19

Polarized deep-inelastic processes and the proton 'spin' crisis

We extend the previous unpolarized deep-inelastic scattering analysis to the case of polarized processes in the aim to study the 'spin' content of the proton and, later on (see next chapter), of the photon from γ-γ scattering.[1] Interest on such processes has been stimulated by the EMC collaboration [253] finding that the first moment of the polarized proton structure function g_1^p is unexpectedly suppressed compared with the naïve quark model prediction (OZI [254] violation), which has provoked an extensive discussion (see e.g. [255–261]) of the parton model interpretation of QCD in deep inelastic scattering processes involving the $U(1)$ axial anomaly [262–264]. We shall be concerned here with the parity-violating part of the hadronic tensor defined in Eq. (15.36) where the structure function is defined as:

$$g_1 \equiv \frac{\nu}{M_p^2} W_3 \ . \tag{19.1}$$

Data [16] are shown in Fig. 19.1.

19.1 The case of massless quarks

We shall discuss here the approach based on a composite operator and proper vertex. This discussion can be consulted in the reprinted paper [260] given in Section 19.4 at the end of this chapter.

19.2 Extension of the method to massive quarks

We extend the previous approach to the case of massive quarks [261]. In this paper, a detailed estimate of the slope of the topological susceptibility using the approach of QCD spectral sum rules in the case of massive quarks is given.[2] The result [261]:

$$\sqrt{\chi'(0)}|_{m_q \neq 0} = (33.5 \pm 3.9)\ \text{MeV}\ , \tag{19.2}$$

[1] It is a pleasure to thank Graham Shore for discussions related to this chapter.
[2] A previous estimate of the slope of the topological charge using QSSR in pure Yang–Mills theory has been done in [265] and confirmed later on by lattice calculations [266].

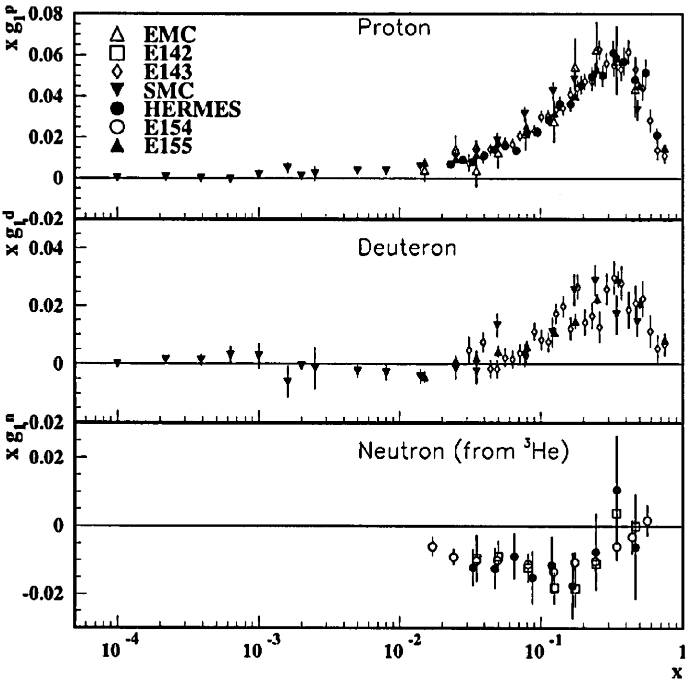

Fig. 19.1. The spin-dependent structure function $xg1$ of proton, deuteron and neutron in DIS of polarized electron/positron versus x and for different Q^2 ranging from 0.01 to 100 GeV^2 (SMC) for proton and deuteron, and from 1 to 17 GeV^2 (E154) for neutron.

compared to the massless quark values of (26.4 ± 4.1) MeV [260] is smaller than the OZI expectation of (43.8 ± 5.0) MeV. As a result, the singlet polarized structure function:

$$a^0 = a^8 \left(\frac{\sqrt{6}}{f_\pi}\right) \sqrt{\chi'(0)|_{Q^2}}\,, \tag{19.3}$$

where:

$$G_A^{(8)} \equiv \frac{1}{2\sqrt{3}} a^8\,, \qquad G_A^{(0)} \equiv a^0\,, \tag{19.4}$$

has the value [261]:

$$\begin{aligned} a^0(Q^2 = 10\ \text{GeV}^2) &= 0.31 \pm 0.02 \\ \Gamma_p^1(Q^2 = 10\ \text{GeV}^2) &= 0.141 \pm 0.005\,, \end{aligned} \tag{19.5}$$

which is about the same as the one obtained in the chiral limit, and confirms the expectation that the result is insensitve to the quark mass values. This result is in agreement with the data, which may also confirm the proposal that the proton spin suppression is a target-independent effect due to the screening of the topological charge of the QCD vacuum.

19.3 Further tests of the universal topological charge screening

The previous proposal can be tested in different processes. This can be done either in semi-inclusive polarized ep scattering (for a review see e.g. [267]) or in the $\gamma\gamma$ polarized process.

19.3.1 Polarized Bjorken sum rule

If the previous proposal requiring an identical suppression of the flavour singlet component for the proton and neutron is correct, one expects that the Bjorken sum rule:

$$\begin{aligned}\delta\Gamma_1^{p-n} &\equiv \Gamma_1^p - \Gamma_1^n \equiv \int_0^1 dx\big[g_1^p(x;Q^2) - g_1^n(x;Q^2)\big] \\ &= \frac{1}{6}g_A\big(1 - a_s - 3.583a_s^2 - 20.215a_s^3\big) + \frac{a_p - a_n}{Q^2}\,, \end{aligned} \tag{19.6}$$

should hold. The $1/Q^2$ higher twist term can be neglected at higher Q^2. Using the experimental value of g_A, one may extract the value of α_s. Instead, we use the previous sum rule and that of the nucleon [260]:

$$\delta\Gamma_1^{p-n}(2\ \mathrm{GeV}^2) \simeq -(0.203 \pm 0.029)\,, \qquad \Gamma_1^n(Q^2 = 2\ \mathrm{GeV}^2) \simeq -(0.022 \pm 0.011)\,, \tag{19.7}$$

from which one can deduce the higher twist terms in units of GeV^2:

$$a_p \simeq -0.117 \pm 0.145\,, \qquad a_n \simeq -0.018 \pm 0.025\,, \tag{19.8}$$

which, although consistent with zero are neverthless interesting.

19.3.2 Semi-inclusive polarized ep scattering

An alternative test of the previous proposal is to perform a DIS experiment on a target other than the nucleon. This can be done by studying a semi-inclusive process in which a single hadron carrying a large target energy fraction is detected in the target fragmentation region. This is shown in the Fig. 19.2.

In terms of the *fracture function* [268] $M_i^{h/N}(x, z, t, Q^2)$ which represents the joint probability distribution for producing a parton i with momentum fraction x and a detected hadron h carrying an energy fraction $z = p_2' \cdot q/p_2 \cdot q$ from a nucleon N (t is the invariant momentum transfer), the lowest order polarized cross-section reads [267]:

$$\frac{d\Delta\sigma^{\mathrm{target}}}{dx\,dQ^2dz\,dt} = \frac{4\pi\alpha^2 y(2-y)}{Q^4}\Delta M_1^{h/N}(x, z, t, Q^2) \tag{19.9}$$

where:

$$\Delta M_1^{h/N} = \sum_i \frac{Q_i^2}{2}\Delta M_i^{h/N}\,, \tag{19.10}$$

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

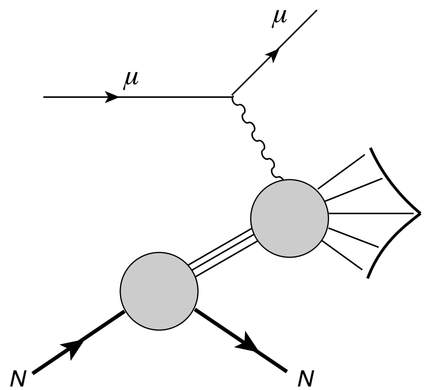

Fig. 19.2. Semi-inclusive process.

is equivalent to the inclusive structure function g_1^N, and where Q_i is the charge of the quark i in units of e. For large $z \to 1$, the fracture function can be modelled by, for example, a single region exchange and reads:

$$\Delta M_1^{h/N} \simeq F(t)(1-z)^{-2\alpha_{\mathcal{R}}(t)} g_1^{\mathcal{R}} \left(\frac{x}{1-z}, t, Q^2 \right) , \tag{19.11}$$

where $g_1^{\mathcal{R}}$ is the structure function of the exchanged region $\mathcal{R}$ with trajectory $\alpha_{\mathcal{R}}(t)$. Independently on the detailed model of the fracture functions, one can predict the ratio of the moments:

$$\begin{aligned} &\frac{\mathcal{M}_1(ep \to e\pi^- X)}{\mathcal{M}_1(en \to e\pi^+ X)} \simeq \frac{2s+2}{2s-1} \simeq \frac{\mathcal{M}_1(ep \to eD^- X)}{\mathcal{M}_1(en \to eD^0 X)} , \\ &\frac{\mathcal{M}_1(ep \to eK^0 X)}{\mathcal{M}_1(en \to eK^+ X)} \simeq \frac{2s+1}{2s-1} , \end{aligned} \tag{19.12}$$

where:

$$\mathcal{M}_1 = \int_0^{1-z} dx \Delta M_1^{h/N} \cdot (x, z, t, Q^2) , \tag{19.13}$$

and:

$$s(Q^2) = \left(\frac{C_1^S}{C_1^{NS}} \right) \left(\frac{\sqrt{2n_f}}{f_\pi} \right) \sqrt{\chi'(0)} . \tag{19.14}$$

C_1^S and C_1^{NS} are ratio fo the singlet and non-singlet Wilson coefficients. In the OZI limit, $s = 1$, such that one expects a large deviation from the previous value of $\chi'(0)$. In the small limit $z \to 0$, the previous ratios reduce to the first moment ratio g_1^p/g_1^n. In the whole range of z, one expects a deviation of about a factor 2.5 from the OZI prediction.

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

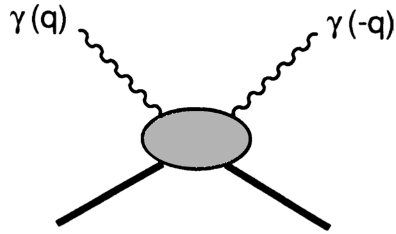

Fig. 1. The two-current matrix element $\langle \mathrm{N}|J_\mu(q)J_\nu(-q)|\mathrm{N}\rangle$.

19.4 Reprinted paper

Target independence of the EMC–SMC effect

S. Narison, G. M. Snore and G. Veneziano

Reprinted from *Nuclear Physics B*, Volume B433, pp. 209–233, Copyright (1995) with permission from Elsevier Science.

