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ISOMORPHISMS BETWEEN SKEW POLYNOMIAL RINGS

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Abstract

Let R be any ring and α any automorphism of R. We determine here all the automorphisms of the skew polynomial ring $R[x, \alpha]$ which fix R elementwise. We then deal with isomorphisms between different skew polynomial rings whose underlying rings are isomorphic.

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1. Introduction

Throughout this paper, all rings have identity and all ring homomorphisms preserve the identity. The skew polynomial ring $R[x, \alpha]$, where α is an automorphism of R, is defined to be the set of all finite sums $r_0 + r_1x + \cdots + r_nx^n$ where $r_i \in R$ for all $i \in \{0, \cdots, n\}$, with addition carried out as with ordinary polynomials, and with multiplication determined by $xr = r^\alpha x$ for all $r \in R$. When R is commutative and α the identity automorphism of R, Gilmer (1968) determined all the automorphisms of $R[x, \alpha]$ which fix R elementwise. Parmenter (to appear) extended this to the case where R is commutative and α is any automorphism of R. The case where α is the identity on R and R is any ring has been handled by Coleman and Enochs (1971). These three results are an immediate consequence of our more general result, Theorem 1, below. There we determine all automorphisms of $R[x, \alpha]$ which fix R elementwise, where R is any ring and α is any automorphism of R. We then extend our investigation to isomorphisms, between different skew polynomial rings, which preserve an underlying ring isomorphism. Our major result here is Theorem 3 which has Theorem 1 as a consequence.

2. Automorphisms of skew polynomial rings

THEOREM 1. *Let R be any ring with identity and let α be any automorphism of R. Then $x \to \sum_{i=0}^{n} r_i x^i$ induces an automorphism of $R[x, \alpha]$ which fixes each element of R if and only if*

(i) $r^\alpha r_i = r_i r^{\alpha^i}$ *for all* $r \in R$ *and all* $i \in \{0, 1, \cdots, n\}$,

(ii) r_1 *is a central unit in* R, *and*

(iii) r_i *is nilpotent for all* $i \in \{2, \cdots, n\}$.

Further, these three conditions hold if and only if $x \to \sum_{i=0}^{n} r_i x^i$ induces a surjective homomorphism of $R[x, \alpha]$ which fixes each element of R.

We prove Theorem 1 by first introducing some preliminary results, the main one of which is Lemma 2 which allows us to concentrate on the special case when $r_0 = 0$ and $r_1 = 1$. This case is dealt with in Theorem 2. Theorem 1 is an immediate consequence of Theorem 2 and Lemma 2. As noted by Parmenter (to appear), there is not always a homomorphism of $R[x, \alpha]$ which fixes each element of R and maps x to $\sum_{i=0}^{n} r_i x^i$. Clearly, there is at most one such homomorphism, and this exists precisely when $(\sum r_i x^i) r = r^\alpha (\sum r_i x^i)$ for all $r \in R$; that is

(1) $$\text{for all } r \in R, \text{ and for all } i \in \{0, \cdots, n\}, r^\alpha r_i = r_i r^{\alpha^i}.$$

This is just condition (i) of Theorem 1. In particular $r^\alpha r_1 = r_1 r^\alpha$ for all $r \in R$, so that r_1 is central.

LEMMA 1. *If t_1 is a central unit of R and if $t_0 \in R$ is such that $t_0 r = r^\alpha t_0$ for all $r \in R$, then $x \to t_0 + t_1 x$ induces an automorphism of $R[x, \alpha]$ which fixes all elements of R.*

PROOF. From (1), $x \to t_0 + t_1 x$ induces a homomorphism of $R[x, \alpha]$ fixing R elementwise. Since t_1 is a unit this homomorphism is injective, and as $x = -t_1^{-1} t_0 (t_0 + t_1 x)^0 + t_1^{-1}(t_0 + t_1 x)$, the homomorphism is surjective.

