fundamental catalogues proposed here and the increase on this basis of the precision of astronomical time determinations, should, in our opinion, be carried out simultaneously with the organization at several time services of parallel observations with photographic zenith tubes, impersonal astrolabes and transit instruments. However, we do not deem it expedient that all the time services observe only with zenith tubes or only with impersonal astrolabes since each of these instruments is characterized by its own specific errors and since at the same time there are still possibilities of increasing the precision of transit instruments.

Taking into account the above stated, the Pulkovo Time Service proposes in the near future to base its work on observations with photoelectric transit instruments and a photographic zenith tube.

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VARIATIONS IN ROTATION OF THE EARTH, RESULTS OBTAINED WITH THE DUAL-RATE MOON CAMERA AND PHOTOGRAPHIC ZENITH TUBES

By WM. MARKOWITZ

U. S. Naval Observatory, Washington, D. C.

Abstract. Comparison of photographic zenith tube (P.Z.T.) observations with time derived from quartz-crystal clocks during 1951 to 1955 and with cesium standards of frequency during 1955 to 1958 indicates that the seasonal variation is nearly the same from year to year. Lunar-tidal inequalities of semi-monthly and monthly periods with amplitudes of about 0ˢ001 each were found. A preliminary value of the Love number, *k*, is derived. Observations made since 1952 with the dual-rate moon position camera are used to derive $\Delta T = \mathrm{ET} - \mathrm{UT}$. Comparison of the P.Z.T. observations and atomic standards at the National Physical Laboratory and the Naval Research Laboratory shows details of the irregular variation from 1955 to 1958.

1. *Introduction.* As is well known, there are three types of variation in speed of rotation of the earth—secular, irregular, and periodic. Although the general nature of these variations has been known for some time, there is a need for determining them in detail.

Variations in speed are determined by comparing Universal Time (UT), which is based on the rotation of the earth, with time based on some other standard. This may be the moon, quartz-crystal clocks, or atomic standards of frequency. Studies of the variation in speed of rotation made at the U. S. Naval Observatory have been based on observations for UT made with the photographic zenith tubes (P.Z.T.'s) at Washington, D. C., and Richmond, Florida, and for Ephemeris Time (ET) with the dual-rate moon position camera at Washington. The clocks used have been the quartz-crystal clocks of the Naval Observatory and of the National Bureau of Standards, and the cesium standards of the National Physical Laboratory, Teddington, and the Naval Research Laboratory, Washington.

The problems treated here concern the stability of the seasonal variation, the determination of the lunar tidal variation, and the determination of the irregular variation. The results obtained permit a comparison to be made between variations in speed determined from astronomical observations and those computed from geophysical and meteorological considerations. A preliminary, but independent, value of the Love number *k* is obtained, and the nature of the irregular variations is shown.

2. *P.Z.T.* The cesium standards are stable to about 1 part in 10^{10}. This stability is so high that for all practical purposes the precision with which changes in speed of rotation of the earth can now be obtained depends entirely upon the accuracy with which UT can be determined. The P.Z.T.'s are well suited for the determination of UT. The instrument is impersonal, the zenith is automatically defined by a basin of mercury, and no instrumental corrections for azimuth, collimation, or level are required (Markowitz, in press). Observations are made at the zenith, where refraction anomalies may be expected to be a minimum. An important advantage of the P.Z.T. in the study of periodic variations in speed of rotation is that the system of star positions used is made internally consistent from the P.Z.T. observations themselves.

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Thus, periodic errors of star catalogues do not introduce spurious periodic terms in the determination of UT.

The operation of two stations, which are placed about one thousand miles apart, is of great advantage, principally because it permits a check on the results to be made. If P.Z.T. No. 3, at Washington, and P.Z.T. No. 2, at Richmond, both indicate the same change in speed of rotation a real change may have occurred. If, however, the changes indicated are not the same the existence of a change is doubtful.

Observations are made on about 200 nights per year at Washington and 300 nights per year at Richmond. The Richmond station is advantageously located, not only in regard to favorable weather conditions, but by being near the equator, at the low latitude of $+25° 37'$. At the equator the speed of stars which transit near the zenith is a maximum and the correction for variation of longitude vanishes.