1. Introduction

The discovery by the EMC Collaboration [1] (see also Ref. [2]) of an unexpected suppression of the first moment of the polarised proton structure function g_1^{p} has provoked an extensive discussion of the parton model interpretation of QCD in deep inelastic scattering processes involving the axial U(1) anomaly. (For reviews, see Refs. [3,4].) While it has so far proved possible with careful redefinitions and interpretations [5] to preserve the essence of the parton model description, it is becoming clear that these processes involve subtle field theoretic properties of QCD which lead beyond both the original and QCD-improved parton approximation. In this paper, we develop an alternative approach to deep inelastic scattering emphasising field theoretic concepts such as the operator product expansion (OPE), composite operator Green functions and proper vertices. This clarifies some of the difficulties encountered in the parton description and gives a new insight into the underlying reason for the EMC result. In particular, our analysis strongly suggests that the observed suppression of the first moment of g_1^{p} is a generic QCD effect related to the anomaly and is actually *independent* of the target. Rather than revealing a special property of the proton structure, the EMC result reflects an anomalously small value of the first moment of the QCD topological susceptibility [6,7].

The essential features of this method are easily described for a general deep inelastic scattering process. The hadronic part of the scattering amplitude is given by the imaginary part of the two-current matrix element $\langle N \mid J_\mu(q)J_\nu(-q) \mid N\rangle$ illustrated in Fig. 1, where J_μ is the current coupling to the exchanged hard proton (or electroweak vector boson) and $\mid N\rangle$ denotes the target. The OPE is used to expand the large Q^2 limit of the product of currents as a sum of Wilson coefficients $C_i(Q^2)$ times renormalised composite operators $\mathcal{O}_i$ as follows (suppressing Lorentz indices):

$$J(q)J(-q) \underset{Q^2\to\infty}{\sim} \sum_i C_i(Q^2)\mathcal{O}_i(0). \tag{1.1}$$

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

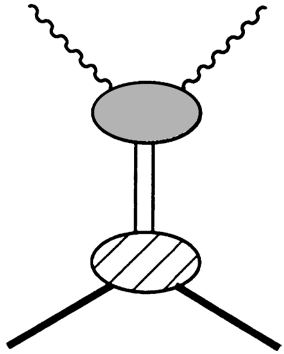

Fig. 2. Decomposition of the matrix element into a composite operator propagator (denoted by the double line) and a proper vertex (hatched).

The dominant contributions to the amplitude arise from the operators $\mathcal{O}_i$ of lowest twist. Within this set of lowest twist operators, those of spin n contribute to the nth moment of the structure functions, i.e.

$$\int_0^1 \mathrm{d}x\, x^{n-1} F(x, Q^2) = \sum_i C_i^n(Q^2)\langle N \mid \mathcal{O}_i^n(0) \mid N\rangle. \tag{1.2}$$

The Wilson coefficients are calculable in QCD perturbation theory, so the problem reduces to evaluating the target matrix elements of the corresponding operators. We now introduce appropriately defined proper vertices $\Gamma_{\tilde{\mathcal{O}}NN}$, which are chosen to be 1PI with respect to a physically motivated basis set $\tilde{\mathcal{O}}_k$ of renormalised composite operators. The matrix elements are then decomposed into products of these vertices with zero-momentum composite operator propagators as follows:

$$\langle N \mid \mathcal{O}_i(0) \mid N\rangle = \sum_k \langle 0 \mid \mathcal{O}_i(0)\tilde{\mathcal{O}}_k(0) \mid (0)\rangle \Gamma_{\tilde{\mathcal{O}}_k NN}. \tag{1.3}$$

This is illustrated in Fig. 2. In essence, what we have done is to split the whole amplitude into the product of a "hot" (high momentum) part described by QCD perturbation theory, a "cold" part described by a (non-perturbative) composite operator propagator and finally a target-dependent proper vertex.

All the target dependence is contained in the vertex function $\Gamma_{\tilde{\mathcal{O}}NN}$. However, these are not unique – they depend on the choice of the basis $\tilde{\mathcal{O}}_k$ of composite operators. This choice is made on physical grounds based on the relevant degrees of freedom, the aim being to parametrise the amplitude in terms of a minimal, but sufficient, set of vertex functions. A good choice can often lead to an almost direct correspondence between the proper vertices and physical couplings such as, e.g., the pion–nucleon coupling $g_{\pi\mathrm{NN}}$. In particular, it will be wise to use, whenever possible, RG-invariant proper vertices.

Despite being non-perturbative, we can frequently evaluate the composite operator Green functions using a combination of exact Ward identities and dynamical approximations (see Sections 2 and 3). On the other hand, because of the target dependence, the proper vertices are not readily calculable from first principles in QCD, so we are in general left

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

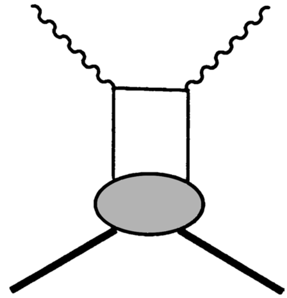

Fig. 3. The original parton model representation of the scattering amplitude.

with a parametrisation of the amplitude in terms of a (hopefully small) set of unknown vertices. These play the rôle of the non-perturbative (i.e. primordial or not-yet-evolved) parton distributions in the usual treatment. Just as for parton distributions, many different QCD processes can be related through parametrisation with the same set of vertex functions.

Now compare this approach with the parton model. In the original parton model, the amplitude is approximated by Fig. 3, describing the scattering of a large Q^2 photon with a parton in the target nucleon. This picture is already sufficient to reveal Bjorken scaling. It may be improved in the context of QCD by including gluonic corrections, exactly as in the OPE, as shown in Fig. 4. These give the logarithmic scaling violations characteristic of perturbative QCD. The total amplitude is therefore factorised into a perturbative scattering amplitude for the hard photon with a parton (quark or gluon) and a parton distribution function giving the probability of finding a particular parton with given fraction x of the target momentum.

The question of whether the full QCD amplitude can be given a natural parton interpretation depends on the composite operators $\mathcal{O}_i$ in the Wilson expansion. For example, if the lowest twist operator for a given process is multilinear in the elementary quark and gluon fields rather than simply quadratic then the diagram of Fig. 4 is not appropriate and the process can only be described in terms of multi-parton distributions [8]. A more subtle problem arises when the operators $\mathcal{O}_i$ are non-trivially renormalised and mix with other composite operators under renormalisation. In this case, the parton interpretation is

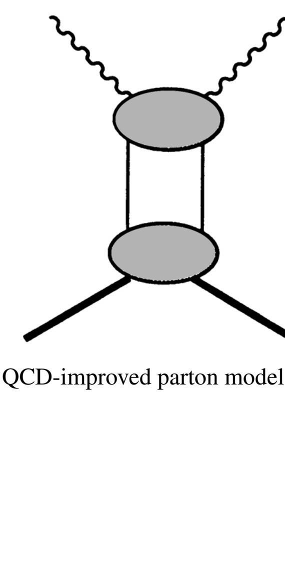

Fig. 4. The QCD-improved parton model representation.

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

preserved by *defining* parton distributions directly in terms of the operator matrix elements (see, e.g., Ref. [8]). This procedure becomes especially delicate [5] in the case of polarised deep inelastic scattering because of the special renormalisation properties of the relevant Wilson operator $J^0_{\mu 5\mathrm{R}}$ due to the axial U(1) anomaly.

In this paper, rather than attempt to interpret the amplitude for polarised deep inelastic scattering in terms of specially defined polarised quark and gluon distributions, we instead focus the analysis on the composite operator level. By splitting the matrix elements in the form of Eq. (1.3), we can exploit chiral Ward identities and the renormalisation group to separate out generic features of QCD manifested in the composite operator propagator from specific properties of the target. In the next section, we see how this clarifies the origin of the suppression of the first moment of g_1^{p} observed in polarised μp scattering.

2. The first moment sum rule for g_1^{p}

Our starting point is the familiar Ellis–Jaffe [9] sum rule for the first moment of the polarised proton structure function g_1^{p}. For $N_{\mathrm{F}} = 3$ and in the $\overline{\mathrm{MS}}$ scheme [10], this reads[1]

$$\begin{aligned}\Gamma_1^{\mathrm{p}}(Q^2) &\equiv \int_0^1 \mathrm{d}x\, g_1^{\mathrm{p}}(x;Q^2) \\ &= \frac{1}{6}\left\{\left(G_{\mathrm{A}}^{(3)}(0) + \frac{1}{\sqrt{3}}G_{\mathrm{A}}^{(8)}(0)\right)\left[1 - \frac{\alpha_{\mathrm{s}}}{\pi} - 3.583\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^2 - 20.215\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^3\right]\right. \\ &\quad \left. + \frac{2}{3}G_{\mathrm{A}}^{(0)}\big(0;Q^2\big)\left[1 - \frac{1}{3}\frac{\alpha_s}{\pi} - 0.550\left(\frac{\alpha_s}{\pi}\right)^2\right]\right\}, \end{aligned} \tag{2.1}$$

where the $G_{\mathrm{A}}^{(a)}$ are form factors in the proton matrix elements of the axial current

$$\big\langle P \mid J^a_{\mu 5\mathrm{R}}(k) \mid P\big\rangle = G_{\mathrm{A}}^{(a)}(k^2)\bar{u}\gamma_\mu\gamma_5 u + G_{\mathrm{P}}^{(a)}(k^2)k_\mu \bar{u}\gamma_5 u, \tag{2.2}$$

and a is an SU(3) flavour index. In our normalisations (see Ref. [7])

$$\begin{aligned} G_{\mathrm{A}}^{(3)} &= \tfrac{1}{2}(\Delta u - \Delta d), \\ G_{\mathrm{A}}^{(8)} &= \frac{1}{2\sqrt{3}}(\Delta u + \Delta d - 2\Delta s), \\ G_{\mathrm{A}}^{(0)} &= \Delta u + \Delta d + \Delta s \equiv \Delta\Sigma. \end{aligned} \tag{2.3}$$

We ignore heavy quarks and, for simplicity, set the light quark masses to zero in the formulae below.

The axial current occurs here since it is the lowest twist, lowest spin, odd-parity operator in the OPE of two electromagnetic currents, i.e.

$$J_\mu(q)J_\nu(-q) \underset{Q^2\to\infty}{\sim} 2\sum_{a=0,3,8} \epsilon_{\mu\nu\alpha}{}^{\beta}\frac{q^\alpha}{Q^2}C^a(Q^2)J^a_{\beta 5\mathrm{R}} + \dots. \tag{2.4}$$

[1] We use the NLO and NNLO coefficients given in Ref. [11]. However, due to our definition (2.5) of the renormalised composite operators, the radiative corrections of the singlet are different from the corresponding terms in Ref. [11], which uses a different renormalisation of the singlet operators.

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

The suffix R emphasises that the current is the *renormalised* composite operator. Under renormalisation, the gluon topological density Q_{R} and the divergence of the flavour singlet axial current $J^0_{\mu 5\mathrm{R}}$ mix as follows [12]:

$$J^0_{\mu 5\mathrm{R}} = Z J^0_{\mu 5\mathrm{B}},$$
$$Q_{\mathrm{R}} = Q_{\mathrm{B}} - \frac{1}{2N_{\mathrm{F}}}(1 - Z)\partial^\mu J^0_{\mu 5\mathrm{B}}, \tag{2.5}$$

where $J^0_{\mu 5\mathrm{B}} = \sum \bar{q}\gamma_\mu\gamma_5 q$ and $Q_{\mathrm{B}} = (\alpha_{\mathrm{s}}/8\pi)\mathrm{tr}(G^{\mu\nu}\tilde{G}_{\mu\nu})$ and we have quoted the formulae for N_{F} flavours. The mixing is such that the combination occurring in the axial anomaly Ward identities, e.g.

$$\langle 0 \mid \left(\partial^\mu J^0_{\mu 5\mathrm{R}} - 2N_{\mathrm{F}} Q_{\mathrm{R}}\right)\tilde{\mathcal{O}}_k \mid 0\rangle + \langle 0 \mid \delta_5 \tilde{\mathcal{O}}_k \mid 0\rangle = 0, \tag{2.6}$$

is not renormalised.

