_4+c_1-2c_2-2c_3); \\ 2b_1+c_1-c_2-c_3, b_1+b_4-c_2-c_3, \\ \tfrac{1}{2}(3b_1+c_1-2c_2-2c_3), \tfrac{1}{2}(1+3b_1+c_1-2c_2-2c_3) \end{bmatrix}$$

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$$+\Gamma\left[\begin{matrix}2-a_2, a_2-1, 1+a_3-a_2, 1+a_4-a_2, a_2-a_3, a_2-a_4;\\ 1+b_1-a_2, 1+c_1+b_1-c_2-a_2, 1+c_1+b_1-c_3-a_2,\\ 1+b_4-a_2, a_2-c_1, a_2-c_2, a_2-c_3, a_2\end{matrix}\right]$$

$$\times {}_4H_4\left[\begin{matrix}1+c_1-a_2, 1+c_2-a_2, 1+c_3-a_2, 1-a_2;\\ 1+b_1-a_2, 1+c_1+b_1-c_2-a_2, 1+c_1+b_1-c_3-a_2,\\ 1+b_4-a_2\end{matrix}\right]$$

$$+ \text{idem } (a_2; a_3, a_4) = 0.$$

If we put $b_4 = a_4$, $a_2 = c_1 + b_1 - c_2$ and $a_3 = c_1 + b_1 - c_3$ in (2.3), we get a relation between two Saalschützian ${}_5F_4$, two nearly-poised ${}_4F_3$ of the second kind and a nearly-poised ${}_4F_3$ series of the first kind. Also, if in this new relation we put $c_1 = 1 + c_3 - b_1 + n$, we get a relation between a terminating nearly-poised ${}_4F_3$ series of the second kind and a terminating Saalschützian ${}_5F_4$ series and, after reversing the terminating Saalschützian ${}_5F_4$ series, we get

$$(2.4) \qquad {}_4F_3\left[\begin{matrix}c_3-n, 1+c_3-b_1, c_2-n, -n;\\ b_1-n, 1+c_3-c_2, a_4-n\end{matrix}\right]$$

$$= \frac{(a_4-c_3)_n}{(a_4-n)_n}\, {}_5F_4\left[\begin{matrix}1+c_3-a_4, b_1-c_2, \tfrac{1}{2}(c_3-n), \tfrac{1}{2}(1+c_3-n), -n;\\ b_1-n, 1+c_3-c_2, \tfrac{1}{2}(1+c_3-a_4-n),\\ 1+\tfrac{1}{2}(c_3-a_4-n)\end{matrix}\right]$$

which is § 4.5 (1) of Bailey **(1)**.

Again, if we reverse the first ${}_4H_4$ series on the left of (2.3) and then put $a_2 = 1$, we get

$$(2.5)\qquad {}_4F_3\left[\begin{matrix}1-b_4, 1-b_1, 1+c_2-c_1-b_1, 1+c_3-c_1-b_1;\\ 1-c_1, \quad 1-c_2, \quad 1-c_3\end{matrix}\right]$$

$$= \Gamma\left[\begin{matrix}1-c_1, 1-c_2, 1-c_3, 2b_1+c_1-c_2-c_3-1;\\ c_1+b_1-c_2-c_3, b_1-c_2, b_1-c_3, 2-b_1-c_1\end{matrix}\right]$$

$$\times {}_5F_4\left[\begin{matrix}1-b_1, 1+c_2-c_1-b_1, 1+c_3-c_1-b_1, \tfrac{1}{2}(1+b_4-c_1-b_1),\\ 1+\tfrac{1}{2}(b_4-c_1-b_1);\\ 1+b_4-c_1-b_1, 1-\tfrac{1}{2}(c_1+b_1), \tfrac{1}{2}(3-c_1-b_1),\\ 2+c_2+c_3-2b_1-c_1\end{matrix}\right]$$