The intercomparison of the P.Z.T.'s shows that systematic errors of the order of 10 milliseconds may occur, which remain for several days or weeks. However, the cesium standards indicate that the adopted UT2, based on the two P.Z.T.'s, is free of noticeable systematic error. For the period from 1955.7 to 1958.0 for, example, ν_U, the frequency of cesium on UT2, was represented by a straight line. The probable error of a monthly mean of ν_U is 4 parts in 10^{10}. The most probable explanation of this linear relation is that the earth was changing its speed uniformly and that the adopted UT2 was practically free of systematic error.

The suitability of the P.Z.T.'s for determining variations of small amplitude is illustrated in the case of the theoretical lunar tidal terms, whose periods are 13.7 and 27.6 days, respectively. The amplitudes of each are about 1 millisecond, and it was formerly considered that these terms could not be found from observation. These terms, however, not only were detected in the P.Z.T. observations of 1951 to 1954, but it became necessary to construct a more detailed mathematical theory in order to be able to compare observation with theory.

P.Z.T. No. 1 was used for time determination at Washington from October 1933 to May 1955. P.Z.T. No. 3, which replaced it at Washington, has been used since May 1954. P.Z.T. No. 2 has been used at Richmond since February 1949. The reduction of the Richmond time sights is made at the Richmond station. The analysis of the results for the two stations, which are tied together by monitoring time pulses from WWV, is made at Washington. There have been no major changes in observing programs or in methods of reduction at either station. Hence, it is possible to use the P.Z.T.'s to study the variations in speed of the rotation of the earth in a homogeneous manner.

3. *Seasonal variation.* The correction for the annual and semiannual terms which comprise the seasonal variations may be written,

$$\begin{aligned}\Delta SV &= A \sin (2\pi t - \theta_1) + B \sin (4\pi t - \theta_2) \\ &= a \sin 2\pi t + b \cos 2\pi t \\ &\qquad + c \sin 4\pi t + d \cos 4\pi t,\end{aligned}$$

where A, B, θ_1, θ_2, a, b, c, and d are constants, and t is the fraction of a year. ΔSV is to be added to UT1, that is, Universal Time corrected for variation in longitude, in order to obtain UT2.

The seasonal variation was first determined reliably by Dr. N. Stoyko, at Paris, in 1937. For the principal coefficient, A, he obtained about 60 milliseconds. In subsequent years similar results were obtained by Stoyko and by other investigators. In 1950 H. F. Finch announced that the Greenwich results for 1943 to 1949 confirmed those of Stoyko. The seasonal variation thus appeared to be stable, with an amplitude of 60 milliseconds for the annual term.

In February 1952, however, H. M. Smith of Greenwich announced that for 1950 and 1951 A had decreased to 33 milliseconds, and for 1952 Smith and R. H. Tucker obtained 20 milliseconds. The stability of the seasonal variation was now questioned.

The apparent change in the annual term about 1950 may be due to the fact that quartz-crystal clocks previous to this time did not have, in general, the reliability of continuous operation and precision which was obtained later. In order to overcome difficulties due to clock stoppages and short runs, various mathematical artifices were resorted to, such as working with the mean of the second differences of the clock corrections of a number of clocks.

These analytical methods did have the merit of showing the existence of the seasonal variation, but were probably not sufficiently rigorous to furnish definitive values. The analysis cannot make up for an intrinsic lack of precision in the clocks.

In 1953 I attempted to determine the seasonal variation from the P.Z.T. observations and the

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clocks at Washington. Let γ_i be the correction to a clock, i. Let ψ_i be that portion of the correction due to drift of the crystal, and let σ be the seasonal variation. Then

$$\gamma_i = \psi_i + \sigma.$$

It was required that for each clock it should be possible to find a function ψ_i which would represent the drift of the crystal for two years or more, and that the σ's obtained from the various clocks should be accordant. I was not able in 1953 to find any Washington clocks which met these requirements. The best clocks were the resonator quartz-crystals of the National Bureau of Standards, which were placed in operation in May 1951. The functions ψ_i could not be well determined in 1953 because the initial accelerations were high.