Since $J^0_{\mu 5\mathrm{R}}$ is renormalised, its matrix elements satisfy renormalisation group equations with an anomalous dimension γ, so that in particular $G^{(0)}_{\mathrm{A}}(0; Q^2)$ depends on the RG scale (which is set to Q^2 in Eq. (2.1)).

As we have emphasised elsewhere, $G^{(0)}_{\mathrm{A}}$ does *not*, as was initially supposed, measure the spin of the quark constituents of the proton. The RG non-invariance of $J^0_{\mu 5\mathrm{R}}$ (a consequence of the anomaly) is itself sufficient to prevent this identification. The interest in the first EMC data [1,2] on polarised μp scattering[2], which allows the following result for $G^{(0)}_{\mathrm{A}}$ to be deduced:

$$G^{(0)}_{\mathrm{A}}(0; Q^2 = 11\,\mathrm{GeV}^2) \equiv \Delta\Sigma = 0.19 \pm 0.17, \tag{2.7}$$

is rather that this value for $G^{(0)}_{\mathrm{A}}$ represents a substantial violation of the OZI rule [13,14], according to which we would expect

$$G^{(0)}_{\mathrm{A}}(0)_{\mathrm{OZI}} = 3F - D \simeq 0.579 \pm 0.021. \tag{2.8}$$

Here, we have used [15,16]

$$F + D \simeq 1.257 \pm 0.008, \qquad F/D \simeq 0.575 \pm 0.016 \tag{2.9}$$

as fitted from hyperon and β-decays. The assumption that the OZI rule is satisfied for $G^{(0)}_{\mathrm{A}}(0)$ is equivalent to the Ellis–Jaffe sum rule prediction for the first moment of g_1^{p}.

It follows immediately from Eq. (2.2) (assuming the absence of a massless pseudoscalar boson in the U(1) channel) that

$$G^{(0)}_{\mathrm{A}}\left(0; Q^2\right)\bar{u}\gamma_5 u = \frac{1}{2M}\langle P \mid \partial^\mu J^0_{\mu 5\mathrm{R}} \mid P\rangle, \tag{2.10}$$

[2] The combined SLAC/EMC data quoted in Ref. [1] gives

$$\Gamma_1^{\mathrm{p}}(Q^2 = 11\,\mathrm{GeV}^2) = 0.126 \pm 0.010 \pm 0.015.$$

The result for $G^{(0)}_{\mathrm{A}}$ in Eq. (2.7) is extracted from the sum rule using the values for F and D given below and the running coupling from tau-decay data [30] (see the remarks after Eq. (3.32)).

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

where M is the proton mass. The anomalous chiral Ward identity then allows $G_{\mathrm{A}}^{(0)}$ to be re-expressed as the forward matrix element of the renormalised gluon topological density Q_{R}, i.e.

$$G_{\mathrm{A}}^{(0)}(0;Q^2)\bar{u}\gamma_5 u = \frac{1}{2M}2N_{\mathrm{F}}\langle P \mid Q_{\mathrm{R}}(0) \mid P\rangle. \tag{2.11}$$

Notice that in terms of bare fields, Q_{R} contains both gluon and quark bilinears. This, together with the explicit factor of α_{s} in the definition of the topological density, is the source of the difficully in giving a natural and unambiguous parton interpretation [5,3,4].

At this point, we apply the method described in the introduction. We choose as the composite operator basis $\tilde{\mathcal{O}}_k$ the set of renormalised flavour singlet pseudoscalar operators, viz. Q_{R} and $\Phi_{5\mathrm{R}}$, where, up to a crucial normalisation factor discussed below, the corresponding bare operator is simply the singlet $i\sum\bar{q}\gamma_5 q$. We then define $\Gamma[Q_{\mathrm{R}},\Phi_{5\mathrm{R}};P,\overline{P}]$ to be the generating functional of proper vertces which are 1PI with respect to these composite fields only. (Here, P and $\overline{P}$ denote interpolating fields for the proton – they play a purely passive rôle in the construction.) Γ is obtained from the QCD generating functional by a Legendre transform with respect to the sources for the composite operators Q_{R} and $\Phi_{5\mathrm{R}}$ only. We may then write (cf. Eq. (1.3))

$$\langle P \mid Q_{\mathrm{R}}(0) \mid P\rangle = \langle 0 \mid Q_{\mathrm{R}}(0)Q_{\mathrm{R}}(0) \mid 0\rangle\Gamma_{Q_{\mathrm{R}}P\overline{P}} + \langle 0 \mid Q_{\mathrm{R}}(0)\Phi_{5\mathrm{R}}(0) \mid 0\rangle\Gamma_{\Phi_{5\mathrm{R}}P\overline{P}}, \tag{2.12}$$

where the propagators are at zero momentum.

The composite operator propagator in the first term in Eq. (2.12) is the zero-momentum limit of an important quantity in QCD known as the topological susceptibility $\chi(k^2)$, viz.

$$\chi(k^2) = \int \mathrm{d}x\, \mathrm{e}^{ik\cdot x} i\langle 0 \mid T^* Q_{\mathrm{R}}(x)Q_{\mathrm{R}}(0) \mid 0\rangle. \tag{2.13}$$

The second term is clearly independent of the normalisation of the renormalised quark bilinear operator $\Phi_{5\mathrm{R}}$. We choose to normalise this operator in such a way that the inverse two-point function $\Gamma_{\Phi_{5\mathrm{R}}\Phi_{5\mathrm{R}}}$, which has to vanish at $k^2=0$, is equal to k^2, the correct normalisation for a free, massless particle. With this normalisation, a straightforward but intricate argument [7] using chiral Ward identities (see Appendix A) shows that the propagator $\langle 0 \mid Q_{\mathrm{R}}\Phi_{5\mathrm{R}} \mid 0\rangle$ at zero momentum is simply the square root of the first moment of the topological susceptibility $\chi(k^2)$. We therefore find

$$\langle P \mid Q_{\mathrm{R}}(0) \mid P\rangle = \chi(0)\Gamma_{Q_{\mathrm{R}}P\overline{P}} + \sqrt{\chi'(0)}\Gamma_{\Phi_{5\mathrm{R}}P\overline{P}}. \tag{2.14}$$

The chiral Ward identities further show that for QCD with massless quarks, $\chi(0)$ actually vanishes. (This is in contrast to pure Yang–Mills theory, where $\chi(0)$ is non-zero and is related to the η'-mass in the large N_{C} resolution of the U(1) problem [17,18].) Only the second term in Eq. (2.14) remains. Remarkably, this means that the matrix element of the renormalised *gluon* density Q_{R} measures the coupling of the proton to the renormalised pseudoscalar *quark* operator $\Phi_{5\mathrm{R}}$. This happens because the composite operator propagator

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

matrix in the pseudoscalar ($Q_{\rm R}$, $\Phi_{5\rm R}$) sector is off-diagonal. We therefore arrive at our basic result [7],

$$G_{\rm A}^{(0)}(0;Q^2)\bar{u}\gamma_5 u = \frac{1}{2M}2N_{\rm F}\sqrt{\chi'(0)}\Gamma_{\Phi_{5\rm R}P\overline{P}}. \tag{2.15}$$

The renormalisation group properties of Eq. (2.15) are central to our argument. With the normalisation of $\Phi_{5\rm R}$ chosen above, it can be shown [7] that the proper vertex $\Gamma_{\Phi_{5\rm R}P\overline{P}}$ is RG invariant and so has no scale dependence. The scale dependence needed to match $G_{\rm A}^{(0)}$ is provided entirely by the topological susceptibility which, as shown in Appendix A, satisfies the RGE

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(\alpha_{\rm s})\alpha_{\rm s}\frac{\partial}{\partial\alpha_{\rm s}} - 2\gamma\right)\chi'(0) = 0. \tag{2.16}$$

The challenge posed by the EMC data is to understand the origin of the OZI violation in $G_{\rm A}^{(0)}$. The OZI approximation applied to the RHS of Eq. (2.15) would require[3] (neglecting flavour SU(3) breaking) $\Gamma_{\Phi_{5\rm R}P\overline{P}} \simeq \sqrt{2}\, g_{\eta_8\rm NN}\bar{u}\gamma_5 u$ while $\sqrt{\chi'(0)} \simeq (1/\sqrt{6})f_\pi$.

Our proposal is that we should expect the source of the OZI violation to lie in RG non-invariant terms, i.e. in $\chi'(0)$. The reasoning is straightforward. In the absence of the U(1) anomaly, the OZI rule would be an exact property of QCD. So the OZI violation is a consequence of the anomaly. But it is the existence of the anomaly that is responsible for the non-conservation and hence non-trivial renormalisation of the axial current $J^0_{\mu 5\rm R}$. We therefore expect to find OZI violations in quantities sensitive to the anomaly, which we identify through their RG dependence on the anomalous dimension γ. This seems reasonable since, if the OZI rule were to be good for such quantities, it would mean approximating a RG non-invariant, scale-dependent quantity by a scale-independent one. If this proposal is correct, we expect $\sqrt{\chi'(0)}$ to be significantly suppressed relative to its OZI approximation of $(1/\sqrt{6})f_\pi$. The proper vertex $\Gamma_{\Phi_{5\rm R}P\overline{P}}$ would behave exactly as expected according to the OZI rule. That is, the Ellis–Jaffe violating suppression of the first moment of $g_1^{\rm p}$ observed by EMC would *not* be a property of the proton at all, but would simply be due to an anomalously small value of the first moment of the QCD topological susceptibility $\chi'(0)$.

In the next section, we attempt to verify this hypothesis by evaluating $\chi'(0)$ using QCD spectral sum rules.

[3] To understand this, we note from Ref. [7] that Eq. (2.15) is equivalently written as one form of the U(1) Goldbergen–Treiman relation, viz.

$$G_{\rm A}^{(0)}(0;Q^2) = F_{\eta\rm OZI}\, g_{\eta_{\rm OZI}\rm NN},$$

where $F_{\eta\rm OZI}$ and $g_{\eta_{\rm OZI}\rm NN}$ are respectively the decay constant and nucleon coupling of a state $|\,\eta_{\rm OZI}\rangle$. $|\,\eta_{\rm OZI}\rangle$ is an *unphysical* state in QCD (i.e. not a mass eigenstate) which in the OZI or large $N_{\rm C}$ limit, in which the anomaly is absent, can be identified as the massless U(1) Goldstone boson. Simple quark counting rules then relate $g_{\eta_{\rm OZI}\rm NN}$ to the η_8–nucleon coupling $g_{\eta_8\rm NN}$. This identification is the origin of our choice of normalisation of $\Phi_{5\rm R}$. In the OZI limit, $\Gamma_{\Phi_{5\rm R}\rm NN}$ becomes the Goldstone boson–nucleon coupling.

