Dynamics of Comets

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Abstract. The gravitational processes affecting the dynamics of comets are reviewed. At great distances from the Sun the motion of comets is primarily affected by the vertical component of the galactic field, as well as by encounters with stars and giant molecular clouds. When comets move in the region of the planets, encounters with these can strongly affect their motion. A good fraction of all periodic comets spend some time in temporary libration about mean motion resonances with Jupiter; some comets can be captured by this planet as temporary satellites. Finally, there is a small number of objects with orbital characteristics quite different from those of all other short-period comets.

Key words: Comets - Dynamics - Solar System

1. Introduction

Comets are ubiquitous members of the solar system: their orbits can in some cases have perihelia smaller than the solar radius and in other — much more frequent — cases can have aphelia so large as to become unbound from the Sun's attraction. This ubiquity, combined with the short physical lifetime when they are in the orbits in which we observe them, due to the sublimation of their volatile component, makes it necessary to use different gravitational models, and different celestial mechanical techniques, in order to study the various aspects of their possible dynamical evolutions. In most cases planetary dynamics can be studied in the framework of the various variants of the gravitational n-body problem, without consideration of the possibility of close encounters, and this applies also to the study of the dynamics of satellites and of asteroids (in this last case there are exceptions, constituted just by the objects customarily said to be on cometary orbits). For comets, on the other hand, close planetary encounters can play a very important role when they move in the planetary region, whereas when they are outside it stellar encounters, gravitational perturbations due to the passage of the solar system through giant molecular clouds and the galactic field all affect their motion. Moreover, when comets move well within the planetary region, additional perturbations appear (the *nongravitational forces*), due to the outgassing of cometary nuclei.

In this review we will shortly summarize the most important gravitational dynamical processes that influence cometary motions, giving examples for some of the cases in which the dynamics of individual objects have been studied in more detail (mostly for short-period comets).

It is customary to divide comets into long-period and short-period ones, basing on the orbital period P; short-period comets are those for which

$$P < 200\text{yr}; \tag{1}$$

among them, a further distinction is made between Halley-type (HT) and Jupiter-family (JF) comets, again on the basis of the orbital period, setting the dividing line at 20 yr. In this way, HT comets are those with

$$20\text{yr} < P < 200\text{yr} \tag{2}$$

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 255–268.

At the end of 1989 155 short-period comets had been observed. Examining this sample, it is possible to note that 20 comets have periods in the range between 20 and 200 yr, and that all of them have values of the Tisserand invariant

$$T = \frac{a_j}{a} + 2\sqrt{\frac{a}{a_j}(1-e^2)}\cos i \tag{3}$$

(a_j is the semimajor axis of Jupiter's orbit) smaller than 2 (Kresák, 1972; Carusi et al., 1986), whereas almost all JF comets have values of T greater than that value, with the exceptions of P/Machholz, P/IRAS and P/Tuttle. This implies that the relative velocity at encounters with Jupiter, given by

$$U = \sqrt{3-T} \tag{4}$$

in units of the planet's orbital velocity, is greater than 1 for HT comets, and less — often much less — than that for the other short-period comets. In the following we will consider as of HT all comets matching the criterion based on the orbital period, plus P/Tuttle, for its rather low value of the Tisserand invariant ($T = 1.60$); the two remaining cases of $T < 2$, P/Machholz and P/IRAS, are quite peculiar objects, as discussed also in the last Section. P/IRAS is under the dynamical control of Saturn, that can be encountered at one of the nodes, and avoids encounters with Jupiter. P/Machholz has a dynamical behaviour dominated by secular perturbations that cause exchanges between the out-of-the-ecliptic and the in-the-ecliptic components of its angular momentum similar to those experienced by some of the HT comets (Green et al., 1990; also, see below); on the other hand its orbital period and aphelion distance fit very well in the Jupiter family, and are so much separated from values typical of HT comets that the inclusion in the latter group would seem questionable.