In October 1954 the resonators were moved to Boulder, Colorado. They had then been running continuously for more than three years, and it was now possible to determine the ψ_i's. For R2 and R3 these are, respectively,

$$\psi_2 = 31282 - 13.098t_1 - 46470e^{-0.00102t_1}, \quad (1)$$
$$\psi_3 = 18663 - 21.506t_1 - 18280e^{-0.00137t_1},$$

where $t_1 = 0$ on August 30.625, 1951. The ψ_i are in milliseconds and t_1 is in days.

The mean for R2 and R3 of $\Delta SV = -\sigma$ is shown in Figure 1. The results of harmonic analysis for ΔSV for individual years are given in lines (1) to (3) of Table I. During the 38 months from August 1951 to October 1954 the maximum difference between the σ's for the two clocks in accumulated time was never greater than 4 milliseconds.

The Essen ring-crystal oscillator, E-43, which

TABLE I. ΔSV, CORRECTION FOR SEASONAL VARIATION
Unit = 0ˢ001

	a	b	A	c	d	B
(1) 1951.7–52.7	+21	−25	33	−4	+6	8
(2) 1952.7–53.7	+28	− 8	29	−7	+9	12
(3) 1953.7–54.7	+24	+20	31	−9	+7	11
(4) 1955	+15	−25	29	−9	+7	11
(5) 1956	+22	−15	27	−7	+8	11
(6) 1957	+24	−15	28	−5	+5	7
(7) Mean	+22.8	−18.0	29.5	−6.8	+7.0	9.7
(8) Solar tides	+ 1.5	0.0		−4.3	−1.6	
(9) (7) − (8)	+21.3	−18.0	27.8	−2.5	+8.6	9.0
(10) Winds	+17.0	−11.0	20.0	−0.9	+0.4	1.0
(11) B.I.H., adopted	+22	−17	27.8	−7	+6	9.2

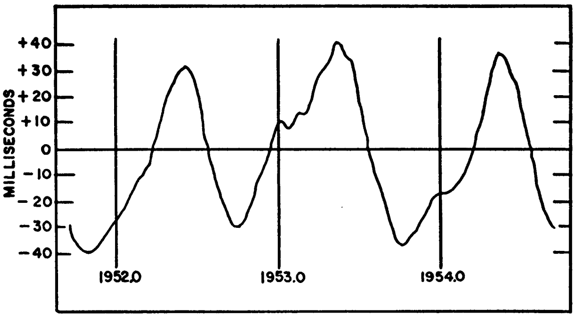

Figure 1. The correction for seasonal variation (ΔSV), 1951.7–1954.7.

was installed at the Naval Observatory in July 1954, was used to determine ΔSV for 1955.0 to 1956.0. The result is given in line (4) of Table I. A preliminary study based on E-56, which was installed in October 1954, furnishes a similar value of ΔSV. A more complete analysis will be made later. The drift of E-43 for the interval from August 1954 to November 1956, in milliseconds, is represented by

$$\psi_{43} = 56730 - 1.184t_2 - 0.00895t_2^2 + 3.8 \times 10^{-6}t_2^3,$$

where $t_2 = 0$ on January 1.625, 1956.

For 1956 and 1957 the solutions were based on the running of the cesium standard of the National Physical Laboratory, Teddington (Essen, Parry, Markowitz and Hall 1958). The solutions are given in lines (5) and (6).

The mean values of the coefficients are given in line (7). Line (11) gives the coefficients which were adopted by the Bureau International de l'Heure, in 1955, for use during 1956 to convert from UT1 to UT2. The same coefficients were retained by the B.I.H. for use during 1957 and 1958. It is evident that the choice of coefficients, made by Stoyko, was a good one.

The seasonal variation as determined with the P.Z.T.'s is seen to be remarkably consistent from year to year with a mean value of 30 milliseconds for A. It appears doubtful that there was a significant change in the value of A about the year 1950, but this possibility cannot be dismissed. It would be interesting to know whether formulas for drift that are valid for two years or more can be found for clocks in use previous to 1950. If such formulas could be found we might be able to obtain definitive results for the earlier years.

4. *Causes of seasonal variation.* Part of the seasonal variation is due to tides in the crust of the earth which are induced by the sun. The

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change in the figure of the earth causes a change in the moment of inertia. Since the angular momentum is constant the speed of rotation changes. Line (8) of Table I gives the theoretical corrections for the solar tides Sa and Ssa. Line (9) gives the observed ΔSV when the tidal terms are removed, that is, the ΔSV which is not accounted for by gravitational theory.