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

3. QCD spectral sum rule estimate of $\chi'(0)$

We now present an estimate of $\chi'(0)$ in QCD with massless quarks using the method of QCD spectral sum rules (QSSR) pioneered by Shifman, Vainshtein and Zakharov [19] and reviewed recently in Ref. [20].

The correlation function $\chi(k^2)$ is defined in Eq. (2.13) and its renormalisation group equation is given in Appendix A. Including the inhomogeneous contact term [21], we have

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(\alpha_s)\alpha_s\frac{\partial}{\partial\alpha_s}-2\gamma\right)\chi(k^2)=-\frac{1}{(2N_F)^2}2\beta^{(L)}k^4, \tag{3.1}$$

with the beta function

$$\beta(\alpha_s)\equiv\frac{1}{\alpha_s}\mu\frac{d}{d\mu}\alpha_s=\beta_1\frac{\alpha_s}{\pi}+\beta_2\left(\frac{\alpha_s}{\pi}\right)^2, \tag{3.2}$$

where, for QCD with N_F flavours, $\beta_1=-\frac{1}{2}(11-\frac{2}{3}N_F)$ and $\beta_2=-\frac{1}{4}(51-\frac{19}{3}N_F)$, and the anomalous dimension [12]

$$\gamma\equiv\mu\frac{d}{d\mu}\log Z=-\left(\frac{\alpha_s}{\pi}\right)^2. \tag{3.3}$$

The extra RG function $\beta^{(L)}$ (so called because it appears in the longitudinal part of the Green function of two axial currents) is given by

$$\frac{1}{(2N_F)^2}\beta^{(L)}=-\frac{1}{32\pi^2}\left(\frac{\alpha_s}{\pi}\right)^2\left(1+\frac{29}{4}\frac{\alpha_s}{\pi}\right). \tag{3.4}$$

The RGE is solved in the standard way, giving

$$\begin{aligned}\chi(k^2,\alpha_s;\mu)=\exp\left(-2\int_0^t dt'\,\gamma(\alpha_s(t'))\right)\Bigg[\chi\left(k^2,\alpha_s(t);\mu\,e^t\right)\\ -2\int_0^t dt''\beta^{(L)}(\alpha_s(t''))\exp\left(2\int_0^{t''}dt'\,\gamma(\alpha_s(t'))\right)\Bigg],\end{aligned} \tag{3.5}$$

where $\alpha_s(t)$ is the running coupling.

The perturbative expression for the two-point correlation function in the $\overline{\mathrm{MS}}$ scheme is [22]

$$\chi(k^2)_{\text{P.T.}}\simeq-\left(\frac{\alpha_s}{8\pi}\right)^2\frac{2}{\pi^2}k^4\log\frac{-k^2}{\mu^2}\left[1+\frac{\alpha_s}{\pi}\left(\frac{1}{2}\beta_1\log\frac{-k^2}{\mu^2}+\frac{29}{4}\right)+\dots\right]. \tag{3.6}$$

The non-perturbative contribution from the gluon condensates (coming from the next lowest dimension operators in the OPE) is [23]

$$\chi(k^2)_{\text{N.P.}} \simeq -\frac{\alpha_s}{16\pi^2}\left[\left(1+\tfrac{1}{2}\beta_1\frac{\alpha_s}{\pi}\log\frac{-k^2}{\mu^2}\right)\langle\alpha_s G^2\rangle - 2\frac{\alpha_s}{k^2}\langle gG^3\rangle\right]. \tag{3.7}$$

The RGE has been used to check the consistency of the leading log approximation in the perturbative expression and to fix the radiative correction in the gluon condensate contribution.

For the QSSR analysis of $\chi'(0)$, we use the subtracted dispersion relations

$$\frac{1}{k^2}\left[\chi(k^2)-\chi(0)\right] = \int_0^\infty \frac{dt}{t}\frac{1}{t-k^2-i\epsilon}\frac{1}{\pi}\text{Im}\,\chi(t) \tag{3.8}$$

and

$$\frac{1}{k^4}\left[\chi(k^2)-\chi(0)-k^2\chi'(0)\right] = \int_0^\infty \frac{dt}{t^2}\frac{1}{t-k^2-i\epsilon}\frac{1}{\pi}\text{Im}\,\chi(t). \tag{3.9}$$

Then, taking the inverse Laplace transform [20] of both sides of the dispersion relations and using the fact that $\chi(0)=0$ in massless QCD, we find[4]

$$\begin{aligned}&\int_0^{t_c}\frac{dt}{t}e^{-t\tau}\frac{1}{\pi}\text{Im}\,\chi(t)\\ &\quad\simeq\left(\frac{\overline{\alpha}_s}{8\pi}\right)^2\frac{2}{\pi^2}\tau^{-2}[1-\exp(-t_c\tau)(1+t_c\tau)]\\ &\qquad\times\left[1+\frac{\overline{\alpha}_s}{4\pi}\left(29+4\beta_1(1-\gamma_E)-8\frac{\beta_2}{\beta_1}\log(-\log\tau\Lambda^2)\right)\right]\\ &\qquad+\frac{\overline{\alpha}_s}{8\pi}\left(\frac{1}{2\pi}\langle\alpha_s G^2\rangle+\frac{\overline{\alpha}_s}{\pi}\tau\langle gG^3\rangle\right)\end{aligned} \tag{3.10}$$

and

$$\begin{aligned}\chi'(0)\simeq&\int_0^{t_c}\frac{dt}{t^2}e^{-t\tau}\frac{1}{\pi}\text{Im}\,\chi(t)-\left(\frac{\overline{\alpha}_s}{8\pi}\right)^2\frac{2}{\pi^2}\tau^{-1}[1-\exp(-t_c\tau)]\\ &\times\left[1+\frac{\overline{\alpha}_s}{4\pi}\left(29-4\beta_1\gamma_E-8\frac{\beta_2}{\beta_1}\log(-\log\tau\Lambda^2)\right)\right]\\ &+\frac{\overline{\alpha}_s}{8\pi}\left(\frac{1}{2\pi}\tau\langle\alpha_s G^2\rangle+\frac{\overline{\alpha}_s}{2\pi}\tau^2\langle gG^3\rangle\right).\end{aligned} \tag{3.11}$$

where $\overline{\alpha}_s$ is the running coupling expressed in terms of the QCD scale Λ from the two-loop relation:

$$\frac{\overline{\alpha}_s^{(2)}}{\pi} = \frac{\overline{\alpha}_s}{\pi}\left(1-\frac{\overline{\alpha}_s}{\pi}\frac{\beta_2}{\beta_1}\log(-\log\ \tau\Lambda^2)\right), \tag{3.12}$$

with

$$\frac{\overline{\alpha}_s}{\pi} = \frac{2}{\beta_1\ \log\ \tau\Lambda^2}. \tag{3.13}$$

[4] For the corresponding results in pure Yang–Mills theory, see Refs. [24,25].

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

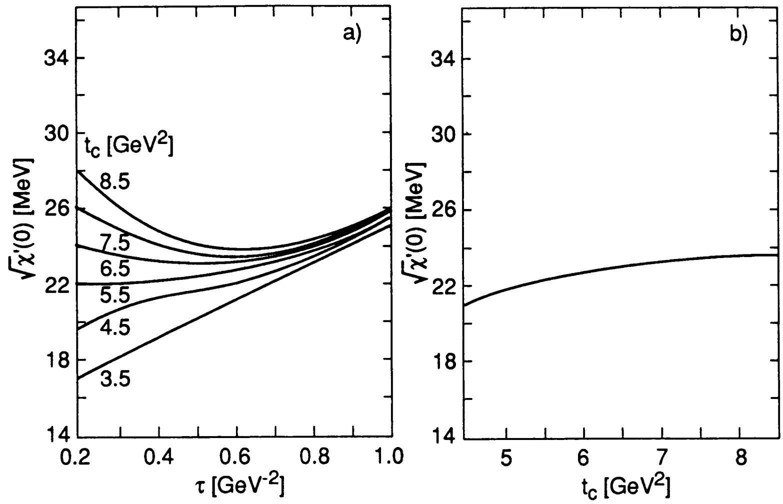

Fig. 5. (a) τ-behaviour of $\sqrt{\chi'(0)}$ for different values of the continuum threshold t_c. (b) Behaviour of different τ-minima versus t_c.

In these expressions, we have cut off the t-integration at some scale t_c and used the perturbation theory approximation to Im $\chi(t)$ for $t > t_c$.

In order to extract a value for $\chi'(0)$ from these sum rules, we keep only the lowest resonance (the η') contribution to the spectral function, i.e. we assume

$$\frac{1}{\pi}\text{Im}\,\chi(t) = 2\tilde{m}^4_{\eta'} f^2_{\eta'} \delta\big(t - \tilde{m}^2_{\eta'}\big) + \text{“QCD continuum”}\ \theta(t - t_c), \tag{3.14}$$

where $\tilde{m}_{\eta'}$ is the mass of the η' extrapolated for massless QCD, viz.

$$\tilde{m}^2_{\eta'} \simeq m^2_{\eta'} - \tfrac{2}{3} m^2_K \simeq (0.87\,\text{GeV})^2. \tag{3.15}$$

To evaluate Eqs. (3.10) and (3.11), we use

$$\Lambda \simeq 350 \pm 100\ \text{MeV} \tag{3.16}$$

for the QCD scale parameter [26],

$$\langle \alpha_s G^2 \rangle \simeq 0.06 \pm 0.02\ \text{GeV}^4 \tag{3.17}$$

from a global fit of the light mesons and charmonium data [20], and parametrise the triple gluon condensate as

$$\langle g^3 G^3 \rangle \simeq 1.5 \pm 0.5\ \text{GeV}^2 \langle \alpha_s G^2 \rangle \tag{3.18}$$

using the dilute gas instanton model [19]. We show the result in Fig. 5a for $\chi'(0)$ plotted versus τ for different values of t_c. In Fig. 5b we show the behaviour of the τ-minima for different t_c. Our optimal result corresponds to the range of values of t_c corresponding to the first appearance of the τ-minimum until the beginning of the t_c stability region. The value

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

of τ at which the stability occurs is around 0.4 to 0.6 GeV^{-2}, which is quite small compared with the light meson systems and is consistent with qualitative expectations [23] of a scale hierarchy in the QSSR analysis of gluonium systems. This small value of τ also ensures that higher dimension operators such as those arising from instanton-like effects will not contribute in the OPE. We deduce

$$\sqrt{\chi'(0)} \simeq 22.3 \pm 3.2 \pm 2.8 \pm 1.3 \text{ MeV}, \tag{3.19}$$

where the first error comes from $\langle \alpha_s G^2 \rangle$, the second one from Λ and the third from the range of t_c-values from 4.5 to 7.5 GeV^2. The effects of the triple gluon condensate and the radiative corrections are relatively unimportant, contributing about (3–10)% to $\chi'(0)$. We add a guessed error of 5% each from the unknown non-perturbative and radiative correction terms. Finally, adding all these errors quadratically, we find the following Laplace sum rule estimate of the first moment of the topological susceptibility evaluated at $\tau \simeq 0.5$ GeV^{-2}:

$$\sqrt{\chi'(0)} \simeq 22.3 \pm 4.8 \text{ MeV}. \tag{3.20}$$

As a check on the validity of this result, we now repeat the analysis using the finite energy sum rule (FESR) local duality version of the spectral sum rules discussed in Ref. [25]. The advantage of the FESR method is that it projects out the effects of the operators of a given dimension [27] (in this case, dimension 4) in such a way that, at the order to which we are working, the FESR analogues of the sum rules (3.10) and (3.11) are not affected by higher dimension operators such as those induced by instanton-like effects.