LEMMA 2. *Suppose $x \to r_0 + r_1 x + \cdots + r_n x^n$ induces a homomorphism ψ of $R[x, \alpha]$ which fixes each element of R. Then ψ is surjective if and only if*

(i) $r^\alpha r_0 = r_0 r$ *for all* $r \in R$,

(ii) r_1 *is a central unit,*

and (iii) $x \to x + r_1^{-1} r_2 x^2 + \cdots + r_1^{-1} r_n x^n$ *induces a surjective homomorphism, μ say, of $R[x, \alpha]$ which fixes every element of R.*

In this case there exists an automorphism θ of $R[x, \alpha]$, which also fixes R elementwise, induced by $x \to r_0 + r_1 x$, such that $\psi = \theta\mu$. Thus ψ is an automorphism precisely when μ is an automorphism.

PROOF. Suppose that ψ is a surjective homomorphism of $R[x, \alpha]$. Then from (1), condition (i) holds and r_1 is central. But since ψ is surjective there exists $s_i \in R$ such that $x = \sum_{i=0}^{m} s_i [x^{\psi}]^i$ for some $m \in \mathbf{N}$. If we equate the coefficients of x and use (1) we obtain

$$1 = \left[\sum_{i=1}^{m} s_i \sum_{j=0}^{i-1} r_0^{i-1-j} (r_0^{\alpha})^j \right] r_1,$$

so that r_1 is a unit. By Lemma 1, θ is an automorphism. Since r_1 is a central unit, we see from (1) that μ is a homomorphism of $R[x, \alpha]$ fixing R elementwise. Also $(x^{\theta})^{\mu} = x^{\psi}$ so $\theta\mu = \psi$ and μ is surjective.

Conversely, suppose that conditions (i) (ii) and (iii) hold. We see from (1) that ψ is indeed a homomorphism of $R[x, \alpha]$ which fixes R elementwise. Since again $\psi = \theta\mu$ we know that ψ is surjective.

LEMMA 3. *If $k \in \mathbf{N}$ and $w_0, w_1, \cdots, w_k$ are nilpotent elements of R such that for all $j \in \{0, \cdots, k\}$, $Rw_j = w_jR$; then $\sum_{j=0}^{k} w_jR$ is a nilpotent ideal of R.*

The proof is an obvious modification of the usual argument when R is commutative.

THEOREM 2. *The map $x \to x + r_2x^2 + \cdots + r_nx^n$ induces a surjective homomorphism ϕ of $R[x, \alpha]$ which fixes R elementwise precisely when it induces an automorphism, and this occurs if and only if*

(i)′ *$r^{\alpha} r_i = r_i r^{\alpha^i}$ for all $r \in R$ and all $i \in \{2, \cdots, n\}$*

(ii)′ r_i is nilpotent for all $i \in \{2, \cdots, n\}$.

PROOF. (a) Assume $x \to x + r_2x^2 + \cdots + r_nx^n$ induces a surjective homomorphism ϕ of $R[x, \alpha]$ which fixes R elementwise, and let

$$x = \sum_{i=0}^{m} s_i [x^{\phi}]^i. \tag{2}$$

We know from (1) that (i)′ holds. We shall now prove that (ii)′ holds.

We first prove that for all $r \in R$ and all $i \in \{0, \cdots, m\}$

$$r^{\alpha} s_i = s_i r^{\alpha^i}. \tag{3}$$

The proof is by induction on i. Equating coefficients of x^0 and x^1 in (2) we see that $s_0 = 0$ and $s_1 = 1$ and so we have proved (3) for $i = 0$ and 1. Suppose now that $2 \leqq j \leqq m$ and that (3) holds for all i such that $1 \leqq i < j$. Equating coefficient of x^j

$$0 = s_j + \sum_{k=1}^{j-1} s_k \left[\sum_{i_1 + \cdots + i_k = j} r_{i_1} r_{i_2}^{\alpha^{i_1}} \cdots r_{i_k}^{\alpha^{(i_1 + \cdots + i_{k-1})}} \right]. \tag{4}$$

But $(r_cx^c)(r_dx^d) = r_x r_d^{\alpha^c} x^{c+d}$ for all $0 \leqq c, d \leqq n$, so we see from (1) that $(r_cx^c)(r_dx^d) = r_d^{\alpha} r_c x^{c+d}$, and extending this we obtain