The best explanation of the non-gravitational annual portion appears to be that it is due to winds. This explanation was first offered by Munk and Miller (1950) in 1950, who obtained a value of 48 milliseconds for A. In 1951, Mintz and Munk (1951), using new data, reduced the value to 15 milliseconds. Later, in 1953, they obtained a slightly larger value of 20 milliseconds, which is not far from the observed value of 30 milliseconds derived here (1954). The coefficients of the 1953 solution are given in line (10). In the same year, Pariyski and Berlyand (1953) also obtained values for A similar to that now being obtained by observation.

The observed non-gravitational semiannual variation has an amplitude of 9 milliseconds. The amplitude of the semiannual effect attributed to winds is only 1 millisecond so that at present there is no adequate explanation of the semiannual variation. One might offer a number of explanations for an annual variation but an unexplained semiannual variation appears to be a more difficult matter for which to account.

5. *Lunar tidal variations*. Tides induced in the crust of the earth by the moon cause variations in speed of rotation in the same manner as the sun does. The principal short period terms are denoted Mf and Mm. The Mf term, due to the varying declination of the moon, has a period of 13.66 days. The Mm term, due to the varying distance of the moon from the earth, has a period of 27.55 days.

The effects of the lunar and solar tidal terms on the variation in speed of rotation were derived by H. Jeffreys in 1928, by F. Andersson in 1937, and by Mintz and Munk in 1953. The developments were not carried out in detail because it was considered that the effects were not capable of being observed.

After deriving the seasonal variation for the period 1951 to 1954 from P.Z.T. observations it appeared that the lunar terms might be detected. The coefficients obtained by observation agreed with those computed from theory within the errors of observation. It was evident, however, that additional theoretical terms were needed. The effective values of the coefficients for any lunation depend on the maximum and minimum declinations of the moon during the lunation.

A new calculation of the solar and lunar tidal effects was made in 1956 by Woolard (1959). The principal terms are as follows:

	Tide	Period
$\Delta t = k[515.0 \sin \Omega$	—	18.6 years
$+2.47 \sin 2\mathrm{L}$	Mf	13.66 days
$+1.02 \sin (2\mathrm{L}-\Omega)$		
$+2.63 \sin g$	Mm	27.55 days
$-0.17 \sin (g+\Omega)$		
$-0.17 \sin (g-\Omega)$		
$+15.29 \sin 2\odot$	Ssa	0.5 year
$+\ 4.88 \sin g']$,	Sa	1.0 year

where k is the Love number, Ω is ascending node of moon, L is mean longitude of moon, g is mean anomaly of moon, $\odot$ is mean longitude of sun, g' is mean anomaly of sun.

Δt is the correction in milliseconds to be added to Universal Time determined on any night in order to remove the lunar and solar tidal effects. It should be noted that the lunar terms are largely removed by a smoothing process when determining the adopted UT2 ordinarily.

The following process was used to determine the lunar terms. Each night of observation gives a correction to a clock, E-43 for example, on the system UT2. The correction of the clock due to drift, ψ_{43}, is subtracted and the residuals are analyzed for terms with arguments 2L and g, respectively.

The coefficients, O, derived from the P.Z.T. observations are given in Table II. The values for the first 3 years differ only slightly from those that were obtained by a different process in 1955.

The computed coefficients, C, were obtained by assigning to k the rounded value 0.300, and to Ω the value for the middle of the interval analyzed.

From the differences $(O - C)$ an external probable error of ± 0.43 millisecond is obtained for an observed coefficient.