The FESR sum rules are

$$\begin{aligned}
&\int_0^{t_c} \frac{dt}{t} \frac{1}{\pi} \text{Im}\, \chi(t) \\
&\quad \simeq \left(\frac{\overline{\alpha}_s}{8\pi}\right)^2 \frac{2}{\pi^2} \frac{t_c^2}{2} \left[1 + \frac{\overline{\alpha}_s}{4\pi}\left(29 - 4\frac{\beta_1}{2} - 8\frac{\beta_2}{\beta_1} \log(-\log\ \tau\Lambda^2)\right)\right] \\
&\qquad + \frac{\overline{\alpha}_s}{8\pi}\left[\frac{1}{2\pi}\langle \alpha_s G^2 \rangle\right]
\end{aligned} \tag{3.21}$$

and

$$\begin{aligned}
\chi'(0) \simeq &\int_0^{t_c} \frac{dt}{t^2} \frac{1}{\pi} \text{Im}\, \chi(t) \\
&- \left(\frac{\overline{\alpha}_s}{8\pi}\right)^2 \frac{2}{\pi^2} t_c \left[1 + \frac{\overline{\alpha}_s}{4\pi}\left(29 - 4\beta_1 - 8\frac{\beta_2}{\beta_1} \log(-\log\ \tau\Lambda^2)\right)\right].
\end{aligned} \tag{3.22}$$

Analysing Eqs. (3.21) and (3.22), we realise that the solution increases monotonically with t_c so that no firm prediction can be made, although the result gives a rough indication of consistency with the previous Laplace sum rule. To overcome this problem, we repeat the analysis using only the FESR (3.22) and using as an extra input the value of the parameter $f_{\eta'}$ extracted from the first Laplace sum rule (3.10). The value of $f_{\eta'}$ is given in Appendix B. This weakens the t_c-dependence of the result and t_c-stability now appears as an inflection

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

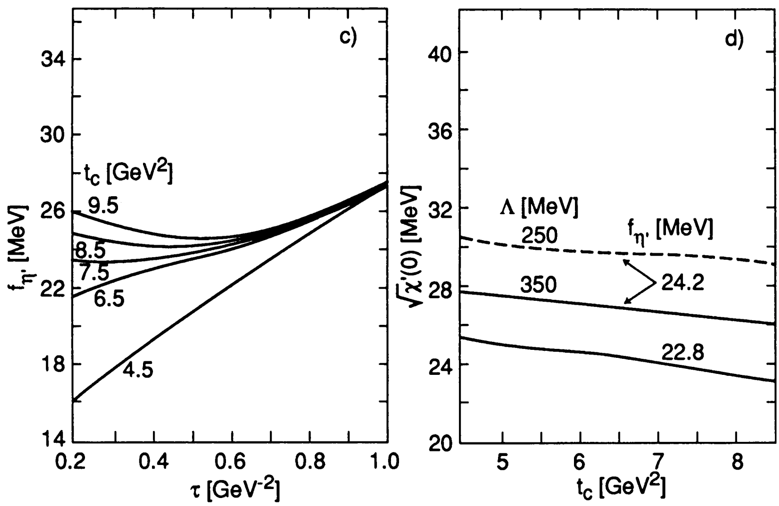

Fig. 6. (a) As Fig. 5a for the parameter $f_{\eta'}$ (b) FESR prediction of $\sqrt{\chi'(0)}$ versus t_c for different values of $f_{\eta'}$.

point. We obtain the result shown in Fig. 6 for different values of $f_{\eta'}$ and Λ, from which we deduce that with $t_c \simeq 6.5$–$9.5\ \text{GeV}^2$,

$$\sqrt{\chi'(0)} \simeq 25.5 \pm 1.5 \pm 2.0 \pm 1.0\ \text{MeV}, \tag{3.23}$$

where the errors come from $f_{\eta'}$, Λ and t_c respectively. Adding the errors quadratically and including a further 5% error from the unknown higher order terms, we obtain at the scale $\tau \simeq 0.5\ \text{GeV}^{-2}$

$$\sqrt{\chi'(0)} \simeq 26.5 \pm 3.1\ \text{MeV}, \tag{3.24}$$

where we have run the result from $t_c = 8\ \text{GeV}^2$ to the scale $\tau = 0.5\ \text{GeV}^{-2}$ using the RGE solution expressed in terms of Λ, viz.

$$\chi'(0;\mu) \simeq \hat{\chi}'(0) \exp\left(\frac{8}{\beta_1^2 \log(\mu/\Lambda)}\right), \tag{3.25}$$

where $\chi'(0)$ is RG invariant. (Notice that the inhomogeneous term proportional to $\beta^{(L)}$ does not contribute to the first moment at $k^2 = 0$.) We see that the FESR result is consistent with the Laplace one.

Taking the average of the Laplace and FESR results, we obtain our final estimate of the first moment of the topological susceptibility at the scale $\tau = 0.5\ \text{GeV}^{-2}$:

$$\sqrt{\chi'(0)} \simeq 25.3 \pm 2.6\ \text{MeV}. \tag{3.26}$$

This result should be compared with that obtained [24,25] in pure $N_C = 3$ Yang–Mills theory using a similar QSSR approach:

$$\sqrt{-\chi'(0)}\ |_{\text{YM}} \simeq 7 \pm 3\ \text{MeV}. \tag{3.27}$$

It is important to notice that this pure Yang–Mills result has been confirmed by lattice

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

calculations [28,29], which is a strong indication of the validity of the methods used in deriving both (3.27) and (3.26). The introduction of massless quarks has changed the sign of $\chi'(0)$ and increased its absolute value by a factor of around 12. From the QSSR analysis, this effect is due mainly to the low value of the η'-mass of 0.87 GeV (massless QCD) which enters into the spectral function, compared with the pseudoscalar gluonium mass of about 1.36–1.66 GeV [24,20] in pure Yang–Mills theory.

To compare with the experimental result on the polarised proton structure function, we use the RGE to run the result for $\chi'(0)$ to the EMC scale of 10 GeV^2. We find

$$\sqrt{\chi'(0)}\,|_{\mathrm{EMC}} \simeq 23.2 \pm 2.4\ \mathrm{MeV}. \tag{3.28}$$

This is smaller by a factor of 1.64 ± 0.17 than the OZI value of $(1/\sqrt{6})f_\pi$. We therefore do indeed find a significant suppression of $\chi'(0)$ relative to its OZI value.

To convert this result into a prediction for the singlet form factor, we take our fundamental expression (2.15) for $G_{\mathrm{A}}^{(0)}$ and equate the proper vertex $\Gamma_{\Phi_{5R}\,P\overline{P}}$ with its OZI expression given by the Goldstone boson–nucleon coupling. In this way, we obtain

$$G_{\mathrm{A}}^{(0)}(0) = G_{\mathrm{A}}^{(0)}(0)_{\mathrm{OZI}} \frac{\sqrt{\chi'(0)}}{(1/\sqrt{6})f_\pi}. \tag{3.29}$$

Using the value of $G_{\mathrm{A}}^{(0)}(0)_{\mathrm{OZI}}$ in Eq. (2.8) and including an additional error of approximately 10% for the use of the OZI approximation for the proper vertex, we arrive at our final prediction:

$$G_{\mathrm{A}}^{(0)}(0; Q^2 = 10\ \mathrm{GeV}^2) \simeq 0.353 \pm 0.052. \tag{3.30}$$

Substituting this result[5] together with

$$\begin{aligned} G_{\mathrm{A}}^{(8)} &\equiv \frac{1}{2\sqrt{3}}(3F - D), \\ G_{\mathrm{A}}^{(3)} &\equiv \tfrac{1}{2}(F + D) \end{aligned} \tag{3.31}$$

into the first moment sum rule (2.1), using the values of F and D from Eq. (2.9), and neglecting the higher twist terms (which are certainly negligible at $Q^2 = 10$ GeV^2), we deduce

$$\Gamma_1^{\mathrm{p}}(10\ \mathrm{GeV}^2) \simeq 0.143 \pm 0.005. \tag{3.32}$$

Here, we have used the coupling $\alpha_{\mathrm{s}}(m_\tau) = 0.347 \pm 0.030$ extracted from tau-decay data [30]. One should notice that the radiative corrections decrease the leading order result by about 12%.

Our result, Eqs. (3.30) and (3.32), certainly goes in the right direction, i.e. that of reducing the prediction from the OZI (Ellis–Jaffe) value. At the time we obtained it, however,

[5] In terms of the quantities Δu, Δd and Δs defined in Eq. (2.3), we have at $Q^2 = 10$ GeV^2 $\Delta u = 0.84 \pm 0.01$, $\Delta d = -0.41 \pm 0.01$, $\Delta s = -0.08 \pm 0.02$.

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

Eq. (3.30) still appeared too high compared to the experimental result (2.7), which would have implied further OZI violations in the proper vertex. Amusingly enough, while this paper was being completed we learned of the new results from the SMC Collaboration which, combined with the earlier proton data, gives the new world average [31]:

$$\Gamma_1^{\mathrm{p}}(10\ \mathrm{GeV}^2) = 0.145 \pm 0.008 \pm 0.011 \tag{3.33}$$

from which we deduce

$$G_{\mathrm{A}}^{(0)}(0; Q^2 = 10\ \mathrm{GeV}^2) \equiv \Delta\Sigma = 0.37 \pm 0.07 \pm 0.10. \tag{3.34}$$

These results are now in excellent agreement with our predictions.

4. Tests of the Bjorken sum rule and estimate of higher twist effects

Recently, the SMC Collaboration at CERN [31,32] and the E142 Collaboration at SLAC [33] have produced data on the polarised neutron structure function g_1^{n}. Since our proposal requires that the flavour singlet suppression is identical for the proton and neutron, we see no reason why the Bjorken sum rule [34],

$$\begin{aligned}\delta\Gamma_1^{\mathrm{p-n}} &\equiv \Gamma_1^{\mathrm{p}} - \Gamma_1^{\mathrm{n}} \\ &\equiv \int_0^1 \mathrm{d}x \left[g_1^{\mathrm{p}}(x;Q^2) - g_1^{\mathrm{n}}(x;Q^2)\right] \\ &\equiv \tfrac{1}{6} g_{\mathrm{A}} \left[1 - \frac{\alpha_{\mathrm{s}}}{\pi} - 3.583\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^2 - 20.215\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^3\right] + \frac{a_{\mathrm{p}} - a_{\mathrm{n}}}{Q^2},\end{aligned} \tag{4.1}$$

should not hold, at least up to flavour SU(2) breaking. Provided the measurements are at sufficiently high Q^2, the higher twist corrections related to the coefficients $a_{\mathrm{p}} - a_{\mathrm{n}}$ can be neglected. Analysis of the combined proton and deuteron data as performed in Ref. [35] gives at $Q^2 = 5\ \mathrm{GeV}^2$ [31]

$$\delta\Gamma_1^{\mathrm{p-n}} \simeq 0.203 \pm 0.029, \tag{4.2}$$

to be compared with the QCD prediction, with $\alpha_{\mathrm{s}}\ (5\ \mathrm{GeV}^2) = 0.32 \pm 0.02$, of

$$\delta\Gamma_1^{\mathrm{p-n}} \simeq 0.176 \pm 0.003 + \frac{a_{\mathrm{p}} - a_{\mathrm{n}}}{Q^2}, \tag{4.3}$$

From this, one can deduce the difference of the higher twist coefficients (in units of GeV^2)[6],

$$a_{\mathrm{p}} - a_{\mathrm{n}} \simeq 0.135 \pm 0.145. \tag{4.4}$$

[6]Keeping the order α_{s} term and using the estimate of the higher twist terms from QCD spectral sum rules, the authors of Ref. [36] found $\Gamma_1^{\mathrm{p-n}} \simeq 0.180 \pm 0.006$, in agreement with the data in Eq. (4.2). Our attitude here is different, as we will extract the size of the higher twist terms from the data in order to test the reliability of the previous theoretical estimate of those terms.