2. Long-period comets and the Oort cloud

2.1. Long-period comets

When comets are discovered, it is customary to compute a preliminary orbit which is parabolic because the majority of them actually move, close to perihelion, in orbits of eccentricity departing very slightly from unity. If there is a sufficient number of observations, a different conic solution can be computed, either a hyperbola or an ellipse (of moderate to low eccentricity if the comet turns out to be a short-period one). A hyperbolic heliocentric motion about the perihelion passage does not mean, in general, that the comet was originally not bound to the Sun, because before entering the planetary region the comet was actually moving about the barycenter of the solar system, feeling the gravitational pull of the Sun plus the planets; the eccentricity of the barycentric orbit, usually called the *original* orbit is in general slightly smaller; in fact, if there are enough observations to allow the computation of the original orbit, it almost always turns out to be elliptical, even if of very long period (10^3 to 10^6 yr); the exceptions are probably due to the presence of nongravitational forces.

The outbound branch of the orbit of such long-period comets, when computed with the same procedure used for the original orbit, can turn out to be hyperbolic, due to the fact that the typical planetary perturbations, mostly due to Jupiter, are rather larger than the typical binding energy, even in the absence of close encounters; since the perturbation distribution is roughly symmetrical about zero, the result is that many long-period comets are ejected from the solar system at each perihelion passage.

2.2. The Oort cloud

In a historical paper, Oort (1950) analyzed the then available sample of original orbits and discussed the apparently puzzling fact that it shows a distinct peak of orbital energies whose width is far smaller than the typical planetary energy perturbation just described. To explain the peak, he argued that the comets in it had been making their first passage through the inner planetary region on the *observed* original orbits, whereas comets outside the peak had been presumably making subsequent passages. The implication is that comets in the energy peak should have been stored for very long time in orbits with very small ($1/a \leq 10^{-4}\mathrm{AU}^{-1}$) binding energies and larger perihelia. The large semimajor axes involved imply aphelion distances twice as large, in excess of several times 10^4 AU, where perturbations by passing stars can unbind definitively the comets from the solar system or shorten their perihelion distances, sending them to visit the inner planetary region in orbits allowing discovery.

In the storage region postulated by Oort the orbital planes of the comets would be randomized by stellar perturbations in much less than the solar system age, and the comets would spend most of the time close to the aphelia of their very elongated orbits (due to Kepler's law), so that the concept of a comet *cloud* surrounding the solar system seems an appropriate visualization of the situation.

Comets would have been implanted in the Oort cloud in the late stages of planetary formation: during the formation of the outer planets — especially that of Uranus and Neptune, that should have taken place on a time scale much longer that that of Jupiter and Saturn — due to the intrinsic inefficiency of the accumulation process, the majority of planetesimals should have been scattered into either hyperbolic or very elongated orbits by successive planetary encounters. Planetesimals that had been put on orbits with aphelia larger than $\simeq 10^4$ AU would have their perihelia decoupled from the planetary region, and their orbital planes randomized, by the action of passing stars, in what is essentially the inverse of the process that sends comets back to the inner planet region starting from the cloud (Fernández, 1985).

The more than forty years passed since Oort's paper have not substantially altered the basic picture described above; however, additional perturbers of the comets in the cloud have been recognized, namely giant molecular clouds and the vertical component of the galactic field, and a massive inner component, together with a trans-neptunian, disk-like comet ring, has been added to the original Oort cloud (sometimes now referred to as the *outer* or *active* Oort cloud).

In particular, the tidal effect of the vertical component of the gravitational field of the Galaxy has been shown to be the most efficient perturber of cometary orbits in the cloud. It acts by secularly changing the orbital angular momentum, and its effect is maximum for orbits whose perihelia are inclined at $\pm 45°$ with respect to the galactic plane, and progressively vanishes towards inclinations $0°$ and $\pm 90°$. Changing the angular momentum while keeping the energy constant results in changing the perihelion distance, that can in this way be brought into the planetary region.

2.3. The inner Oort cloud and the trans-neptunian belt

The passage of the solar system through a giant molecular cloud results in a gravitational perturbation of practically all the comets in the Oort cloud. As a consequence, similarly to the stellar encounter case, comets can be ejected to interstellar space or injected into the planetary region, with a net loss for the cloud's cometary population. The frequency and strength of the encounetrs with molecular clouds are not yet assessed with certainty, but anyway it seems that they would be sufficient to strongly deplete the Oort cloud well within the solar system's lifetime. This is why the existence of an inner, more massive cloud has been postulated; when stripping comets away from the outer cloud, molecular clouds would also pump up the semimajor axes of comets in the inner cloud, effectively inflating it enough as to replenish the outer, ejected layers. Numerical simulations (Duncan et al., 1987) nicely show that the same process forming the outer cloud described before, i.e. planetary scattering of the aphelia followed by stellar decoupling of the perihelia, in fact leads to the formation of the required inner cloud.

