(such as the reviewer!) in the areas of the book. Did any non-specialist read the manuscript? If they had, surely they would have pointed out that "Euler's $\phi$-function" is not defined. In the list of basic notation at the front $\phi(n)$ is described as "Euler's totient function". So what is it? We are not told. While Chapter 6 is devoted to the Möbius $\mu$-function, the function is not defined anywhere in the book. While these are well-known functions, the reader is entitled to a reminder of their definitions. But what of "Jordan's divisor function $J_k$" and its "unitary analogue $J_K^*$"? No non-specialist could be expected to know what these are. Again we are not told. There are many other examples of undefined terms and notation in the book and this flaw seriously detracts from the usefulness of the book. It could however be easily remedied by making the list of basic notation at the front of the book into a thoroughly comprehensive list of definitions. And why does the book have no index? A detailed contents and author index (in the manner of Hardy and Wright) is no substitute – an inexcusable omission in a reference book. Finally, readers paying the top-of-the-range price of £179 will expect a book of top-of-the-range production quality. The typesetting in the book, while adequate, is far from top quality, being sometimes very cramped (as on page 71) and sometimes very spread out (as on page 551). The English, while adequate, would have benefitted from a native-speaker's red pencil.

C.J. SMYTH

MACDONALD, I.G. *Symmetric functions and Hall polynomials* (2nd edition) (Clarendon Press, Oxford, 1995), x + 475 pp, 0 19 853489 2, £55.

The first edition of Macdonald's book *Symmetric Functions and Hall Polynomials* appeared in 1979. It was slim (180 pages), magnificently written, and became the definitive book on the subject for a wide range of workers. Sixteen years later a second edition has appeared. It is no longer slim. Two new chapters have been added together with a rich selection of extra material and these changes demand a new review.

As with the first edition over half of the book is devoted to the first chapter, which gives an account of symmetric functions. Symmetric functions impinge on many disciplines and this will be the chapter that most readers consult. What material has been added to produce a chapter whose length rivals that of the original book? Although new text appears, the great majority of added material consists in further examples at the end of each section. As a rule of thumb the number of examples in each section has doubled and this has produced a veritable treasure trove. Doubly stochastic matrices (§1 Ex. 13), Muirhead's inequalities (§2 Ex. 18) and Kac's treatment of quadratic reciprocity (§3 Ex. 17) make an appearance alongside many more gems. A welcome consequence of this abundance of examples is that the references in this edition (sadly lacking in the first) have been expanded many-fold; this will greatly benefit those workers wishing to pursue the veins of enquiry opened by these examples. Macdonald has also made improvements to the text which were suggested in earlier reviews. Section §6, which deals with the transition matrices between the various symmetric functions, now gives a direct description of those involving the power-sum symmetric functions; Appendix A contains an elementary and self-contained account (§8) of the polynomial representations of $GL_m(k)$ for $k$ algebraically closed and of characteristic 0. The treatment of the inner (or internal) product of symmetric functions has been expanded. There is also an entirely new Appendix B dealing with the characters of wreath products $G \sim S_n$; this is treated in the same fashion as the new material in §7 and provides welcome preparation for Chapter IV.

As for the later chapters, Chapter II, "Hall Polynomials", sees changes in §4, which now provides an explicit formula for $g_s(t)$ in terms of basic hypergeometrics. This chapter also contains an appendix by A. Zelevinsky (the Russian translator of the first edition) giving a further proof of Hall's theorem. Chapter III, "Hall–Littlewood Symmetric Functions", sees some slight rewriting of §2 and §5; the last example of the earlier §7 (which deals with the case $t = -1$ of these functions) is now promoted to a section of its own (§8) on Schur's $Q$-functions.

Here again the examples have been greatly extended and we see several inspired by the mathematical physics community. Vertex operators appear in §5 Ex. 8 while §6 Ex. 7 comes from Kirillov and Reshetikin's treatment of the Bethe Ansatz and Young Tableaux. Although Chapter IV ("The Characters of $GL_n$ over a Finite Field") and Chapter V ("The Hecke Ring of $GL_n$ over a Local Field") are unchanged, their references and notes have been updated.

The final two chapters are new. Chapter VI, "Symmetric Functions with Two Parameters", generalises the one parameter family of Hall–Littlewood functions to yield a family containing the Schur functions of Chapter I and Jack's symmetric functions. This chapter, both in its text and examples, encapsulates much recent research and will be of significant interest not only to mathematicians but also to those workers (conformal field theorists, statistical mechanists and the like) who deal with multi-dimensional integrals and orthogonal polynomials. We find here various generalisations of Selberg's integral

$$\frac{1}{n!}\int_0^1 \cdots \int_0^1 \prod_i x_i^{a-1}(1-x_i)^{b-1} \prod_{i<j} |x_i - x_j|^{2c} dx_1 \ldots dx_n$$
$$= \prod_{j=0}^{n-1} \frac{\Gamma(a+jc)\Gamma(b+jc)\Gamma((j+1)c)}{\Gamma(a+b+(n+j-1)c)\Gamma(c)},$$

many of which have been inspired by Macdonald. In this context Greg W. Anderson's elegant proof [1] of the above formula was one of the few references I felt lacking.

The final chapter treats zonal polynomials via Gelfand pairs and zonal spherical functions. These polynomials have long been of interest to statisticians and more recently have found application by algebraic combinatorialists studying association schemes. The material here contains not only a useful compilation of results dispersed in the literature but many new results as well. Readers interested in such material should also note that Audrey Terras has a draft sequel (*Fourier analysis on finite groups and applications*) to her two volume *Harmonic analysis on symmetric spaces and applications* (Springer-Verlag, 1985, 1988) dealing with similar material, though with a different emphasis.

The second edition appears remarkably free of misprints, which is important for peripatetic workers wishing to use this as a reference volume. (The only misprint I am aware of, carried over from the first edition, is in (ii) p. 205 – a rather innocuous typographical error.) Macdonald should be congratulated for responding to his earlier reviewers and producing a more self-contained and better-referenced text; the extra examples will delight and aid many. This volume will undoubtedly become the standard reference on the subject until well into the next century and should be purchased by all research libraries. Knowing the impecunious nature of postgraduate life I am well aware that the £55 cost of the book represents much food and drink: this is one of those rare books I would recommend to students able to partake of its rich fare.

H. BRADEN

## REFERENCE

1. G. W. Anderson, A Short Proof of Selberg's Generalized Beta Formula, *Forum Math.* **3** (1991), 415–417.

Kałuża, R. *Through a reporter's eyes: the life of Stefan Banach* (Birkhäuser, Boston–Basel–Berlin, 1996), 167 pp., 3 7643 3772 9 (hardcover), £17.

Stefan Banach (1892–1945), one of the founders of modern functional analysis, was a leading figure in that group of extremely able Polish mathematicians who flourished in the period

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