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

We can pursue an analogous analysis for the first moment of the neutron structure function, which satisfies the sum rule (cf. Eq. (2.1))

$$\begin{aligned}
&\Gamma_1^{\mathrm{n}}(Q^2)\\
&\quad\equiv \int_0^1 \mathrm{d}x\, g_1^{\mathrm{n}}(x;Q^2)\\
&\quad= \frac{1}{6}\left\{\left(-G_{\mathrm{A}}^{(3)}(0)+\frac{1}{\sqrt{3}}G_{\mathrm{A}}^{(8)}(0)\right)\left[1-\frac{\alpha_{\mathrm{s}}}{\pi}-3.583\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^2-20.215\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^3\right]\right.\\
&\qquad\left.+\tfrac{2}{3}G_{\mathrm{A}}^{(0)}(0;Q^2)\left[1-\frac{1}{3}\frac{\alpha_{\mathrm{s}}}{\pi}-0.550\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^2\right]\right\}+\frac{a_{\mathrm{n}}}{Q^2},
\end{aligned}\tag{4.5}$$

where we have included the higher twist contribution. Evaluating this quantity at $Q^2 = 2\ \mathrm{GeV}^2$, where the SLAC data are available, we find

$$\Gamma_1^{\mathrm{n}}(2\,\mathrm{GeV}^2) \simeq -(0.031 \pm 0.006) + \frac{a_{\mathrm{n}}}{Q^2}. \tag{4.6}$$

Comparing this with the SLAC data [33],

$$\Gamma_1^{\mathrm{n}}(2\,\mathrm{GeV}^2) \simeq -(0.022 \pm 0.011), \tag{4.7}$$

and using Eq. (4.4), we can extract the coefficients of the higher twist terms. In units of GeV^2, we find

$$\begin{aligned}
a_{\mathrm{p}} &\simeq -0.117 \pm 0.145,\\
a_{\mathrm{n}} &\simeq 0.018 \pm 0.025.
\end{aligned}\tag{4.8}$$

These values of the higher twist terms are consistent with the previous determinations [37,38] from QCD spectral sum rules. However, these sum rules would be affected by a more general choice of the nucleon interpolating field [20] (the one used in Refs. [37,38] is not the optimal one) and by the well-known [20] large violation by a factor 2–3 of the vacuum saturation of the four-quark condensate, which is assumed in Refs. [37,38] to be satisfied to within (10–20)%. In addition, radiative corrections, which are known to be large in the baryon sum rules [20], can also be important here. More accurate data on the Bjorken and neutron sum rules, and/or a measurement of the proton sum rule at lower Q^2, are needed to improve the results in Eq. (4.8), which are necessary to test the validity of the QCD spectral sum rule predictions in Ref. [38].

5. Further discussion

In this paper, we have presented evidence that the experimentally observed suppression of the first moment of the polarised proton structure function g_1^{p} (the so-called EMC "proton

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

spin" crisis) is a target-independent effect reflecting a suppression of the first moment of the QCD topological susceptibility $\chi'(0)$ relative to the OZI expectation. Not only does $G_A^{(0)}(0)$ not measure the quark spin, its suppression is not even a property of the proton structure.

It would be interesting to test this hypothesis directly by polarised deep inelastic scattering experiments on other targets not simply related to the proton by flavour symmetry. We have already studied the case of a photon target and have presented elsewhere [39] a new sum rule for the first moment of the polarised photon structure function g_1^γ measurable in polarised e^+e^- colliders. However, this turns out to be a special case because the electromagnetic U(1) anomaly contributes at leading order and so the g_1^γ sum rule does not display the suppression mechanism described here. Another possibility is to consider semi-inclusive processes in which a particular hadron with a fraction z of the incoming momentum is observed in the target fragmentation region. It was recently suggested [40] that such cross sections should be described in terms of new, non-perturbative hybrid functions $M(z, x, Q)$, called "fracture functions". To the extent that an OPE can be used, it would be possible to represent M in terms of the forward matrix element of a composite operator between a suitable proton-plus-hadron state. In this case, one would again factorise M into a composite propagator of the usual type and a proper vertex involving four external hadron legs. If the suppression of the polarised structure function indeed originates from the propagator, as we suggest, such a suppression should also be found at the level of the (less inclusive) fracture functions.

So far, we have only considered the first moment of g_1^p. Of course, we would like to extend our approach to higher moments and discuss the full x-dependence of the structure function. This would require knowledge of the renormalisation properties and composite operator Green functions of the higher spin axial currents and gluon densities [41], together with the associated proper vertices.

Another possible line of development would be to try to develop techniques to estimate the proper vertices themselves, rather than just the composite operator Green functions. To the extent that the quenched approximation may be trusted for the proper vertices, lattice calculations could already be suitable for the task, and QCD spectral sum rule techniques could be used in conjunction to check the validity of that approximation. We recall that, in contrast, the use of the quenched approximation directly for the matrix elements of the operator Q can be shown to be completely unreliable since these are affected by low-lying poles that should disappear after dynamical quark loops are added. This is another example of how the apparent complication introduced by our splitting of matrix elements into propagators and proper vertices can ultimately pay off.

Finally, it would be interesting to attempt to apply this analysis of deep inelastic scattering using proper vertices to other QCD processes normally described in the language of the parton model rather than in terms of the OPE. Semi-inclusive deep inelastic scattering is one such example, but many other interesting possibilities can be considered, especially in the context of hadron–hadron collisions.

Appendix A

Chiral Ward identities and the renormalisation group

The anomalous chiral Ward identities for Green functions of the pseudoscalar operators Q_{R} and $\Phi_{5\mathrm{R}}$ are (for zero quark masses)

$$ik_\mu\langle 0 \mid J^0_{\mu 5\mathrm{R}}(k)Q_R(-k) \mid 0\rangle - 2N_{\mathrm{F}}\langle 0 \mid Q_{\mathrm{R}}(k)Q_{\mathrm{R}}(-k) \mid 0\rangle = 0, \tag{A.1}$$

$$ik_\mu\langle 0 \mid J^0_{\mu 5\mathrm{R}}(k)\Phi_{5R}(-k) \mid 0\rangle - 2N_{\mathrm{F}}\langle 0 \mid Q_{\mathrm{R}}(k)\Phi_{5\mathrm{R}}(-k) \mid 0\rangle + \langle 0 \mid \delta_5\Phi_{5\mathrm{R}}(-k) \mid 0\rangle = 0. \tag{A.2}$$

So, at zero momentum, assuming there is no physical massless U(1) boson,

$$\langle 0 \mid Q_{\mathrm{R}}(0)Q_{\mathrm{R}}(0) \mid 0\rangle = 0, \tag{A.3}$$

showing that the topological susceptibility $\chi(0)$ vanishes for massless QCD, and

$$\langle 0 \mid Q_{\mathrm{R}}(0)\Phi_{5\mathrm{R}}(0) \mid 0\rangle = -\frac{1}{2N_{\mathrm{F}}}2\langle\Phi_{\mathrm{R}}\rangle, \tag{A.4}$$

where $\langle\Phi_{\mathrm{R}}\rangle$ is the VEV of the scalar partner of $\Phi_{5\mathrm{R}}$ and is non-vanishing because of the quark condensate.

The field $\Phi_{5\mathrm{R}}$ is normalised such that the two-point proper vertex $\Gamma_{\Phi_{5\mathrm{R}}\Phi_{5\mathrm{R}}} = k^2$. This means that $\Gamma_{\Phi_{5\mathrm{R}}\Phi_{5\mathrm{R}}}$ is (minus) a component of the inverse propagator matrix in the pseudoscalar sector, i.e.

$$\Gamma_{\Phi_{5\mathrm{R}}\Phi_{5\mathrm{R}}} = \langle 0 \mid Q_{\mathrm{R}}Q_{\mathrm{R}} \mid 0\rangle(\langle 0 \mid Q_{\mathrm{R}}\Phi_{5\mathrm{R}} \mid 0\rangle^2 - \langle 0 \mid Q_{\mathrm{R}}Q_{\mathrm{R}} \mid 0\rangle\langle 0 \mid \Phi_{5\mathrm{R}}\Phi_{5\mathrm{R}} \mid 0\rangle)^{-1}. \tag{A.5}$$

Expanding to lowest order in k^2 gives

$$\Gamma_{\Phi_{5\mathrm{R}}\Phi_{5\mathrm{R}}} = \chi'(0)\langle 0 \mid Q_{\mathrm{R}}(0)\Phi_{5\mathrm{R}}(0) \mid 0\rangle^{-2}k^2 + \mathrm{O}(k^4), \tag{A.6}$$

where we have written $\langle 0 \mid Q_{\mathrm{R}}(k)Q_{\mathrm{R}}(-k) \mid 0\rangle = \chi'(0)k^2 + \mathrm{O}(k^4)$. We therefore deduce

$$\langle 0 \mid Q_{\mathrm{R}}(0)\Phi_{5\mathrm{R}}(0) \mid 0\rangle = \sqrt{\chi'(0)}, \tag{A.7}$$

as quoted in Eq. (2.15).

The renormalisation group equation for the topological susceptibility follows from the definition of the renormalised composite operators, Eq. (2.4), and the chiral Ward identities. The Ward identity for the two-current Green function is

$$ik^\mu\langle 0 \mid J^0_{\mu 5\mathrm{R}}(k)J^0_{\nu 5\mathrm{R}}(-k) \mid 0\rangle - 2N_{\mathrm{F}}\langle 0 \mid Q_{\mathrm{R}}(k)J^0_{\nu 5\mathrm{R}}(-k) \mid 0\rangle = 0. \tag{A.8}$$

Combining Eqs. (A.1), (A.8) and (2.4), we find straightforwardly

$$\langle 0 \mid Q_{\mathrm{R}}(k)Q_{\mathrm{R}}(-k) \mid 0\rangle = Z^2\langle 0 \mid Q_{\mathrm{B}}(k)Q_{\mathrm{B}}(-k) \mid 0\rangle + \dots. \tag{A.9}$$

The dots denote the extra divergences associated with contact terms in the two-point Green functions of composite operators. Taking these into account (see Refs. [21,7] for full details)

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

we find the full RGE for $\chi(k^2)$,

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(\alpha_s)\alpha_s\frac{\partial}{\partial\alpha_s}-2\gamma\right)\chi(k^2)=-\frac{1}{(2N_F)^2}2\beta^{(L)}(\alpha_s)k^4, \tag{A.10}$$

where $\beta^{(L)}$ is a new RG function. The inhomogeneous term does not contribute at zero momentum, however, and the required RGE (2.13) for $\chi'(0)$ follows immediately.