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(5) $$(r_{i_1}x^{i_1})\cdots(r_{i_k}x^{i_k}) = r_{i_k}^{\alpha^{k-1}}\cdots r_{i_2}^{\alpha} r_{i_1} x^j$$

for all $i_1,\cdots,i_k \in \{0,\cdots,n\}$ with $i_1+\cdots+i_k = j$. Thus from (4) and (5) we obtain

$$0 = s_j + \sum_{k=1}^{j-1} s_k \Big[\sum_{i_1+\cdots+i_k=j} r_{i_k}^{\alpha^{k-1}}\cdots r_{i_1} \Big].$$

Now for any $r \in R$, we see from (1) that

$$(r_{i_k}^{\alpha^{k-1}}\cdots r_{i_1}) r^{\alpha^j} = r^{\alpha^{j+k-(i_1+\cdots+i_k)}}(r_{i_k}^{\alpha^{k-1}}\cdots r_{i_1}).$$

But since $i_1+\cdots+i_k = j$,

$$(r_{i_k}^{\alpha^{k-1}}\cdots r_{i_1}) r^{\alpha^j} = r^{\alpha^k}(r_{i_k}^{\alpha^{k-1}}\cdots r_{i_1}).$$

Hence

$$s_j r^{\alpha^j} = -\sum_{k=1}^{j-1} s_k r^{\alpha^k} \Big[\sum_{i_1+\cdots+i_k=j} r_{i_k}^{\alpha^{k-1}}\cdots r_{i_1} \Big] = r^{\alpha} s_j,$$

since, by the induction assumption, $s_k r^{\alpha^k} = r^{\alpha} s_k$ for $1 \leqq k \leqq j-1$. This completes the proof of (3).

We next show that $s_i r_{b_i}^{\alpha^{i-1}}\cdots r_{b_1}$ is nilpotent for all $1 \leqq i \leqq m$ with $b_j \in \{1,\cdots,n\}$ and $\sum_{j=1}^{i} b_j \geqq 2$. (Here we are using the notation $r_1 = 1$.) Notice that (ii)$'$ is a consequence of the case $i = 1$ (since $s_1 = 1$) and so the proof will be complete when we have proved this assertion. We shall use induction on $\sum_{j=1}^{i} b_j$. Assume that $2 \leqq k \leqq mn$, and that for all $1 \leqq i \leqq m$ and all $b_1,\cdots,b_i \in \{1,\cdots,n\}$ such that $\sum_{j=1}^{i} b_j > k$, $s_i r_{b_i}^{\alpha^{i-1}}\cdots r_{b_1}$ is nilpotent. Note that for $k = mn$ this is no assumption, since $\sum_{j=1}^{i} b_j \leqq mn$. Now consider any $1 \leqq i \leqq m$ and any $b_1,\cdots,b_i \in \{1,\cdots,n\}$ with $\sum_{j=1}^{i} b_j = k$. If we equate the coefficients of x^k in (2) we see that

(6) $$s_i r_{b_i}^{\alpha^{i-1}}\cdots r_{b_1} + \sum_{j=0}^{m} s_j t_j = 0$$

where, if $j \neq i$, t_j is the coefficient of x^k in $[x^{\phi}]^j$ and is given by

$$t_j = \sum \{ r_{c_j}^{\alpha^{j-1}}\cdots r_{c_1} \mid c_1+\cdots+c_j = k \},$$

and

$$t_i = \sum \{ r_{c_i}^{\alpha^{i-1}}\cdots r_{c_1} \mid c_1+\cdots+c_i = k \text{ and } (c_1,\cdots,c_i) \neq (b_1,\cdots,b_i) \}.$$

Consider a term $s_j r_{c_j}^{\alpha^{j-1}}\cdots r_{c_1}$ occuring in $s_j t_j$ for some $0 \leqq j \leqq m$, and let

$$q = (s_i r_{b_i}^{\alpha^{i-1}}\cdots r_{b_1})(s_j r_{c_j}^{\alpha^{j-1}}\cdots r_{c_1})(s_i r_{b_i}^{\alpha^{i-1}}\cdots r_{b_1}).$$

Note that by (1) and (3), $Rq = qR$. Suppose first that there exists $1 \leqq e \leqq$