Although the combination inner cloud — outer cloud ensures a durable enough reservoir for the continous supply of long-period comets, there are serious doubts about its ability to supply short-period comets at the observed rate as well. To start the multi-stage capture process that eventually leads to short-period comets of the Jupiter family we must have comets with perihelia just at the outskirts of the planetary region. Their capture by Neptune is extremely inefficient unless their orbital inclinations are very low. The question of the efficiency of the capture process has an immediate bearing on the number of comets with which we have to start, and thus with the population of the the reservoir involved. A trans-neptunian disk of comets, such as that originally proposed by Kuiper (1951) as a natural by-product of planetary formation, as opposed to a *very* massive inner core of the Oort cloud, is being actively studied as the possible main source of short-period comets, and the numerical simulations by Quinn et al. (1990), although made with a questionable increase of the planetary masses, in order to shorten an otherwise prohibitively long computation, point in this direction.

3. Short-period comets

3.1. Halley-type comets

The high relative velocity of HT comets at close encounters with Jupiter mentioned in the Introduction has the consequence that the effects of the encounters are less pronounced than in the case of JF comets, and in fact numerical integrations of the

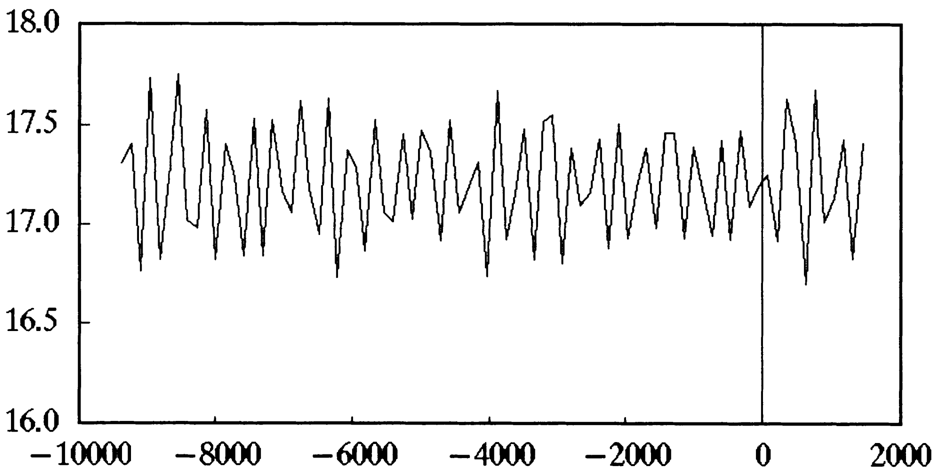

Fig. 1. Time evolution of the heliocentric semimajor axis a, in AU, of P/Brorsen-Metcalf for 4,000,000 days (about 11,000 years) backwards, starting from 1.0 February 1585 (JD 2,300,000.5)

motion of HT comets show a much smaller variability of the orbital elements. This variability is mostly due to indirect planetary perturbations experienced when the comet passes perihelion (Carusi et al. 1986; 1987a).

An important feature of the dynamics of HT comets is the presence, in many cases, of librations about mean motion resonances with Jupiter like the 1/5, 1/6 and 1/7 (Carusi et al. 1987a,b). These librations have been found by numerical integrations over about 10^4 yr, and apparently take place for the majority of HT comets with orbital period between 60 and 90 yr and on direct orbits; indeed some of them seem to experience very stable librations, although it is clear that the duration of the librating motion has to be possibly long, but anyway temporary. Figure 1 shows the behaviour of the heliocentric semimajor axis of the orbit of P/Brorsen-Metcalf, that has been found to librate about the 1/6 resonance with the mean motion of Jupiter for at least 11,000 yr (Carusi et al., 1987b).

Kozai (1979) found that some HT comets, including P/Halley, have the argument of perihelion librating, because of secular perturbations. In the case of P/Halley, the libration would prevent close encounters with Jupiter, a feature with important consequences on studies of the dynamical lifetime of this comet and of the associated meteor streams. However, as pointed out by Kozai himself, the comet appears to be at the border of the libration region in phase space (as it should be, since P/Halley cannot have avoided Jupiter encounters forever in the past), and in fact numerical integrations (Carusi et al., 1988a) show indeed the possibility of a break-up of the libration within about 10^4 yr in the past. This is shown in Figure 2, where the nodal distances of P/Halley are plotted for a time span of about 11,000 yr in the past: towards the end of the backward integration, taken from Carusi et al.