The value of k is uncertain. Dr. P. Melchior has pointed out that the value obtained from earth tide experiments is 0.19 ± 0.06 whereas the value obtained from the Chandlerian motion of the pole is 0.28 ± 0.03. There is thus a discordance between the terrestrial and astronomical determinations. Dr. Melchior noted that when enough data have been accumulated we may use the lunar tidal terms to obtain an independent value of k. It is interesting to see what

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TABLE II. COEFFICIENTS OF LUNAR TIDAL TERMS
Unit = 0.001

	(1) sin 2L		(2) cos 2L		(3) sin g		(4) cos g	
	O	C	O	C	O	C	O	C
1951.7–52.7	+1.0	+1.0	−0.2	+0.2	+0.4	+0.7	−1.0	0
1952.7–53.7	+1.5	+0.9	+0.5	+0.2	+0.9	+0.7	+0.6	0
1953.7–54.7	+0.7	+0.8	+1.3	+0.3	+0.5	+0.8	−0.2	0
1955	+0.5	+0.7	0.0	+0.3	+1.1	+0.8	+0.4	0
1956	+0.3	+0.6	+0.3	+0.3	+2.7	+0.8	+0.5	0
1957	−0.4	+0.5	+1.0	+0.2	+1.6	+0.9	+0.1	0

value we can obtain with the material at hand. We may obtain k from the relation $k = 0.300\ O/C$. The weight of each determination is C^2.

The solutions are:

		k	Wt.
From	sin 2L term,	0.27±0.07,	3.6
	cos 2L term,	0.57±0.21	0.4
Combined,	2L terms,	0.30±0.07	4.0
From	g term,	0.47+0.07	3.7
From	all terms	0.38±0.05	7.7

p.e. of unit weight = ± 0.13

The values of k obtained here are in reasonable agreement with the value derived from the Chandlerian motion. The value obtained from the Mf tide agrees closely, although the Mm value disagrees by about twice the probable error. It is too early, however, to say whether there is a discordance. More data must be obtained, and the effects of water tides on the direction of the vertical must be evaluated.

From a study of the coefficients it is found that the observed monthly tide is in phase with the tide predicted by theory, with a probable error of ±1 day. The observed fortnightly tide, however, appears to occur 0.5 day in advance of the theoretical, with a probable error of ±0.5 day. We may conclude that within the errors of observation the observed lunar tides in the crust of the earth are in phase with the theoretical.

6. *Irregular variation.* The existence of irregular variations in the speed of rotation of the earth was found, in the past, chiefly from a comparison of the orbital motion of the moon with the rotation of the earth. It was not feasible, however, to detect changes in speed that occurred within a short time in this manner. In fact, it was not even certain how these changes took place. It was formerly thought that irregular changes took place abruptly, but in 1952 Brouwer (1952) gave another interpretation of the irregular changes. On his hypothesis we should expect to observe changes in acceleration but not sudden changes in rate. According to Brouwer the observed changes in speed may be the accumulation of small random changes.

With the development of the cesium standard of frequency it becomes possible to determine the nature of the irregular variation in detail and choose between the two interpretations. A joint program for determining the frequency of cesium in terms of the second of Ephemeris Time is being carried out by L. Essen and J. V. L. Parry of the National Physical Laboratory, Teddington, and W. Markowitz and R. G. Hall of the Naval Observatory, Washington (1958). As a preliminary step, the frequency of cesium in terms of UT2 was determined for each month from June 1955 to January 1958. The results indicate that during this interval the speed of rotation changed gradually and not suddenly. From September 1955 to January 1958 the mean speed of rotation of the earth underwent a practically uniform deceleration of 50 parts in 10^{10} per year, equivalent to an increase in the length of the day of 0.43 millisecond per year. There is some indication of a change in acceleration about August or September 1955 but there are not enough data to confirm this.

7. *Moon camera.* Ephemeris Time is defined by the orbital motion of the earth about the sun, but is obtained in practice from the orbital motion of the moon about the earth. Since 1952 the U. S. Naval Observatory has been using the dual-rate moon position camera to determine Ephemeris Time (Markowitz 1954, in press). The camera tracks the moon and surrounding stars simultaneously at their respective rates by use of a dark plane-parallel glass filter. The image of the moon, which passes through this filter, is shifted in position by tilting the filter during the exposure. It is thus possible to hold the moon fixed with relation to the stars. The epoch of observation is the instant when the filter is parallel to a light colored glass plate which is in front of the star field. Exposures are for 10 or 20 seconds.

The plates are measured in a machine which has x and y-screws and an accurate rotating

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stage. A stroke of a ratchet lever rotates the stage 6°. The stars are first measured in x and y. The radii of the moon are then measured every 6° along the bright limb, using only the y-screw. The plate is rotated 180° from its original position and the stars are measured again. It is now possible to determine the positions of the stars and of the center of circle of best fit of the moon, with respect to the center of rotation of the stage in x and in y. This determines the apparent position of the moon with respect to the stars.