Appendix B

Decay constants and the η'

We can estimate the parameter $f_{\eta'}$ appearing in the spectral expansion using the first Laplace QSSR, Eq. (3.10). $f_{\eta'}$ is defined by

$$\langle 0 \mid J^0_{\mu 5R}(k) \mid \eta'\rangle = ik_\mu f_{\eta'}, \tag{B.1}$$

and is RG non-invariant. On shell (see Ref. [7], Appendix D), the scale dependence is due entirely to the anomalous dimension γ of the axial current so, using Eq. (3.3) and expressing the result in terms of the QCD scale Λ, we may write

$$f_{\eta'}(\mu)=\hat{f}_{\eta'}\exp\left(\frac{4}{\beta_1^2\log(\mu/\Lambda)}\right), \tag{B.2}$$

where $\hat{f}_{\eta'}$ is RG invariant. From the QSSR (3.10), we find the τ-stability starts at $t_c\simeq$ 6.5 GeV^2, while the t_c-stability is reached for t_c larger than 9.5 GeV^2. In this region, the radiative corrections are about 10% of the lowest order term, while the $\langle g^3G^3\rangle$ one contributes about 10%. Under such conditions, our optimal result at $\tau\simeq 0.6\,\mathrm{GeV}^{-2}$ is (see Fig. 6a)

$$f_{\eta'}\simeq 24.1\pm 0.6\pm 3.4\pm 0.3\,\mathrm{MeV}, \tag{B.3}$$

where the first error comes from $\langle\alpha_s G^2\rangle$, the second from Λ and the third from the range of t_c-values between 6.5 and 9.5 GeV^2. Adding a 5% error from the unknown QCD terms, adding the different errors quadratically and running to the EMC scale, we obtain

$$f_{\eta'}\mid_{\mathrm{EMC}}\simeq 23.6\pm 3.5\,\mathrm{MeV}. \tag{B.4}$$

This value is strongly suppressed relative to the OZI prediction of $\sqrt{6}\,f_\pi$ for the η'-decay constant.

However, as has been shown in Refs. [42,7], this $f_{\eta'}$ is *not* the η'-decay constant measured in, e.g., the decay $\eta'\to\gamma\gamma$. In fact, the analogues of the current algebra formulae

$$f_\pi g_{\pi gg}=\frac{1}{\pi}\alpha_{\mathrm{em}} \tag{B.5}$$

and

$$f_\pi g_{\pi\mathrm{NN}} = m_\mathrm{N} g_\mathrm{A} \tag{B.6}$$

in the flavour singlet sector are [42,7]

$$F g_{\eta'\gamma\gamma} + \frac{1}{2N_\mathrm{F}} F^2 m_{\eta'}^2 g_{G\gamma\gamma}(0) = \frac{4}{\pi}\alpha_\mathrm{em} \tag{B.7}$$

and

$$F g_{\eta'\mathrm{NN}} + \frac{1}{2N_\mathrm{F}} F^2 m_{\eta'}^2 g_{\mathrm{GNN}}(0) = 2m_\mathrm{N} G_\mathrm{A}^{(0)}(0). \tag{B.8}$$

Here, F is the RG-invariant decay constant defined by

$$F = \frac{2\langle\phi_\mathrm{R}\rangle}{m_{\eta'}} \left(\int \mathrm{d}x\, i\langle 0 \mid T^* \phi_{5\mathrm{R}}^0(x)\phi_{5\mathrm{R}}^0(0) \mid 0\rangle \right)^{-1/2}, \tag{B.9}$$

where $\phi_{5\mathrm{R}}^0 = i\Sigma\bar{q}\gamma_5 q$ and $\langle\phi\rangle = \Sigma\langle\bar{q}q\rangle$. The extra terms $g_{\mathrm{G}\gamma\gamma}$ and g_GNN appearing in Eqs. (B.7), (B.8) (which are properly defined as proper vertices [42,7]) may be thought of as the couplings of the gluonic component of the η'. They arise because the η' is not a Goldstone boson in the U(1) channel and so the naive current algebra extensions of Eqs. (B.5), (B.6) are not valid. At first sight, therefore, Eqs. (B.7) and (B.8) are not predictive since $g_{\mathrm{G}\gamma\gamma}$ and g_GNN are unknown. However, if we follow our proposal that OZI violations are associated with RG-non-invariant quantities we can make predictions.

Taking Eq. (B.7) first, we have shown [42] that $g_{\mathrm{G}\gamma\gamma}$ is RG invariant. Since in the OZI limit this term is absent, we therefore expect $g_{\mathrm{G}\gamma\gamma}$ to be small, and so to a good approximation we predict

$$F g_{\eta'\gamma\gamma} = \frac{4}{\pi}\alpha_\mathrm{em}. \tag{B.10}$$

Since F is RG invariant, we expect it to be well approximated by its OZI value $\sqrt{6}f_\pi$. Experimentally (see Ref. [43]), the relation (B.10) is very well satisfied.

In Eq. (B.8), on the other hand, g_GNN is not RG invariant so we do not expect this term to be small. In fact, this equation is just a rewriting of the U(1) GT formula quoted in the text, for which our proposal is successful.

An important test of our picture of the pattern of OZI breaking is therefore to evaluate the RG-invariant decay constant F from first principles and check that it is close to the OZI prediction of $\sqrt{2N_\mathrm{F}}f_\pi$. Again, we can use QCD spectral sum rules.

We require the zero-momentum limit $\Phi_5(0)$ of the two-point correlation function

$$\Phi_5(k^2) = \int \mathrm{d}x\, \mathrm{e}^{ik\cdot x} i\langle 0 \mid T^* \phi_{5\mathrm{R}}^0(x)\phi_{5\mathrm{R}}^0(0) \mid 0\rangle \tag{B.11}$$

for QCD with 3 flavours and massless quarks. However, as there is a smooth behaviour of the two-point correlator when the common light quark mass m_R goes to zero, we shall work (for convenience) with the RG-invariant correlation function

$$\Psi_5(k^2) \equiv 4m_\mathrm{R}^2 \Phi_5(k^2), \tag{B.12}$$

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

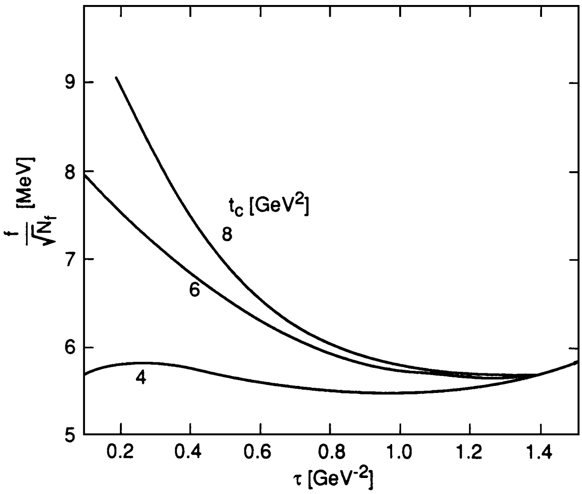

Fig. B.1. As Fig. 5a for the parameter f.

where m_R is the average of the renormalised u and d quark masses. Now, in perturbation theory, the difference between this flavour singlet correlation function and the corresponding non-singlet one appears only at $O(\alpha_s^2)$ from the double-triangle anomaly-type diagrams. Similarly for the non-perturbative condensate terms, the difference is only of $O(\alpha_s^2)$ arising from the equivalent diagrams. Instanton-like effects appear as higher dimension operators. So, at the order we are working, we can simply use the expression for the isotriplet (pion) correlation function in QCD discussed in the literature [20].

The first Laplace sum rule to two loops reads [20]

$$\begin{aligned}
&\int_0^{t_c} \mathrm{d}t\ \mathrm{e}^{-t\tau} \frac{1}{\pi} \mathrm{Im}\Psi_5(t) \\
&\simeq \frac{3N_F}{2\pi^2}\overline{m}^2(\tau)\Bigg\{\tau^{-2}[1-\exp(-t_c\tau)(1+t_c\tau)] \\
&\quad\times\left\{1-\frac{2}{\beta_1 L}\left[\tfrac{11}{3}+2\gamma_E-\frac{2}{\beta_1}\left(\tilde{\gamma}_2-\tilde{\gamma}_1\frac{\beta_2}{\beta_1}\right)+2\frac{\tilde{\gamma}_1\beta_2}{\beta_1^2}\log L\right]\right\} \\
&\quad+\left(\tfrac{1}{3}\pi\langle\alpha_s G^2\rangle+\tfrac{896}{81}\pi^3\rho\alpha_s\langle\overline{u}u\rangle^2\tau\right)\Bigg\},
\end{aligned} \tag{B.13}$$

where $L = -\log \tau\Lambda^2$ and [20]

$$\begin{aligned}
&\rho\alpha_s\langle\overline{u}u\rangle^2 \simeq (3.8\pm 2.0)\times 10^{-4}\,\mathrm{GeV}^6, \\
&\overline{m}(\tau) \equiv \tfrac{1}{2}(\overline{m}_u+\overline{m}_d)(\tau) \simeq \left(-\tfrac{1}{2}\log\tau\Lambda^2\right)^{\tilde{\gamma}_1/\beta_2}(12.1\pm 1.0)\ \mathrm{MeV}.
\end{aligned} \tag{B.14}$$

As before, we parametrise the spectral function keeping only the lowest (η') resonance, i.e.

$$\frac{1}{\pi}\,\mathrm{Im}\Psi_5(t) = 2\tilde{m}_{\eta'}^4 f^2\delta\big(t-\tilde{m}_{\eta'}^2\big) + \text{“QCD continuum”}\ \theta(t-t_c), \tag{B.15}$$

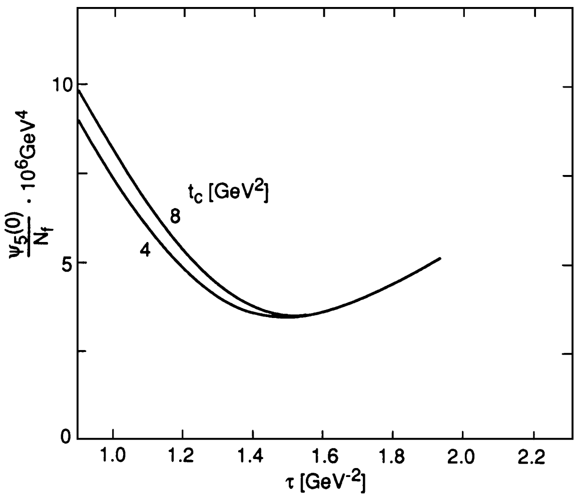

Fig. B.2. As Fig. 5a for $\Psi_5(0)$.