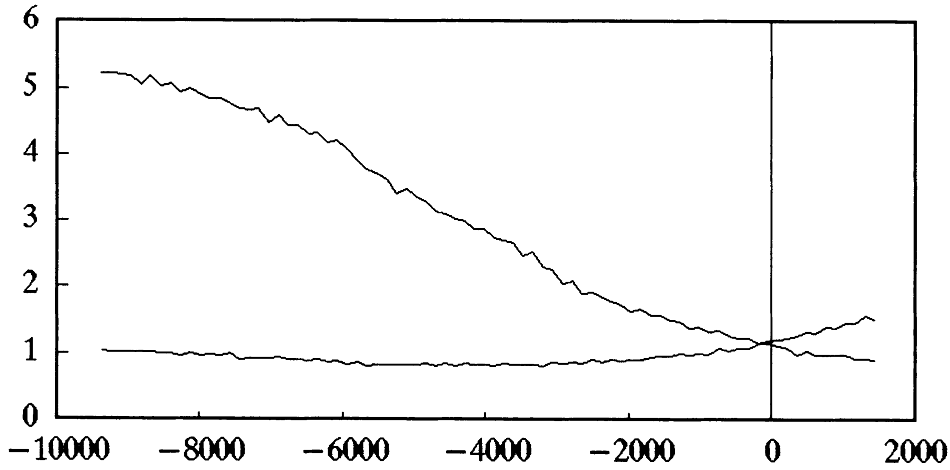

Fig. 2. Time evolution of the nodal distances, in AU, of P/Halley for the same time span of Figure 1

(1988a), close encounters with Jupiter become possible.

The capture into HT orbits is probably through a single encounter with Jupiter, which brings the comet from a nearly parabolic orbit to one with semimajor axis in the appropriate range. The single-encounter mechanism of capture is the first one that has been studied in the attempt to understand the origin of short-period comets. It was initially thought that short-period comets are captured from observed long-period comets (i.e. from comets with perihelia small enough to allow discovery) through a single, deep jovian encounter. Newton (1893) showed that if this were the case, then one quarter of all short-period comets should be on retrograde orbits. The numerical work of Everhart (1969; see also Everhart, 1976) confirmed Newton's finding. About one quarter of HT comets are on retrograde orbits: indeed, they seem to be the end product of the dynamical process once thought to be the one leading to all periodic comets.

3.2. Jupiter-family comets

3.2.1. Dynamical characteristics of Jupiter-family comets

Due to the high degree of chaoticity of the orbits of JF comets, which is reflected in the mobility of these objects in the phase space of orbital elements, it is rather difficult to reliably classify them into dynamically homogeneous groups; a qualitative subdivision of them, based on their dynamical behaviours, has been attempted by Carusi and Valsecchi (1987). Before describing the dynamical processes responsible for the chaotic behaviour, let us give a look to the distributions of some key orbital parameters.

Figure 3 shows the distribution of orbital periods of all JF comets at the three

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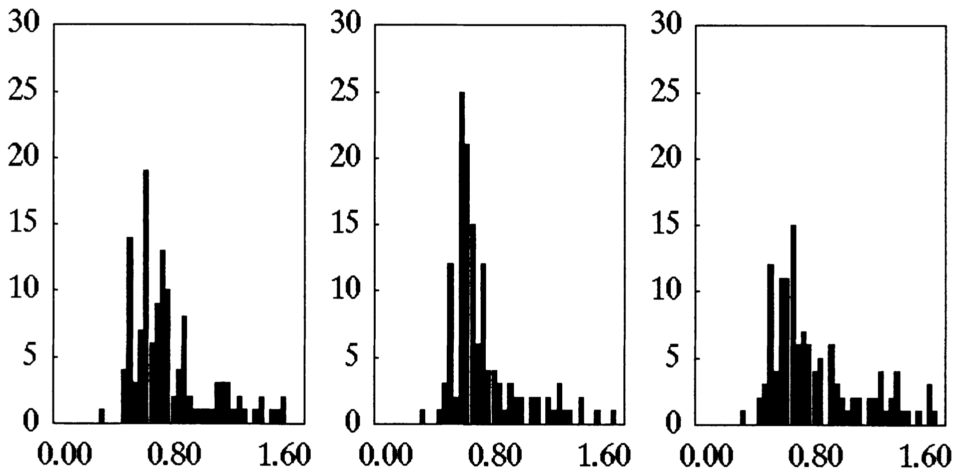