$$+\Gamma\left[\begin{matrix}1-c_1, 1-c_2, 1-c_3, 1+c_2+c_3-c_1-2b_1,\\ 3b_1+b_4+c_1-2c_2-2c_3-1;\\ 1-b_1, 1+c_2-c_1-b_1, 1+c_3-c_1-b_1, b_1+b_4-c_2-c_3,\\ 3b_1+c_1-2c_2-2c_3\end{matrix}\right]$$

$$\times {}_5F_4\left[\begin{matrix}c_1+b_1-c_2-c_3, b_1-c_2, b_1-c_3, \tfrac{1}{2}(3b_1+b_4+c_1-2c_2-2c_3),\\ \tfrac{1}{2}(3b_1+b_4+c_1-2c_2-2c_3-1);\\ 2b_1+c_1-c_2-c_3, b_1+b_4-c_2-c_3, \tfrac{1}{2}(3b_1+c_1-2c_2-2c_3),\\ \tfrac{1}{2}(1+3b_1+c_1-2c_2-2c_3)\end{matrix}\right]$$

which is (1) of § 6.5 of **(1)**.

3. In this section, I consider certain transformations of nearly-poised series of the type ${}_5H_5$. If we take $M = 5$ in (2.1) and then reverse the first ${}_5H_5$ series in it and put $C_5 = 0$, we get the following relation between a nearly-poised ${}_5F_4$ series of the first kind and four nearly-poised ${}_5H_5$ series:

$$\Gamma\begin{bmatrix} a_2, a_3, a_4, a_5, 1-a_2, 1-a_3, 1-a_4, 1-a_5; \\ b_1, c_1+b_1-c_2, c_1+b_1-c_3, c_1+b_1-c_4, b_5, 1-c_1, 1-c_2, \\ 1-c_3, 1-c_4 \end{bmatrix}$$

$$\times {}_5F_4\begin{bmatrix} 1-b_5, 1-b_1, 1+c_2-c_1-b_1, 1+c_3-c_1-b_1, \\ 1+c_4-c_1-b_1; \\ 1-c_1, 1-c_2, 1-c_3, 1-c_4 \end{bmatrix}$$

(3.1)
$$+\Gamma\begin{bmatrix} 2-a_2, a_2-1, 1+a_3-a_2, 1+a_4-a_2, 1+a_5-a_2, a_2-a_3, \\ a_2-a_4, a_2-a_5; \\ 1+b_1-a_2, 1+c_1+b_1-c_2-a_2, 1+c_1+b_1-c_3-a_2, \\ 1+c_1+b_1-c_4-a_2, \\ 1+b_5-a_2, a_2-c_1, a_2-c_2, a_2-c_3, a_2-c_4, a_2 \end{bmatrix}$$

$$\times {}_5H_5\begin{bmatrix} 1+c_1-a_2, 1+c_2-a_2, 1+c_3-a_2, 1+c_4-a_2, 1-a_2; \\ 1+b_1-a_2, 1+c_1+b_1-c_2-a_2, 1+c_1+b_1-c_3-a_2, \\ 1+c_1+b_1-c_4-a_2, 1+b_5-a_2 \end{bmatrix}$$

$$+ \text{idem } (a_2; a_3, a_4, a_5) = 0.$$

Now if we first put $a_2 = c_1 + b_1 - c_2$ in (3.1) and then in the new relation put $c_1 = 2c_2 - b_1 - 1$, we get the following relation between a nearly-poised ${}_5F_4$ series of the first kind and three nearly-poised ${}_5H_5$ series:

$$\Gamma\begin{bmatrix} a_3, a_4, a_5, 2-c_2, 1-a_3, 1-a_4, 1-a_5; \\ b_1, 2c_2-c_3-1, 2c_2-c_4-1, b_5, 2+b_1-2c_2, 1-c_2, 1-c_3, 1-c_4 \end{bmatrix}$$

$$\times {}_5F_4\begin{bmatrix} 1-b_5, 1-b_1, 2-c_2, 2+c_3-2c_2, 2+c_4-2c_2; \\ 2+b_1-2c_2, 1-c_2, 1-c_3, 1-c_4 \end{bmatrix}$$