Corrections for parallax are applied to obtain the geocentric right ascension and declination of the moon. The *Improved Lunar Ephemeris, 1952–1959*, is entered with these quantities and the Ephemeris Time is taken out. The Universal Time of the epoch of observation is known, so that we obtain ΔT = ET − UT.

The reduction of the moon observation has been programmed for the electronic computer by my colleague, Dr. R. G. Hall. The reduction of all plates taken since the camera was placed in operation in June 1952 has been completed. The star places used are those taken from the *Yale Zone Catalogues*. Systematic corrections *N30-Yale*, have been determined, but have not yet been applied. A provisional correction of $+0^s18$ was applied to the ΔT's for the difference in equinox.

The results obtained thus far show the existence of systematic effects in both the longitude and latitude of the moon, which are probably due to the systematic departures of the moon from a sphere. The period of the libration in longitude of the moon is an anomalistic month, or 27.55 days, and we may expect to find variations in ΔT which are dependent upon the mean anomaly of the moon, g.

The data were divided into periods of 14 lunations, a period which is nearly equal to 15 anomalistic months, and analyzed for corrections of the form $H \sin g + K \cos g$. The solutions are given in Table III. It is probable that the change in the coefficients from one period to the next is due to the libration in latitude, whose period is a nodical month, or 27.21 days. A more detailed analysis will be made later, after corrections for irregularity of the limb being derived by Dr. C. B. Watts have been applied.

TABLE III. CORRECTIONS FOR TERMS IN g

Lunations	Interval	H	K
364–381	1952.5–1953.8	$+1^s17$	-0^s27
382–395	1953.8 1955.0	−0.08	+0.07
396–409	1955.0 1956.1	+0.47	−0.06
410–423	1956.1 1957.2	+0.29	−0.32
424–437	1957.2 1958.4	−0.23	−0.87

TABLE IV. ΔT FROM MOON CAMERA

Epoch	Nights	ΔT_O	ΔT_C
1952.75	17	29^s66	29^s58
1953.25	16	30.99	30.72
.75	39	30.56	30.32
1954.25	26	30.22	30.10
.75	24	29.67	29.74
1955.25	23	30.00	30.22
.75	19	30.24	30.14
1956.25	23	30.40	30.56
.75	19	30.26	30.23
1957.25	23	30.69	30.50
.75	20	31.20	31.37
1958.25	22	31.05	30.96

Table IV gives the semiannual means of ΔT. ΔT_O gives the values initially obtained and ΔT_C gives the values when corrected by the solutions given in Table III.

The values of ΔT_C are plotted in Figure 2, which also shows other solutions for ΔT. The solution by Brouwer (1952), ΔT_B, is a smoothed one, based on transit circle observations and occultations. The ones marked $\Delta T_6''$ are recent values obtained with 6-inch Transit Circle of the Naval Observatory. The latest results are in press. The ones marked ΔT_{oc} are from recent occultation results.

The following corrections were applied to place the ΔT's on a system defined by the equinox of N30; ΔT_B (on Newcomb), -1^s09; $\Delta T_6''$ and ΔT_{oc} (on FK3), $+0^s27$.

The curve marked "ΔA" gives ΔA = AT − UT, where AT denotes atomic time. This curve was obtained by integrating the frequency of cesium in terms of Universal Time, previously reported (Essen, Parry, Markowitz and Hall 1958). The two constants of integration, which correspond to the value of AT at an initial epoch and the frequency of cesium in cycles per second of Ephemeris Time, were selected so as to pro-

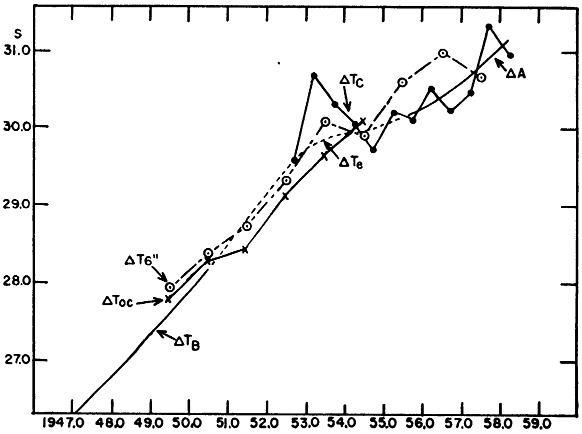

Figure 2. Comparisons of UT with Ephemeris and Atomic Time (AT). ΔT = ET − UT and ΔA = AT − UT For symbols, refer to text.