Fig. 3. Histograms of the distribution of orbital period P, in units of Jupiter's period, for all known JF comets at three epochs: left, 1.0 February 1585; center, 10.0 October 1995; right, 17.0 June 2406

epochs 1.0 February 1585, 10.0 October 1995, 17.0 June 2406 (JD 2,300,000.5, JD 2,450,000.5 and JD 2,600,000.5 respectively). These data are taken from the integration of 155 short-period comets between 1585 and 2406 carried out for the second edition of the catalogue *Long-Term Evolution of Short-Period Comets* (Carusi et al., 1991b), which includes all short-period comets discovered up to 1990.0. A first comment to be made concerns the number of comets whose periods are in excess of 1.67 Jupiter's periods (20 years). They are: 29 in 1585, 21 in 1995, 28 in 2406. The figure of 1995 corresponds to the one in 1990.0, if we remember that P/Tuttle ($P < 20$ yr) is excluded from that number but P/Lexell (not of HT) is included. Since the number of HT comets does not vary in the given time span of 821 years (always 20 with $P > 20$ yr and one with $P < 20$ yr) the two figures referring to the number of comets with $P > 20$ yr in 1585 and 2406 give us a measure of the mobility of comet orbits, at least in their semiaxis. In fact, in 1585 nine observed JF comets had periods exceeding 20 years, in 1995 only one (P/Lexell), in 2406 eight; only two comets have $P > 20$ yr at more than one of the three epochs, namely P/Oterma (in 1585 and 2406) and P/Lexell (in 1995 and 2406). As noted by Carusi et al. (1987a), these comet orbits have perihelia close to the orbit of Jupiter, and rather large Tisserand invariants with respect to Saturn. This fact is indicative of their stepwise capture in short-period orbits (see below).

In Figure 4 the values of the Tisserand invariant larger than 2 are reported, for the same three epochs. It is evident from the figure that the larger the value of T, the larger the number of comets, at least up to values around 2.9. In this sense we may say that, on the average, the Tisserand invariants of the JF comets are rather large; this means that, during encounters with Jupiter, their velocities

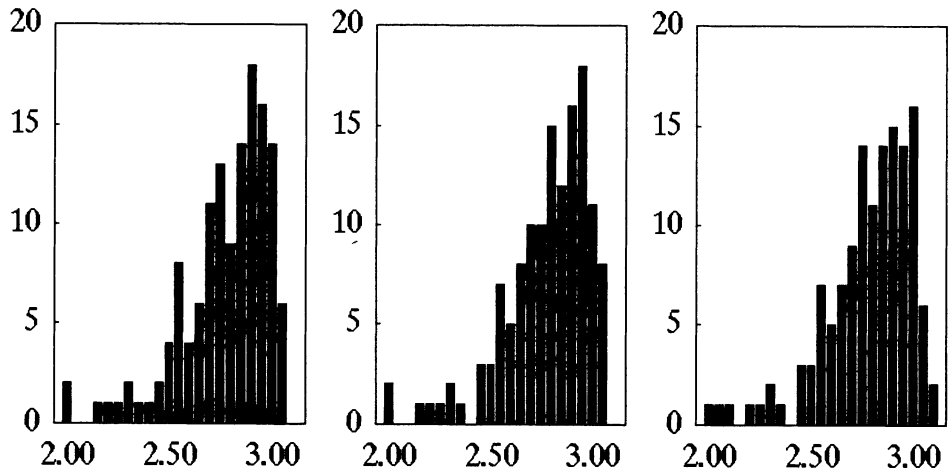

Fig. 4. Same as Figure 3 for the Tisserand invariant T

relative to the planet are pretty low. Moreover, although obviously decreasing very steeply, the number of comets with $T > 3$ is remarkable: 8 in 1585 and 1995, 6 in 2406. At the same time, the number of comets with $T > 2.83$ (the limiting value for escape from the solar system under the action of Jupiter) are 67, 65 and 67, respectively, or about 50% of the whole sample of JF comets. With the exception of a few marginal cases, therefore, these last objects are forced to remain under the control of Jupiter for a long time, a thing which may have strong implications for their rate of physical decay. When the orbit of a comet reaches those of the other outer planets, however, there is always the possibility that a close encounter with them decreases the Tisserand invariant with respect to Jupiter, so that this planet may become able to eject the comet from the system.