(3.2)
$$+\Gamma\begin{bmatrix} 2-a_3, a_3-1, 1+a_4-a_3, 1+a_5-a_3, 1+a_3-c_2, \\ a_3-a_4, a_3-a_5; \\ 1+b_1-a_3, 2c_2-c_3-a_3, 2c_2-c_4-a_3, 1+b_5-a_3, 1+b_1 \\ +a_3-2c_2, a_3-c_2, a_3-c_3, a_3-c_4, a_3 \end{bmatrix}$$

$$\times {}_5H_5\begin{bmatrix} 2c_2-b_1-a_3, 1+c_2-a_3, 1+c_3-a_3, 1+c_4-a_3, 1-a_3; \\ 1+b_1-a_3, c_2-a_3, 2c_2-c_3-a_3, 2c_2-c_4-a_3, 1+b_5-a_3 \end{bmatrix}$$

$$+ \text{idem } (a_3; a_4, a_5) = 0.$$

Since the nearly-poised ${}_5F_4$ series of the first kind on the left of (3.2) can be expressed in terms of two Saalschützian ${}_5F_4$ series [cf. (**1**, § 6.5)], we get the following relation between two Saalschützian ${}_5F_4$ series and three nearly-poised ${}_5H_5$ series:

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$$\Gamma\left[\begin{matrix} a_3, a_4, a_5, 2-c_2, 1-a_3, 1-a_4, 1-a_5, 2c_2+b_1-c_3-c_4-2; \\ b_1, 2c_2-c_3-1, 2c_2-c_4-1, b_5, 1-c_2, b_1-c_3, b_1-c_4, \\ 2c_2-c_3-c_4-1,\ 3-2c_2 \end{matrix}\right]$$

$$\times\ {}_5F_4\left[\begin{matrix} 1-b_1, 2+c_3-2c_2, 2+c_4-2c_2, \tfrac{1}{2}(1+b_5-2c_2), 1+\tfrac{1}{2}(b_5-2c_2); \\ 2+b_5-2c_2, 1-c_2, \tfrac{3}{2}-c_2, 3+c_3+c_4-b_1-2c_2 \end{matrix}\right]$$

$$+\frac{(1+b_5-2c_2)}{2(1-c_2)}\Gamma\left[\begin{matrix} a_3, a_4, a_5, 2-c_2, 1-a_3, 1-a_4, 1-a_5, 2c_2+b_5 \\ +2b_1-2c_3-2c_4-3, 2+c_3+c_4-b_1-2c_2; \\ b_1, 2c_2-c_3-1, 2c_2-c_4-1, b_5, 1-b_1, 2+c_3 \\ -2c_2, 2+c_4-2c_2, 1-c_2, b_5+b_1-c_3-c_4, \\ 2(c_2+b_1-c_3-c_4-1) \end{matrix}\right] \tag{3.3}$$

$$\times\ {}_5F_4\left[\begin{matrix} b_1-c_3, b_1-c_4, 2c_2-c_3-c_4-1, c_2+b_1-c_3-c_4 \\ +\tfrac{1}{2}(b_5-3), c_2+b_1+\tfrac{1}{2}b_5-c_3-c_4-1; \\ b_5+b_1-c_3-c_4, c_2+b_1-c_3-c_4-1, c_2-c_3-c_4+b_1-\tfrac{1}{2}, \\ 2c_2+b_1-c_3-c_4-1 \end{matrix}\right]$$

$$+\ \Gamma\left[\begin{matrix} 2-a_3, a_3-1, 1+a_4-a_3, 1+a_5-a_3, 1+a_3-c_2, \\ a_3-a_4, a_3-a_5; \\ 1+b_1-a_3, 2c_2-c_3-a_3, 2c_2-c_4-a_3, 1+b_5-a_3, 1+b_1 \\ +a_3-2c_2, a_3-c_2, a_3-c_3, a_3-c_4, a_3 \end{matrix}\right]$$

$$\times\ {}_5H_5\left[\begin{matrix} 2c_2-b_1-a_3, 1+c_2-a_3, 1+c_3-a_3, 1+c_4-a_3, 1-a_3; \\ 1+b_1-a_3, c_2-a_3, 2c_2-c_3-a_3, 2c_2-c_4-a_3, 1+b_5-a_3 \end{matrix}\right]$$