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$\min(i,j)$ such that $c_e > b_e$. Then

$$q \in (s_i r_{b_i}^{\alpha^{i-1}} \cdots r_{b_{e+1}}^{\alpha^e}) R r_{c_e}^{\alpha^{e-1}} R (r_{b_{e-1}}^{\alpha^{e-2}} \cdots r_{b_1})$$
$$= s_i r_{b_i}^{\alpha^{i-1}} \cdots r_{b_{e+1}}^{\alpha^e} r_{c_e}^{\alpha^{e-1}} r_{b_{e-1}}^{\alpha^{e-2}} \cdots r_{b_1} R.$$

But $(b_1 + \cdots + b_{e-1}) + c_e + (b_{e+1} + \cdots + b_i) > k$ and so by assumption $s_i r_{b_i}^{\alpha^{i-1}} \cdots r_{b_{e+1}}^{\alpha^e} r_{c_e}^{\alpha^{e-1}} r_{b_{e-1}}^{\alpha^{e-2}} \cdots r_{b_1}$ is nilpotent, and thus, by Lemma 3, q is nilpotent. The remaining possibility is that for all $1 \leqq e \leqq \min(i,j)$, $c_e \leqq b_e$. Since $\Sigma_{l=1}^j c_l = k = \Sigma_{l=1}^i b_l$, it follows that $j \geqq i$. Indeed $j \neq i$ as otherwise $(c_1, \cdots, c_i) = (b_1, \cdots, b_j)$ which is a contradiction. Thus $j > i$ and there exists $1 \leqq e \leqq i$ such that $c_l = b_l$ for all $1 \leqq l < e$ and $c_e < b_e$. Then

$$q \in R s_j r_{c_j}^{\alpha^{j-1}} \cdots r_{c_{e+1}}^{\alpha^e} R r_{b_e}^{\alpha^{e-1}} \cdots r_{b_1} R$$
$$= s_j r_{c_j}^{\alpha^{j-1}} \cdots r_{c_{e+1}}^{\alpha^e} r_{b_e}^{\alpha^{e-1}} \cdots r_{b_1} R.$$

Now $(b_1 + \cdots + b_e) + (c_{e+1} + \cdots + c_j) > k$. Hence, by induction, q is nilpotent. By multiplying (6) on the left and right by $s_i r_{b_i}^{\alpha^{i-1}} \cdots r_{b_1}$ and by Lemma 3 it follows that $(s_i r_{b_i}^{\alpha^{i-1}} \cdots r_{b_1})^3$ is nilpotent. This completes the proof by induction.

(b) Assume conditions (i)′ and (ii)′ hold. From (i)′ and (1), $x \to x + r_2 x^2 + \cdots + r_n x^n$ induces a homomorphism ϕ of $R[x, \alpha]$ which fixes each element of R. If $\Sigma_{j=0}^p c_j [x^\phi]^j = 0$, it is routine to show by induction on j that $c_j = 0$ for $0 \leqq j \leqq p$. Thus ϕ is injective.

It only remains to prove that ϕ is surjective. Consider the ideal T_1 generated by $S_1 = \{r_a^{\alpha^b} \mid 2 \leqq a \leqq n \text{ and } b \in \mathbb{Z}\}$ and the ideal T_2 generated by $S_2 = \{r_a^{\alpha^b} \mid 2 \leqq a \leqq n \text{ and } 0 \leqq b \leqq n-1\}$.

Note that $T_1^\alpha \subseteq T_1$. From (ii)′, $r_a^{\alpha^b}$ is nilpotent for all $r_a^{\alpha^b} \in S_2$, and from (i)′, $r_a^{\alpha^b} R = R r_a^{\alpha^b}$. Thus from Lemma 3, T_2 is nilpotent. Notice that, from (1), for all $r_{a_1}^{\alpha^{b_1}}, r_{a_2}^{\alpha^{b_2}} \in S_1$

$$r_{a_1}^{\alpha^{b_1}} r_{a_2}^{\alpha^{b_2}} = r_{a_2}^{\alpha^{b_2 - a_1 + 1}} r_{a_1}^{\alpha^{b_1}} = r_{a_1}^{\alpha^{b_1 - a_2 + 1}} r_{a_2}^{\alpha^{b_2 - a_1 + 1}},$$