Figure 5 gives the distribution of the perihelion distances. Here, we may note in 1995 a marked concentration of perihelion distances between 1 and 2 AU from the Sun, a fact mainly due to observational selection, because many comets have been discovered recently due to major perturbations which have put them closer to the Sun. The distributions at the two extreme dates are quite similar, providing a better picture of a normal behaviour of this population. We may also note that only three comets have at present perihelia outside the orbit of Jupiter, while this number amounted to 11 in 1585 and will be 8 in 2406. A concentration towards the orbit of Jupiter at the present epoch appears also in the distribution of aphelion distances, where the number of comets with aphelia outside the orbit of Saturn is 13, 5 and 11 in 1585, 1995 and 2406, respectively. It has to be noted, in this respect, that in 2406 there is a comet (P/Helin) with orbit formally hyperbolic, as noted also by Nakamura and Yoshikawa (1991).

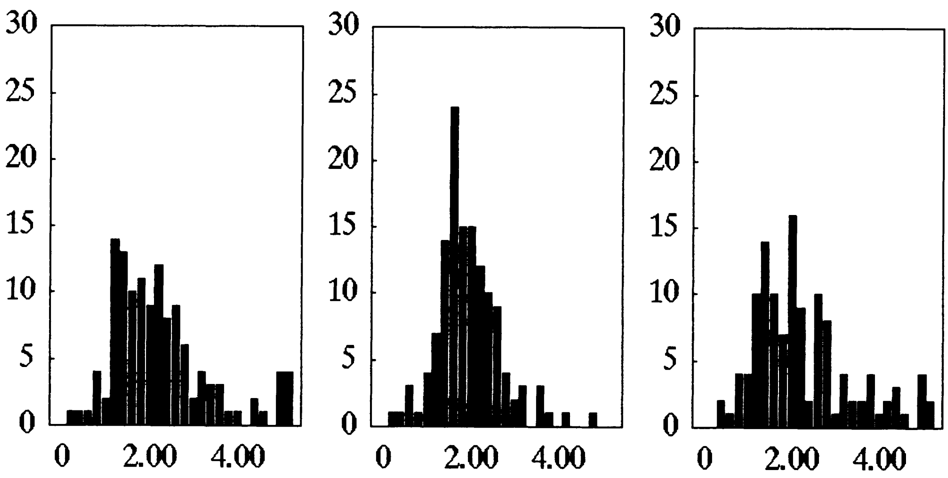

Fig. 5. Same as Figure 3 for the perihelion distance q, in AU

3.2.2. Capture processes: close encounters and temporary satellite captures

The principal dynamical process responsible for the supply of short-period comets is represented by one or more close encounters with the outer planets. We have already noted that HT comets are likely to have been captured in their present orbits by a single encounter with Jupiter. JF comets, on the other hand, are most probably captured through a multi-stage process, such as that described by Everhart (1977, 1982), although single-encounter captures cannot be ruled out completely. Multi-stage capture should take place on orbits characterized by large values of the Tisserand invariant with respect to the planet(s) controlling the motion at every time, allowing low-velocity, more efficient planetary encounters. During these interactions, a considerable reduction of the orbital period may transfer the co… from outside to inside the orbit of the planet involved; if the new orbit intersects (or is tangent to) the one of the next planet toward the Sun, this in general takes over the dynamical control of the comet, and the process may repeat until the object is firmly under the control of Jupiter. The process can also be reversed at any stage, as shown by the case of P/Lexell (Carusi et al., 1982b).

Encounters of JF comets with Jupiter are characterized — as we have seen — by generally low relative velocities, making the original orbits much more sensitive to the planetary perturbations at relatively large distances. The degree of sensitivity of comet orbits to small perturbations depends on the orbit itself: in general, the larger the value of the Tisserand invariant, the larger the sensitivity, but other factors, like the presence of temporary librations (see below), may help in stabilizing, at least temporarily, the motion of the comet.