$$+\ \text{idem}\ (a_3;\ a_4, a_5) = 0.$$

If we take $b_5 = a_5$, $a_4 = 2c_2 - c_4 - 1$ and $a_3 = 2c_2 - c_3 - 1$ in (3.3), we get a relation between two Saalschützian ${}_5F_4$, two nearly-poised ${}_5F_4$ of the second kind and a nearly-poised ${}_5F_4$ series of the first kind. Also, if we put $2 + c_3 - 2c_2 = -n$ in this new transformation, we get a relation between a terminating nearly-poised ${}_5F_4$ series of the second kind and a terminating Saalschützian ${}_5F_4$ series and after reversing the terminating Saalschützian ${}_5F_4$ series, we get

$$ {}_5F_4\left[\begin{matrix} c_3-n, 1+\tfrac{1}{2}(c_3-n), 1+c_3-b_1, c_4-n, -n; \\ \tfrac{1}{2}(c_3-n), b_1-n, 1+c_3-c_4, a_5-n \end{matrix}\right] \tag{3.4}$$

$$= \frac{(a_5-c_3-1-n)(a_5-c_3)_{n-1}}{(a_5-n)_n}$$

$$\times\ {}_5F_4\left[\begin{matrix} b_1-c_4, 1+c_3-a_5, 1+\tfrac{1}{2}(c_3-n), \tfrac{1}{2}(1+c_3-n), -n; \\ b_1-n, 1+c_3-c_4, \tfrac{1}{2}(3+c_3-a_5-n), 1+\tfrac{1}{2}(c_3-a_5-n) \end{matrix}\right]$$

which is (2) of § 4.5 of **(1)**.

Again, if we reverse the first ${}_5H_5$ series in (3.3) and put $a_3 = 1$, we get the following relation:

$$(3.5)\qquad {}_5F_4\left[\begin{matrix}1-b_5,\ 2-c_2,\ 1-b_1,\ 2+c_3-2c_2,\ 2+c_4-2c_2;\\ 1-c_2,\ 2+b_1-2c_2,\ 1-c_3,\ 1-c_4\end{matrix}\right]$$

$$= \Gamma\left[\begin{matrix}2+b_1-2c_2,\ 1-c_3,\ 1-c_4,\ 2c_2+b_1-c_3-c_4-2;\\ b_1-c_3,\ b_1-c_4,\ 2c_2-c_3-c_4-1,\ 3-2c_2\end{matrix}\right]$$

$$\times\ {}_5F_4\left[\begin{matrix}1-b_1,\ 2+c_3-2c_2,\ 2+c_4-2c_2,\ \tfrac{1}{2}(1+b_5-2c_2),\ 1+\tfrac{1}{2}(b_5-2c_2);\\ 2+b_5-2c_2,\ 1-c_2,\ \tfrac{3}{2}-c_2,\ 3+c_3+c_4-b_1-2c_2\end{matrix}\right]$$

$$+\ \frac{(1+b_5-2c_2)}{2(1-c_2)}\ \Gamma\left[\begin{matrix}2c_2+b_5+2b_1-2c_3-2c_4-3,\\ 2+c_3+c_4-b_1-2c_2,\ 2+b_1-2c_2,\ 1-c_3,\ 1-c_4;\\ 1-b_1,\ 2+c_3-2c_2,\ 2+c_4-2c_2,\\ b_5+b_1-c_3-c_4,\ 2(c_2+b_1-c_3-c_4-1)\end{matrix}\right]$$

$$\times\ {}_5F_4\left[\begin{matrix}b_1-c_3,\ b_1-c_4,\ 2c_2-c_3-c_4-1,\ c_2+b_1+\tfrac{1}{2}(b_5-3)\\ -c_3-c_4,\ c_1+b_1+\tfrac{1}{2}b_5-c_3-c_4-1;\\ b_5+b_1-c_3-c_4,\ \ c_2+b_1-c_3-c_4-1,\ \ c_1+b_1-c_3-c_4-\tfrac{1}{2},\\ 2c_2+b_1-c_3-c_4-1\end{matrix}\right]$$

which is the generalization of (2) of §4.6 of **(1)**.