and

$$r_{a_1}^{\alpha^{b_1}} r_{a_2}^{\alpha^{b_2}} = r_{a_2}^{\alpha^{b_2}} r_{a_1}^{\alpha^{b_1 + a_2 - 1}} = r_{a_1}^{\alpha^{b_1 + a_2 - 1}} r_{a_2}^{\alpha^{b_2 + a_1 - 1}},$$

so that either the first or the second exponent can be made to lie between 0 and $n-1$. Thus $T_1^2 \subseteq T_2$ and so T_1 is nilpotent. Now let $g_1(x) = x^\phi + \Sigma_{i=2}^n r_i [x^\phi]^i$. Thus $g_1(x) = x + \Sigma_{i=2}^{m_1} s_{i,1} x^i$ for some $m_1 \in \mathbf{N}$ and $s_{i,1} \in T_1^2$. Continuing this process, given any $j \geqq 2$, define

$$g_j(x) = g_{j-1}(x) + \sum_{i=2}^{m_{j-1}} s_{i,j-1} [g_{j-1}(x)]^i,$$

so that $g_j(x) = x + \Sigma_{i=2}^{m_j} s_{i,j} x$ for some $m_j \in \mathbf{N}$, and $s_{i,j} \in T_1^{2j}$. In particular, since

T_1 is nilpotent, there exists $t \geqq 1$ with $g_t(x) = x$. Therefore, as $g_t(x)$ is just a linear combination of powers of x^ϕ, ϕ is surjective.

3. Isomorphisms between skew polynomial rings

Here we deal with isomorphisms between skew polynomial rings whose underlying rings are isomorphic. We first prove two preliminary results, and then use these and Theorem 1 to prove Theorem 3.

THEOREM 3. *Let A and B be any rings with an isomorphism ϕ: $A \to B$, and let α and β be automorphisms of A and B respectively. Then the map $x \to b_0 + b_1x + \cdots + b_nx^n$ extends ϕ to an isomorphism ϕ: $A[x, \alpha] \to B[x, \beta]$ if and only if*

(i) $a^{\alpha\phi}b_i = b_ia^{\phi\beta^i}$ *for all $a \in A$ and all $0 \leqq i \leqq n$,*

(ii) b_1 *is a unit, and*

(iii) b_i *is nilpotent for all $2 \leqq i \leqq n$.*

Further these three conditions occur precisely when ϕ is a surjective homomorphism.

When ν is a unit in a ring B, we denote by i_ν the inner automorphism of B given by $b^{i_\nu} = \nu b \nu^{-1}$ for all $b \in B$.

LEMMA 4. *Let A and B be any rings with automorphisms α and β respectively and let $\phi: A \to B$ be any isomorphism. If the map $x \to b_0 + b_1x + \cdots + b_nx^n$ extends ϕ to a surjective homomorphism between $A[x, \alpha]$ and $B[x, \beta]$, then b_1 is a unit and $\phi^{-1}\alpha\phi = \beta i_{b_1}$.*

PROOF. Since ϕ is surjective there exist $a_0, \cdots, a_m \in A$ such that $(\sum_{i=0}^m a_ix^i)^\phi = x$. That is, $\sum_{i=0}^m a_i^\phi(b_0 + \cdots + b_nx^n)^i = x$, and when we equate coefficients of x we see that

$$\sum_{i=0}^m \left[\sum_{j=0}^{i-1} a_i^\phi b_0^j b_1 (b_0^\beta)^{i-1-j}\right] = 1. \tag{7}$$

But ϕ is a homomorphism and therefore $a^{\alpha\phi}x^\phi = x^\phi a^\phi$ for all $a \in A$. Hence $a^{\alpha\phi}b_i = b_ia^{\phi\beta^i}$ for all $a \in A$ and all $0 \leqq i \leqq n$. In particular for $i = 1$ we see that

$$a^{\alpha\phi}b_1 = b_1a^{\phi\beta}. \tag{8}$$

It can be seen, on substituting in (7) that

$$\sum_{i=0}^m \left[\sum_{j=0}^{i-1} a_i^\phi b_0^j (b_0^{\phi^{-1}\alpha\phi})^{i-1-j}\right] b_1 = 1.$$

Hence b_1 is a right unit and from (8), b_1 is a unit. Thus $a^{\alpha\phi} = b_1a^{\phi\beta}b_1^{-1}$ for all

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$a \in A$, so that since $\phi : A \to B$ is an isomorphism $b^{\phi^{-1}\alpha\phi} = b_1 b^{\beta} b_1^{-1}$ for all $b \in B$. Hence $\phi^{-1}\alpha\phi = \beta i_{b_1}$.