It has been noted (Carusi and Valsecchi, 1982; Greenberg et al., 1988) that the time sequence of the approaching branch of a close encounter may be consid-

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ered as composed by three phases: an initial phase where the motion is essentially heliocentric, with perturbations by the planet increasing with time but unable to produce qualitative changes in the orbit of the comet; then a second phase where the perturbations by the Sun and the planet are of the same relevance; and a third phase during which the comet is practically under the control of the planet, with the Sun as a perturber. This sequence is repeated after the maximum approach, in reversed order. In all close encounters the first and third phases are clearly recognizable, with durations and effects that depend upon the size and orientation of the relative velocity vector, but the recognizability and duration of the second phase, which is the most critical, can vary substantially from case to case. The second phase is completely ignored in the schematizations of encounters as scattering (for example in the Öpik description, Öpik, 1976) or double-two-body problems (where the motion is simply heliocentric inside and planetocentric outside a given sphere of action), and this is one of the reasons why these techniques fail to reproduce in detail the most complicated encounters.

During the second phase the motion of the comet is contemporarily governed by the Sun and the planet, and it is not surprising that, in a competitive situation like this, very small changes in the geometry of the problem can lead to completely different results. As shown in the case of horseshoe patterns (see for example Everhart, 1973; Carusi et al., 1982a), the outcome of encounters with a sizable second phase may even result in an abortion of the encounter itself, with the comet forced to recede from the planet in a horseshoe behaviour.

In other cases, especially among comets on orbits nearly tangent to that of Jupiter in either their perihelion or aphelion, the relative velocity at the end of the second phase is so low that the planetocentric energy becomes negative: the comet then undergoes a temporary satellite capture. Examples of these events have been reported many times in literature, with a range of possibilities that go from the well known encounter of P/Oterma with Jupiter in 1934-1939 (Carusi et al., 1985a), with the comet following a simple, non-closed jovicentric pattern, to the long-lasting capture of P/Helin-Roman-Crockett (Tancredi et al., 1990), during which it performs five revolutions about Jupiter. According to the integration by Carusi et al. (1991b), which corresponds to that of orbit B of Tancredi et al. (1990), the comet undergoes a satellite capture of duration of about 18 years between 2065 and 2088, with jovicentric orbital elements ($a \simeq 0.16$ AU, $e \simeq 0.55$) not far from those of Sinope, the outermost retrograde jovian satellite ($a = 0.158$ AU, $e = 0.28$).

As already said, the outcomes of close encounters depend very much on the initial conditions. In numerical simulations this translates into the precise knowledge of the orbital parameters at some time preceding the encounter. However, as shown by many investigations in recent years (see, e.g., Carusi et al., 1985b and 1988b; Rickman and Froeschlé, 1988), the very presence of close encounters makes any attempt to reconstruct the long-term evolution of comet orbits a very difficult task. To the intrinsically chaotic nature of the motion one has to add the possible presence of nongravitational effects; only in a very limited number of cases, when the perihelion distance is rather large and the nongravitational effects negligible, the gravitational reconstruction of orbits can be taken with some confidence; unfortunately, these conditions are met mainly by comets with large values of the Tisserand invariant, which are also very sensitive to numerical errors or uncertainties in the starting orbit.

In general, therefore, one cannot rely on orbit integrations which extend past a close encounter. An exception to this rule is the recent linkage of comet P/d'Arrest with comet 1678 La Hire (Carusi et al., 1991a): in this case, however, the integration is reliable, in spite of four close encounters with Jupiter occurring between 1678 and 1851 (discovery of P/d'Arrest), because the nongravitational effects on this comet are extremely stable in time, and because of the existence of observations on both sides of the encounters.

3.2.3. Temporary librations about mean motion resonances

In the previous Section we have noted that the onset of temporary librations may help stabilizing the motion of a short-period, JF comet. These events take place when the semiaxis of a comet orbit is close to some low-order resonance with Jupiter's motion (we have no records on librations of real comets about resonances with the other outer planets). The perturbations due to encounters with Jupiter at both ends of the libration cycle lead to a regular oscillation of the semimajor axis a about the resonance.

A somewhat different mechanism, described in Carusi et al. (1987a), is responsible for librations of HT comets about high-order resonances with Jupiter of the type 1/n (with n between 5 and 7). This does not consist of direct perturbations during encounters with the planet (which in general do not occur), but rather to the shifts in orbital energy originated from the displacement of the centre of motion from the Sun to the solar system barycentre, as the comet approaches or leaves the planetary system (mainly the orbit of Jupiter).