4. The sum of a nearly-poised ${}_3H_3$. Taking $M = 3$ and $a_2 = c_1 + b_1 - c_2$ in (2.1) and then putting $c_1 = 2c_2 - b_1 - 1$ in the new transformation, we get the following relation between two nearly-poised ${}_3H_3$ series:

$$(4.1)\qquad {}_3H_3\left[\begin{matrix}2c_2-b_1-1,\ c_2,\ c_3;\\ b_1,\ c_2-1,\ b_3\end{matrix}\right]$$

$$= \Gamma\left[\begin{matrix}1+a_3-c_2,\ b_1,\ b_3,\ 2+b_1-2c_2,\ 1-c_2,\ 1-c_3;\\ 1+b_1-a_3,\ 1+b_3-a_3,\ 1+b_1+a_3-2c_2,\ a_3-c_2,\ a_3-c_3,\ 2-c_2\end{matrix}\right]$$

$$\times\ {}_3H_3\left[\begin{matrix}2c_2-b_1-a_3,\ 1+c_2-a_3,\ 1+c_3-a_3;\\ 1+b_1-a_3,\quad c_2-a_3,\quad 1+b_3-a_3\end{matrix}\right].$$

If we put $b_3 = a_3$ in (4.1), we get a relation between a nearly-poised ${}_3H_3$ and a summable nearly-poised ${}_3F_2$ series [cf. § 6.4 (2) of **(1)**]. Hence, we get

$$(4.2)\qquad {}_3H_3\left[\begin{matrix}2c_2-b_1-1,\ c_2,\ c_3;\\ b_1,\ \ c_2-1,\ \ a_3\end{matrix}\right]$$

$$= \frac{(1+a_3+c_3-2c_2)}{2(1-c_2)}\ \Gamma\left[\begin{matrix}b_1,\ a_3,\ 2+b_1-2c_2,\ 1-c_3,\\ 2b_1+a_3-c_3-2c_2-1;\\ 1+b_1+a_3-2c_2,\ a_3-c_3,\\ b_1-c_3,\ 2(b_1-c_2)\end{matrix}\right].$$

If we put $b_1 = 1$ in (4.2), we get

$$(4.3)\qquad {}_3F_2\left[\begin{matrix}2(c_2-1),\ c_2,\ c_3;\\ c_2-1,\ a_3\end{matrix}\right]$$

$$= (1+a_3+c_3-2c_2)\ \Gamma\left[\begin{matrix}a_3,\ 1+a_3-c_3-2c_2;\\ 2+a_3-2c_2,\ a_3-c_3\end{matrix}\right].$$

Again, if we put $c_3 = -n$ in (4.3), we get

$$(4.4)\quad {}_3F_2\left[\begin{matrix}2(c_2-1),\ c_2,\ -n;\\ c_2-1,\ a_3\end{matrix}\right] = \frac{(1+a_3-2c_2-n)(2+a_3-2c_2)_{n-1}}{(a_3)_n}$$

which is § 4.5 (1.1) of **(1)**.

Also, if we put $a_3 = 1$ in (4.2), we again get the sum of a nearly-poised ${}_3F_2$ series of the first kind.

It may be remarked that (4.2) can be obtained more easily by using the identity*

$$(4.5)\quad K \times {}_3H_3\left[\begin{matrix}1+\tfrac{1}{2}K,\ b,\ a;\\ \tfrac{1}{2}K,\ 1+K-b,\ w\end{matrix}\right]$$
$$= 2(K-b)\,{}_2H_2\left[\begin{matrix}b,\ a;\\ K-b, w\end{matrix}\right] - (K-2b)\,{}_2H_2\left[\begin{matrix}b,\ a;\\ 1+K-b,\ w\end{matrix}\right]$$

and summing the two ${}_2H_2$ series on the right of (4.5).

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