LETTER TO THE EDITOR

Dear Editor,

Sums and maxima in stationary sequences

In our recent paper on the joint limiting distributions of sums and maxima in stationary sequences (Anderson and Turkman (1991)) the discussion was restricted to sequences of random variables which are unbounded above and which lie in the max domain of attraction of Gumbel or certain Frechet extreme value distributions. The purpose of this letter is to observe that our main result extends easily to random variables which are bounded above and which lie in the Weibull-type or Gumbel domains of attraction. (We use the notation Λ, Φ and Ψ respectively for the Gumbel, Frechet and Weibull-type extreme value distribution functions, where $\Lambda(x)=\exp(-e^{-x})$, $(-\infty<x<\infty)$; $\Phi_\alpha(x)=\exp(-x^{-\alpha})$, $(x\geqq 0)$ for $\alpha>0$; and $\Psi_\alpha(x)=\exp(-(-x)^\alpha)$, $(x\leqq 0)$ for $\alpha>0$.) Specifically, the following extended version of the paper's Theorem 1 holds.

Theorem. Suppose the sequence $\{X_i\}$ is stationary, has zero mean and finite variance, and satisfies:

(i) *$\{X_i\}$ is strong mixing and has positive extremal index;*

(ii) *for some constants $a_n>0$, with $\lim_{n\to\infty} a_n=\infty$, $c_n>0$ and d_n,*

$$\tilde{S}_n=S_n/a_n \xrightarrow{\mathrm{d}} \mathcal{N}(0,1)$$

$$\tilde{M}_n=(M_n-d_n)/c_n \xrightarrow{\mathrm{d}} G$$

marginally, where G is one of the extreme value distributions Λ, Φ_α for some $\alpha>2$, or Ψ_α for some $\alpha>0$;

(iii) *$\{X_i\}$ satisfies the $D'(a_n,u_n)$ condition.*

Then

$$\lim_{n\to\infty} E[\exp(it\tilde{S}_n)\chi(\tilde{M}_n\leqq x)]=\exp(-t^2/2)G(x),$$

so that $\tilde{S}_n$ and $\tilde{M}_n$ are asymptotically independent.

The proof of the paper's Theorem 1 uses right-unboundedness of the random variables X only in Lemma 2.2, asserting that $E\{|X|\chi(X>u_n)\}=O(u_n/n)$. But if $x_0=\sup\{x:F(x)<1\}<\infty$, where F is the distribution function of X, then, since $u_n\nearrow x_0$,

$$E\{|X|\chi(X>u_n)\}\leqq\max\{|u_n|,|x_0|\}P(X>u_n)=O(1/n)$$

and so the conclusion of Lemma 2.2 is seen to hold under the present wider conditions.

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The discussion of m-dependent sequences in the paper made no use of unboundedness, and so also carries over immediately.

The need to extend the paper's results became apparent during the fitting of models derived from the earlier theorem to data on extreme winds, and we are grateful to Mr D. Walshaw for alerting us to it.

Reference

ANDERSON, C. W. AND TURKMAN, K. F. (1991) The joint limiting distributions of sums and maxima in stationary sequences. *J. Appl. Prob.* **28**, 33–44.

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