The integration of motion of short-period comets over 821 years (Carusi et al., 1985a) has shown that a large fraction of them spends some time in librations of the types just described. Comets have been found to librate about almost all the resonances with Jupiter's period (above the 2/1) of the form $(p+q)/p$, with $p \leq 4$ and $q \leq 5$. In a few cases this process lasts for the whole time span of the integration, while in a greater number of cases it is limited to just one or two libration cycles. In addition, several cases of resonances in orbits of period greater than that of Jupiter (apart from the HT comets resonances described before) have been found.

3.3. Comets with peculiar orbits

There are comets whose orbital behaviour is unique in some respect, in the sense that their peculiarity is not exhibited by any other short-period comet. This is for example the case of P/Encke, whose aphelion distance — currently of 4.09 AU — is the smallest among short-period comets. Coupling this small value to the second largest eccentricity ($e = 0.85$) among JF comets, it turns out that the orbital period of P/Encke is the smallest on record ($P = 3.3$ yr). The problem then arises of what has been the mechanism responsible for putting this comet in an orbit which does not allow effective close encounters with Jupiter any more. In the hypothesis that P/Encke has followed in the past the same dynamical routes of all other comets, the supposed mechanism should have decoupled the aphelion of the comet from the orbit of Jupiter in a short time span compared with the typical timescale of deep, catastrophic encounters. Moreover, the Tisserand invariant of P/Encke is very large (3.03), as a consequence of the small aphelion and of the low inclination. It is questionable, however, whether this value is representative of the orbit before the decoupling or, rather, the consequence of a slow but continuous aphelion reduction process.

Another comet very peculiar by its orbital characteristics is P/Machholz. As shown by Green et al. (1990) with a 3-body integration over 4000 yr, and confirmed by Carusi et al. (1991b) for the 821 years of their integration with a more realistic model of the solar system, this comet is librating due to secular perturbations, and its perihelion distance oscillates between roughly 1.0 and less than 0.1 AU in more than 4000 years. Since the semimajor axis of the orbit is rather stable, this oscillation implies a simultaneous increase of eccentricity and decrease of inclination, the latter parameter varying between 0° and 80°. At the minimum of the oscillation, around the year 2450, the perihelion distance of P/Machholz will be as small as 0.03 AU; the question then arises of how many libration cycles the comet can have performed in the past, surviving the likely strong outgassing produced at such a small distance from the Sun. Indeed, as noted by Green et al. (1990), the degree of activity that P/Machholz exhibits now seems to put the comet in between P/Encke and 3200 Phaeton, the asteroid presumed to be the parent body of the Geminid meteor stream.

A final example of a comet in a peculiar dynamical situation is P/IRAS. This comet has a rather inclined orbit (46°) which does not allow encounters with Jupiter, due to the unfavourable geometry; however, the descending node of P/IRAS is at the heliocentric distance of Saturn. The comet, as a matter of fact, has recently undergone (around 1950) the deepest encounter with this planet found in Carusi et al. (1985a). It is just because of this encounter with Saturn that the value of the Tisserand invariant of P/IRAS with respect to Jupiter has dropped below the conventional value of 2, that divides HT from JF comets. In fact, since at least a few centuries, and for some centuries in the future, P/IRAS is definitely under the dynamical control of Saturn, rather than of Jupiter, a unique situation among short-period comets.

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Discussion

Cl.Froeschlé – I would like to point out that the project Spaceguard performed by Milani *et. al.* shows that some Appolo-Amor asteroids may become a cometary type object after some 10^5 years and then go back to an asteroid type orbit.
W.Landgraf – There exists the possibility that comet Halley was captured by the Earth, rather than by Jupiter. The comet always approached very close the Earth (for example in 837) during its history and, particularly, at $\sim$ 1400 BC, acording to my backward integrations. The orbit of the comet came very close to that of the Earth. The energy change of $\Delta a \sim -0.06$ is rather small and can be produced by Earth without problems.
A.Carusi – The suggested capture by Earth is possible, but very improbable.
H.U.Keller – I am astonished that you give a talk on the dynamics of comets without mentioning "non-gravitational" forces. Can you comment on their importance and how they may change the picture you described?
A.Carusi – Nongravitational forces are extremely important in the reconstruction of orbits of observed comets. They are less important in describing processes that affect comet motion, or in model experiments. Nongravitational forces are, anyway, unknown for most comets and applicable only to the limited period covered by observations.