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vide the best fit with the ΔT_C's from the moon camera for the interval June 1955 to June 1958.

A solution of the means of ΔT_C from 1954.25 to 1958.25 gives a value of $+0^{s}.12$ per year^2 for $\Delta T''$. The cesium standard gives $+0^{s}.16$ per year^2. The agreement is satisfactory.

The table below gives the values of ΔT and its derivatives from the solution of Brouwer for 1941.5 to 1950.5, and from the solution based on the moon camera and cesium standard for 1955.5 to 1958.0.

	ΔT	$\Delta T'$	$\Delta T''$
1941.5	23.62	$+0^{s}.43$/year	$+0^{s}.02$/year^2
1950.5	28.20	+0.59	+0.02
1955.5	30.20	+0.20	+0.16
1958.0	31.10	+0.58	+0.16

These values indicate that $\Delta T''$ was negative at some time between 1950.5 and 1955.5 and that two changes of sign occurred. Since it was possible to represent the running of clocks R2 and R3 from 1951.7 to 1954.7 by formulas (1) no very drastic changes in acceleration could have occurred in that interval. The dotted curve in Figure 2, marked ΔT_e, represents an estimate of the variation in ΔT from 1950.5 to 1955.5, based on all the data now available. A more definitive variation will be obtained after corrections for the profile of the moon have been applied.

8. *Variations, 1955 to 1958.* Since September 1956 a cesium standard, No. A6, of the Naval Research Laboratory, Washington, has also been used for comparison with UT2 for studying the variations in speed of rotation. This standard has a quartz-crystal clock associated with it, whose frequency is kept matched to that of the standard. The cesium beam is not operated continuously, but the combination serves as an atomic clock. Daily comparisons of astronomical time from the P.Z.T.'s versus atomic time are made with the Naval Research Laboratory whereas only monthly comparisons of frequency are made with Teddington.

The value of the frequency of cesium obtained by comparing the cesium standard at the National Physical Laboratory with the observations of the moon made at the Naval Observatory (Markowitz, Hall, Essen and Parry 1958) is

$$\nu_E = 9\,192\,631\,770 \text{ cps (of ET)}.$$

Universal Time does not enter in the determination of ν_E, since it cancels out in forming the difference

$$\Delta T - \Delta A = (\text{ET} - \text{UT}) - (\text{AT} - \text{UT}).$$

We may therefore use the above value of ν_E to express the length of the day.

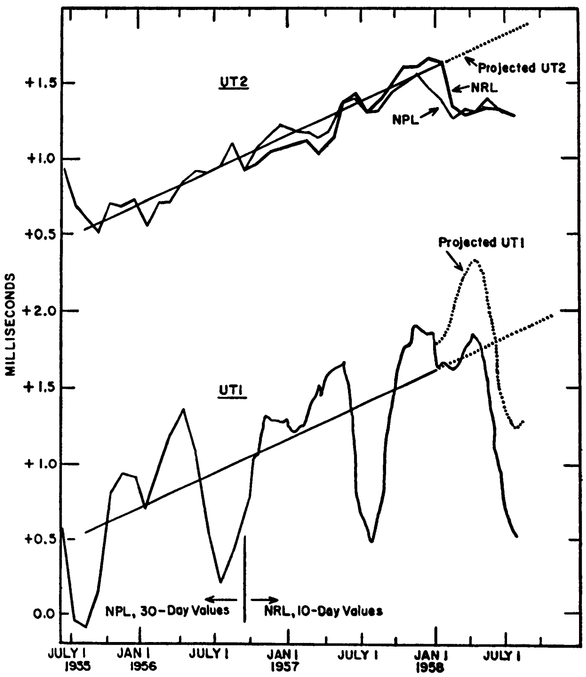

Figure 3. Variation in length of day, derived from P.Z.T.'s and cesium standards. Ordinate is excess over 86400 seconds of Ephemeris Time in length of day.