where the unknown parameter f, which is defined by

$$2m_{\mathrm{R}}\langle 0 \mid \phi^0_{5\mathrm{R}} \mid \eta'\rangle = \sqrt{2} f \tilde{m}^2_{\eta'}, \tag{B.16}$$

can be estimated from the sum rule (B.13). We study the τ-and t_{c}-behaviours of f in Fig. B.1. The τ-stability starts for $t_{\mathrm{c}} \simeq 4\ \mathrm{GeV}^2$, while stability in t_{c} appears above $t_{\mathrm{c}} \simeq 7\ \mathrm{GeV}^2$, a range which is equal to the one for the correlation function for $Q(x)$. The value for the τ-stability of about 0.9 GeV^{-2} is typical of light quark correlation functions. At the minimum, we obtain

$$f = \sqrt{N_{\mathrm{F}}}(5.55 \pm 0.08 \pm 0.65 \pm 0.35 \pm 0.06 \pm 0.03)\ \mathrm{MeV}, \tag{B.17}$$

where the errors come respectively from t_{c}, Λ, $\overline{m}$, $\langle \alpha_{\mathrm{s}} G^2\rangle$ and $\rho\alpha_{\mathrm{s}}\langle \overline{u}u\rangle^2$. Adding these errors quadratically, we deduce

$$f = \sqrt{N_{\mathrm{F}}}(5.55 \pm 0.75)\ \mathrm{MeV}. \tag{B.18}$$

With this value for f, we are now able to estimate $\Psi_5(0)$ itself using a second Laplace sum rule [44,25]:

$$\begin{aligned}\Psi_5(0) \simeq & \int_0^{t_{\mathrm{c}}} \frac{\mathrm{d}t}{t} \mathrm{e}^{-t\tau} \frac{1}{\pi} \mathrm{Im}\Psi_5(t) - \frac{3N_{\mathrm{F}}}{2\pi^2} \overline{m}^2(\tau) \Bigg(\tau^{-1}[1 - \exp(-t_{\mathrm{c}}\tau)] \\ & \times \left\{ 1 - \frac{2}{\beta_1 L}\left[\tfrac{11}{3} + 2\gamma_{\mathrm{E}} - \frac{2}{\beta_1}\left(\tilde{\gamma}_2 - \tilde{\gamma}_1 \frac{\beta_2}{\beta_1}\right) + 2\frac{\tilde{\gamma}_1\beta_2}{\beta_1^2}\log L \right]\right\} \\ & + \left(\tfrac{1}{3}\pi \langle \alpha_{\mathrm{s}} G^2\rangle + \tfrac{1}{2}\tfrac{896}{81}\pi^3 \rho\alpha_{\mathrm{s}}\langle \overline{u}u\rangle^2 \tau\right)\Bigg),\end{aligned} \tag{B.19}$$

where $\tilde{\gamma}_1$, $\tilde{\gamma}_2$ are the coefficients in the anomalous dimension for the light quark mass. For three flavours, $\tilde{\gamma}_1 = 2$ and $\tilde{\gamma}_2 = \frac{91}{12}$. The sum rule analysis of this quantity shows a strong t_c-dependence and the τ-stability only appears at unrealistic values of t_c larger than 8 GeV^2. In order to circumvent this difficulty, we work with a combination of the sum rules (B.13) and (B.19) which has been used successfully in the past for measuring the deviation from pion and kaon PCAC to a good accuracy [44]. The combined sum rule reads

$$\begin{aligned}\Psi_5(0) \simeq \int_0^{t_c} \frac{dt}{t} e^{-t\tau}(1 - t\tau)\frac{1}{\pi}\operatorname{Im}\Psi_5(t) - \frac{3N_F}{2\pi^2}\overline{m}^2(\tau)\Bigg(\tau^{-1}[t_c\tau\exp(-t_c\tau)] \\ \times \left\{1 - \frac{2}{\beta_1 L}\left[\tfrac{11}{3} + 2\gamma_E - \frac{2}{\beta_1}\left(\tilde{\gamma}_2 - \tilde{\gamma}_1\frac{\beta_2}{\beta_1}\right) + 2\frac{\tilde{\gamma}_1\beta_2}{\beta_1^2}\log L\right]\right\} \\ + \tau\left(\tfrac{2}{3}\pi\langle\alpha_s G^2\rangle + \tfrac{3}{2}\tfrac{896}{81}\pi^3\rho\alpha_s\langle\overline{u}u\rangle^2\tau\right)\Bigg). \qquad \text{(B.20)}\end{aligned}$$

This sum rule is studied in Fig. B.2. The position of the stability is almost insensitive to the value of t_c due to some cancellations amongst the perturbative terms. However, this feature also implies that the stability is obtained at values of τ larger than in the previous cases, making the result sensitive to the errors on the four-quark condensates, which affects the accuracy of the result. We deduce

$$\Psi_5(0) \simeq N_F(3.70 \pm 0.90 \pm 0.30 \pm 0.70 \pm 2.00) \times 10^{-6}\ \text{GeV}^4, \qquad \text{(B.21)}$$

where the errors are due to f, Λ, $\langle\alpha_s G^2\rangle$ and $\rho\alpha_s\langle\overline{u}u\rangle^2$. Adding these errors quadratically, we obtain

$$\sqrt{\Psi_5(0)} \simeq \sqrt{N_F}(1.92 \pm 0.53) \times 10^{-3}\ \text{GeV}^2. \qquad \text{(B.22)}$$

Using this value in Eq. (B.9) (with $\tilde{m}_{\eta'}$), after multiplying the numerator and denominator by the overall $2m_R$ factor and using Dashen's formula for $m_R\langle\phi_R\rangle$, we finally find

$$F \simeq (1.55 \pm 0.43)\sqrt{2N_F} f_\pi, \qquad \text{(B.23)}$$

to be compared with the OZI prediction of $\sqrt{2N_F} f_\pi$.

This result is again in broad agreement with our expectations, although of course the errors are much too large to draw a definitive conclusion. Nevertheless, this confirmation can be taken as providing extra support for the reliability of the estimate in the text for $\chi'(0)$.

Acknowledgements

One of us (G.M.S.) would like to thank John Ellis and the TH Division, CERN for their hospitality during several visits.

https://doi.org/10.1017/9781009290296.027 Published online by Cambridge University Press

References

[1] J. Ashman et al., Phys. Lett. B 206 (1988) 364; Nucl. Phys. B 328 (1990) 1.
[2] G. Baum et al., Phys. Rev. Lett. 51 (1983) 1135.
[3] G. Altarelli, Lectures at the Int. School of Subnuclear physics, Erice, preprint CERN-TH.5675/90.
[4] S.D. Bass and A.W. Thomas, J. Phys. G 19 (1993) 925.
[5] G. Altarelli and G.G. Ross, Phys. Lett. B 212 (1988) 391.
[6] G.M. Shore and G. Veneziano, Phys. Lett. B 244 (1990) 75.
[7] G.M. Shore and G. Veneziano, Nucl. Phys. B 381 (1992) 23.
[8] R.G. Roberts, The structure of the proton (Cambridge U. P., Cambridge, 1990).
[9] J. Ellis and R.L. Jaffe, Phys. Rev. D 9 (1974) 1444; D 10 (1974) 1669.
[10] J. Kodaira, Nucl. Phys. B 165 (1980) 129.
[11] S.A. Larin and J.A.M Vermaseren, Phys. Lett. B 259 (1991) 213; S.A. Larin, preprint CERN-TH.7208/94.
[12] D. Espriu and R. Tarrach, Z. Phys. C 16 (1982) 77.
[13] G. Veneziano, Lecture at the Okubofest, May 1990, preprint CERN-TH.5840/90.
[14] G. Veneziano, Mod. Phys. Lett. A 4 (1989) 1605.
[15] M. Bourquin et al., Z. Phys. C 21 (1983) 27.
[16] Z. Dziembowski and J. Franklin, J. Phys. G 17 (1991) 213; F.E. Close and R.G. Roberts, Phys. Lett. B 302 (1993) 533.
[17] E. Witten, Nucl. Phys. B 156 (1979) 269.
[18] G. Veneziano, Nucl. Phys. B 159 (1979) 213.
[19] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385, 448.
[20] S. Narison, QCD spectral sum rules, Lecture notes in physics, Vol. 26 (World Scientific, Singapore, 1989).
[21] G.M. Shore, Nucl. Phys. B 362 (1991) 85.
[22] A.L. Kataev, N.V. Krasnikov and A.A. Pivovarov, Nucl. Phys. B 198 (1982) 508.
[23] V.A. Novikov et al., Nucl. Phys. B 237 (1984) 525.
[24] S. Narison, Z. Phys. C 26 (1984) 209.
[25] S. Narison, Phys. Lett. B 255 (1991) 101.
[26] Particle Data Group, Phys. Rev. D 45 (1992) Part 2.
[27] R.A. Bertlmann, G. Launer and E. de Rafael, Nucl. Phys. B 250 (1985) 61.
[28] G. Briganti, A. Di Giacomo and H. Panagopoulos, Phys. Lett. B 253 (1991) 427.
[29] A. Di Giacomo, Lecture at the QCD 90 Workshop, Montpellier July 1990, Nucl. Phys. B (Proc. Suppl.) 23B (1991).
[30] E. Braaten, A. Pich and S. Narison, Nucl. Phys. B 373 (1992) 581; ALEPH Collaboration, D. Buskulic et al., Phys. Lett. B 307 (1993) 209; S. Narison, Talk given at the Fubini fest, Torino 1994, preprint CERN-TH.7188/94, and references therein.
[31] SMC Collaboration, P. Shanahan, Talk given at the 29th rencontre de Moriond on QCD and high-energy hadronic interactions, Meribel, Savoie, March 1994; D. Adams et al., CERN-PPE/94-57 (1994), to be published.
[32] SMC Collaboration, B. Aveda et al., Phys. Lett. B 302 (1993) 533.
[33] E142 Collaboration, P.L. Anthony et al., Phys. Rev. Lett. 71 (1993) 959.
[34] J.D. Bjorken, Phys. Rev. 148 (1966) 1467; D 1 (1970) 1376.
[35] SMC Collaboration, B. Aveda et al., Phys. Lett. B 320 (1994) 400.
[36] J. Ellis and M. Karliner, Phys. Lett. B 313 (1993) 131.

[37] I.I. Balitsky, V.M. Braun and A.V. Kolesnichenko, 7 Phys. Lett. B 242 (1990) 245; B 318 (1993) 648 (E)
X. Ji and M. Unrau, MIT-CTP-2232 (1993).
[38] G.C. Ross and R.G. Roberts, Phys. Lett. B 322 (1994) 425.
[39] S. Narison, G.M. Shore and G. Veneziano, Nucl. Phys. B 391 (1993) 69.
[40] L. Trentadue and G. Veneziano, Phys. Lett. B 323 (1994) 201.
[41] S.D. Bass, Z. Phys. C 55 (1992) 653.
[42] G.M. Shore and G. Veneziano, Nucl. Phys. B 381 (1992) 3.
[43] M.S. Chanowitz, in Proc. VI Int. Workshop on Photon-Photon Collisions, Lake Tahoe, 1984, ed. R. Lander, World Scientific, Singapore, 1984.
[44] S. Narison, Phys. Lett. B 104 (1981) 485.