LEMMA 5. *Let $\phi : A \to B$ be an isomorphism, and let α and β be automorphisms of A and B respectively. Further, let ν be a unit of B such that $\phi^{-1}\alpha\phi = \beta i_{\nu}$. Then the map $\theta : A[x,\alpha] \to B[x,\beta]$ determined by $x \to \nu x$ and $a \to a^{\phi}$ for all $a \in A$ is an isomorphism.*

PROOF. Now $x^{\theta}a^{\theta} = \nu x a^{\phi} = \nu a^{\phi\beta}x$ for all $a \in A$, and $(a^{\alpha})^{\theta}x^{\theta} = a^{\alpha\phi}\nu x$ for all $a \in A$. But $\alpha\phi = \phi\beta i$, and therefore $x^{\theta}a^{\theta} = (a^{\alpha})^{\theta}x^{\theta}$. Thus θ is a homomorphism. It is easily seen that θ is injective. Indeed, suppose $(\sum_{i=0}^{m} a_i x^i)^{\theta} = 0$ for some $a_0, \cdots, a_m \in A$. Then $\sum_{i=0}^{m} a_i^{\phi}\nu\nu^{\beta}\cdots\nu^{\beta^{i-1}}x^i = 0$, and as ν is a unit, $a_i^{\phi} = 0$ for all $i \in \{0, \cdots, m\}$. Hence $a_i = 0$ for all $i \in \{0, \cdots, m\}$. It only remains to show that θ is surjective. This can readily be seen as $(\nu^{-1})^{\phi^{-1}}x$ is mapped to x.

PROOF OF THEOREM 3. First assume that $x \to b_0 + b_1 x + \cdots + b_n x^n$ extends ϕ to a surjective homomorphism between $A[x,\alpha]$ and $B[x,\beta]$ which we shall also denote by ϕ. Then by Lemma 4, b_1 is a unit and $\phi^{-1}\alpha\phi = \beta i_{b_1}$. By Lemma 5, there is an isomorphism $\theta : A[x,\alpha] \to B[x,\beta]$, which also extends $\phi : A \to B$ and is determined by $x \to b_1 x$.

$$\begin{tikzcd} A[x,\alpha] \arrow[rr, "\phi"] \arrow[dr, "\theta"'] & & B[x,\beta] \\ & B[x,\beta] \arrow[ur, "\psi"'] & \end{tikzcd}$$

Now define $\psi : B[x,\beta] \to B[x,\beta]$ by $\phi = \theta\psi$. Then ψ is a surjective homomorphism and fixes B elementwise. Hence, from Theorem 1, ψ is an automorphism (and so ϕ is an isomorphism), and as b_1 is a unit and $\phi^{-1}\alpha\phi = \beta i_{b_1}$, the conditions of Theorem 1 yield the three required conditions of Theorem 3.

Conversely suppose the three conditions of Theorem 3 hold. From conditions (ii) and (iii), b_1 is a unit and $\phi^{-1}\alpha\phi = \beta i_{b_1}$. Hence, by (i) and Lemma 5, $\theta : A[x,\alpha] \to B[x,\beta]$ is an isomorphism. Define $\psi : B[x,\beta] \to B[x,\beta]$ by $b^{\psi} = b$ for all $b \in B$ and $x \to b_1^{-1}(b_0 + b_1 x + \cdots + b_n x^n)$. Now condition (i) of Theorem 3 implies that the nilpotence of $b_1^{-1}b_i$ follows from that of b_i and it follows from Theorem 1 that ψ is an automorphism. But $\phi = \theta\psi$ and therefore ϕ is an isomorphism.

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