Figure 3 shows the excess over 86400 seconds of ET, obtained by comparing UT1 and UT2, as determined with the P.Z.T.'s, with atomic time based on cesium standards located at the National Physical Laboratory, Teddington and the Naval Research Laboratory, Washington.

The curve marked UT2 gives the mean length of the day with periodic terms removed, that is, the irregular variation. The curve marked UT1 includes the seasonal and irregular variations, but not the lunar-tidal terms.

A rapid decrease of about $0^{s}.0003$ in the length of the day about the beginning of 1958 was indicated by both the Washington and Richmond P.Z.T.'s. There is a possibility, however, that what occurred was only a change in acceleration about this time.

It is a pleasure to acknowledge the help of many members of the staff in carrying out these studies, in particular, Dr. R. G. Hall. Mr. G. C. Whittaker, Director of the Richmond station, maintained a continual flow of results. Others who participated in making the observations and reductions include Miss Ida Ray, Miss Martha Sherman, Mrs. Doris Stanley, Mr. J. Siegel, Mr. W. Weston, Miss Suzanne Ellis, and Miss Norma Grimes at Washington, and Mr. D. Monger, Mr. R. Medford, and Mr. J. Rende at Richmond.

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EPHEMERIS TIME

By G. M. CLEMENCE
Columbia University, New York, N. Y., and U. S. Naval Observatory, Washington, D. C.

Abstract. In this article is described the measurement of time from a more general point of view than has been taken before. It is shown that mean solar time and ephemeris time are two particular applications of the general principle of measuring time by observing angular motions, a principle that can be applied to any angular motion whatever, provided that an adequate theory of the motion is available.

Space and time may be measured in a fashion that makes the errors of measurement independent. That is, a measure of distance and a measure of time may be defined in such a way that an error in a measurement of distance will not produce any error in the measurement of time, and vice versa. It is true that the measurements must be restricted to the local frame of reference. Special relativity teaches us that the errors are no longer strictly independent if two observers in relative motion exchange and combine the results of their observations. But this restriction does not destroy the principle of independence, it means only that one observer must have a care in interpreting the observations of another.

It is easily possible to define the fundamental units of measurement in such a way that the errors in measurements of time and distance are not independent. For example, the fundamental unit of length might be taken as the meter and the fundamental unit of velocity as the velocity of light. The unit of time would then be a derived one, obtained as the quotient of distance by velocity, and then any error in measurements of length would be carried over into practical determinations of time. This procedure would not, however, destroy the duality of space and time, which is intrinsic and, for any one observer, absolute.

An alternative choice of fundamental units is to take the meter as unit of length, and the time required for light to travel a distance of one meter as the unit of time. In this case also the errors would not be independent; practical measurements of time-intervals would be affected by the error in the length of the meter-bar used, as well as by the accidental errors of measurement of the time-intervals themselves.

The non-independence of the errors in the two cases mentioned is a result of defining one thing in terms of another. Let us consider a third case of a different sort. Suppose that the wave length of some monochromatic electromagnetic radiation is taken as the unit of length, and the inverse of the corresponding frequency as the unit of time. In this case the errors may be independent or not, according to the techniques of measurement employed. If no reference is made to frequency when measuring lengths, and if no reference is made to wave length when measuring time-intervals, then the errors are independent. On the other hand, the product of a wave length by the corresponding frequency is an absolute constant, the velocity of light. If the value of this constant is known, it then becomes possible to infer lengths from measurements of frequency, and vice versa; and if this technique is employed, the errors will not be independent. With this choice of units it would become necessary to examine the techniques used in every experiment, in order to ascertain whether the errors are independent or not.

Until now, the fundamental units of length and time have always been defined in such a way as to preserve the intrinsic independence of the errors of measurement. In particular, the recent redefinition of the second preserves the independence. It is impossible to infer anything about time from measurements of distance, and vice versa. I remark, however, that it has been a matter of choice rather than necessity. It would have been possible to define the unit of distance as the distance through which a body would fall in a unit of time, under the action of gravity; and if this had been done, the errors of measurement would not have been independent.

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